Scatterings by Higher Spin Exchange

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in collaboration with

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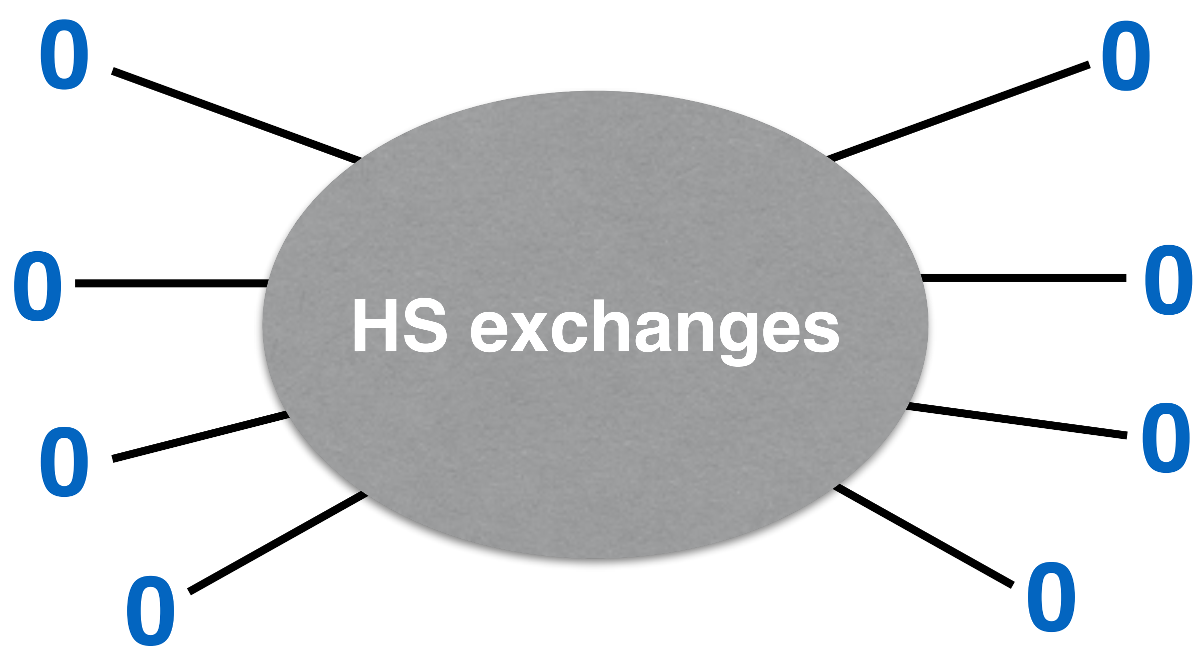
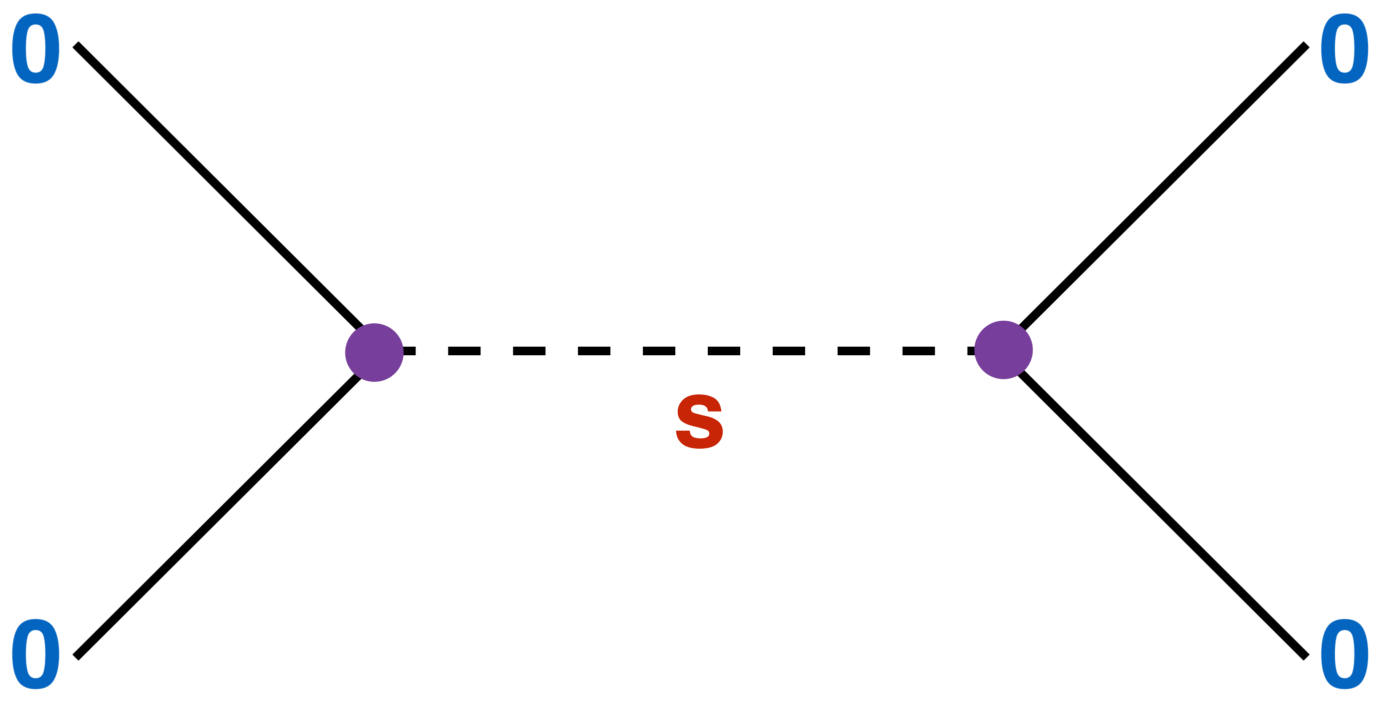
we are interested in



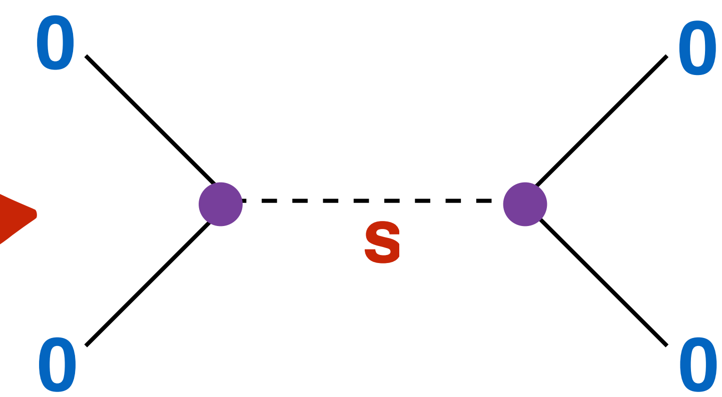
- Massless Scalar

we are interested in



- Massless Scalar
- Some HS Theory
- Proper Interactions

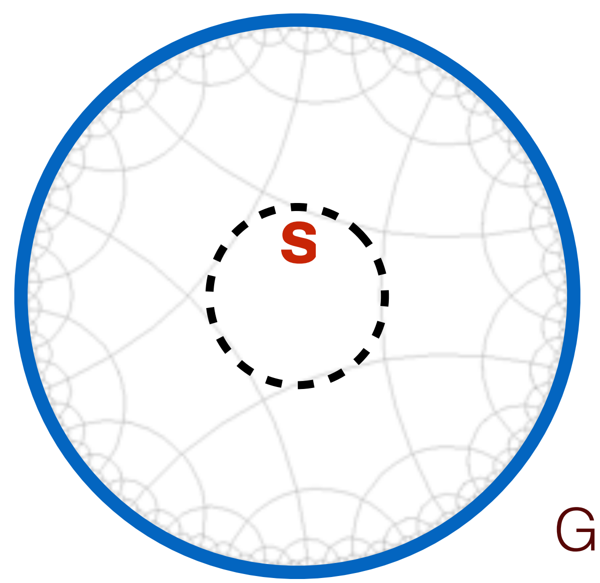
Motivations



Other simple example of spin sum !!

- Summation over Spins

$$\sum_s$$



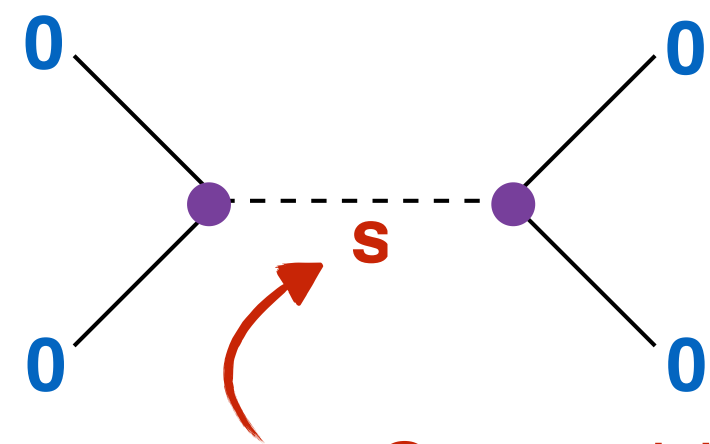
$$= \cancel{0}_s > 0$$

Giombi, Klebanov (et al), Tseytlin, … (2013~)

Why this happens? because of free CFT

How this happens? magic of zeta fn regularization

Motivations

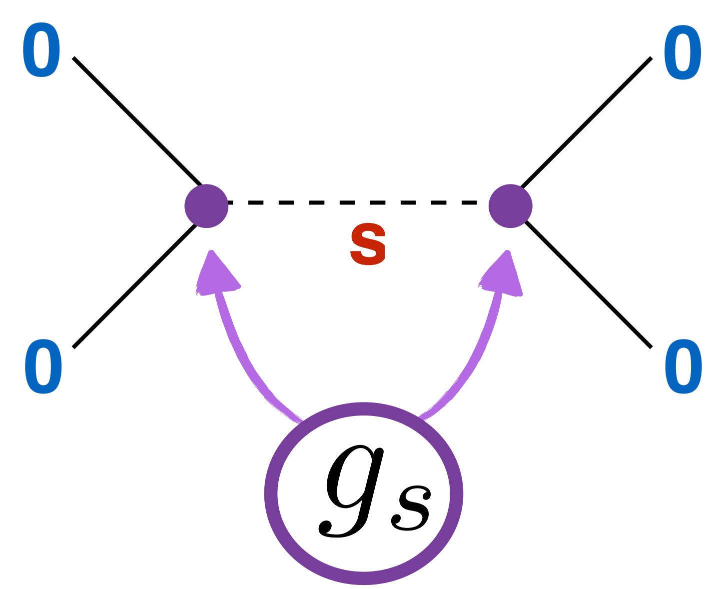


- Some HS Theory

- Summation over Spins

- Conformal HS Theory / Induced HS Theory
  - ✦ Generic Massless HS — Bekaert, EJ, Mourad (2009)
  - ✦ Effective HS Theory — `` (2010)

Massless HS Exchanges



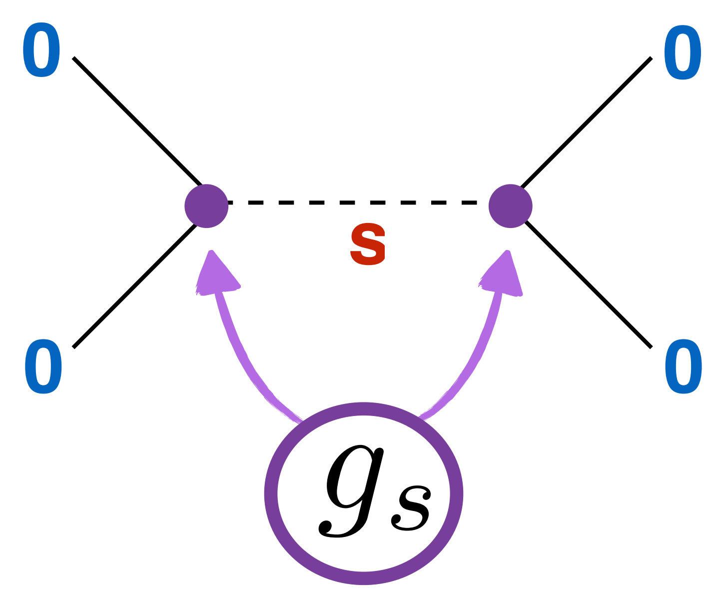
$$a(z) = \sum_{s=0}^{\infty} \frac{(g_s)^2}{s!}\, z^s$$

- Current Coupling: fixed up to coupling constant
- Propagator: massless one

Amplitude

$$A(s,t,u) = -\frac{\lambda^{-2}}{t}\left[a\Big(-\frac{\lambda^2}{8}\big(\sqrt{s}+\sqrt{-u}\big)^2\Big) + a\Big(-\frac{\lambda^2}{8}\big(\sqrt{s}-\sqrt{-u}\big)^2\Big) - a_0\right]$$

Massless HS Exchanges



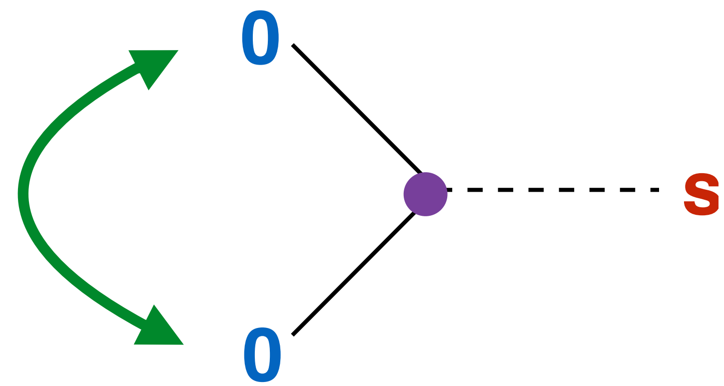
$$a(z) = \sum_{s=0}^{\infty} \frac{(g_s)^2}{s!} z^s$$

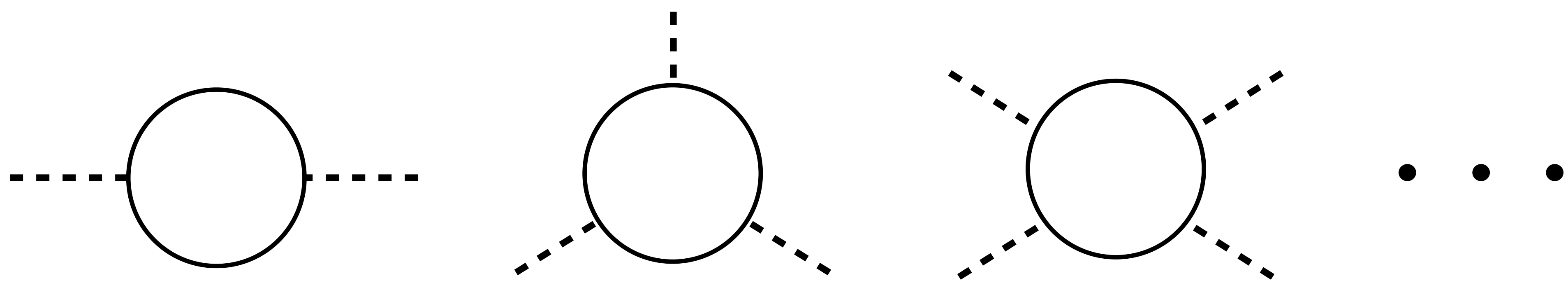
However,

- Coupling Constants are Arbitrary
- Various No-Go's for Massless HS

Effective HS Action

integrate over scalar



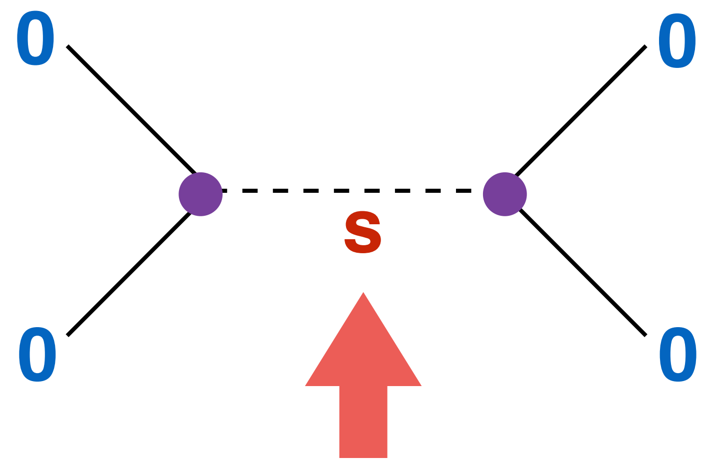


Effective HS action

$$W[h] = W_{\mathrm{nonloc}}[h] + \ln\frac{\Lambda}{\mu}\boxed{W_{\mathrm{loc}}[h]}$$

Conformal HS Action

Fradkin-Tseyltin, Segal, Vasiliev, Shaynkman



$$W[h] = W_{\text{nonloc}}[h] + \ln\frac{\Lambda}{\mu}\, W_{\text{loc}}[h]$$

- HS Theory :

  I) Conformal HS

  II) Induced HS Theory

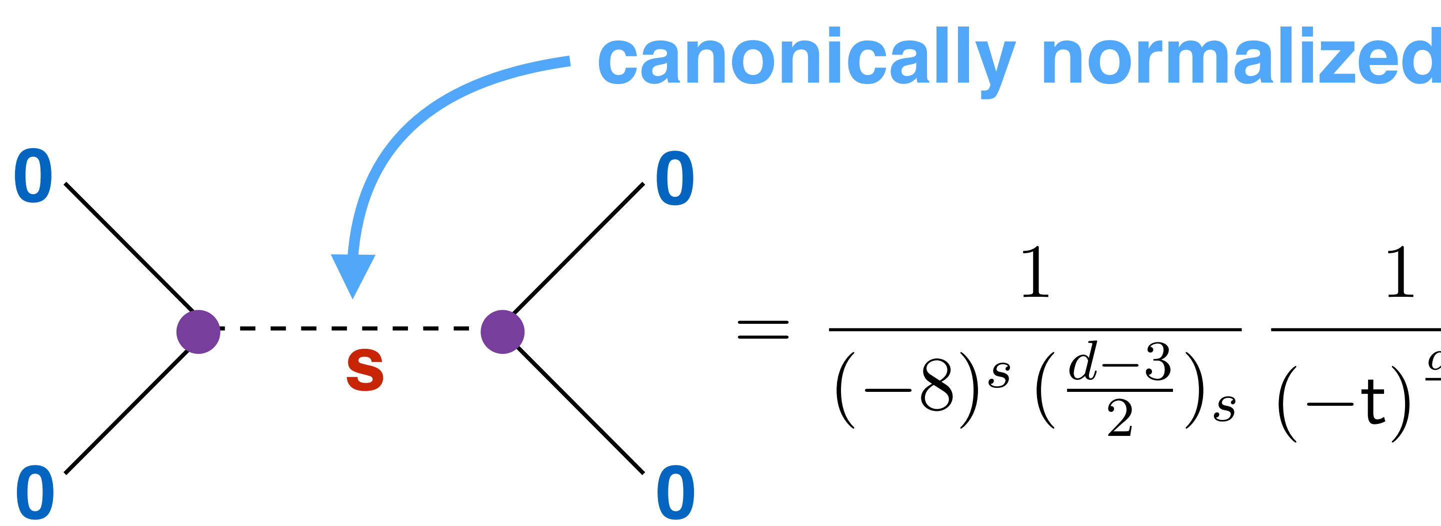
for tree diagrams,

effectively the same

- Coupling Constants are **fixed** up to a factor **g**

$$S[\phi, h] = \int d^dx \left[(\partial\phi)^2 + \sum_s \phi\, \partial^s \phi\, h_s\right] + \frac{1}{g^2} S_{\text{CHS}}[h]$$

Conformal HS Exchanges

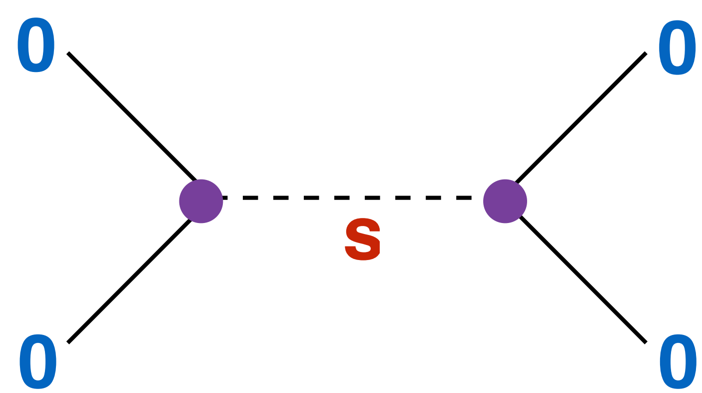


$$= \frac{1}{(-8)^s \left(\frac{d-3}{2}\right)_s} \frac{1}{(-\mathsf{t})^{\frac{d-4}{2}}} C_s^{\left(\frac{d-3}{2}\right)}\left(\frac{\mathsf{s}-\mathsf{u}}{\mathsf{s}+\mathsf{u}}\right)$$

- **HS** generalization of
  - ✦ Charged Scalar + **QED**
  - ✦ Conformal Scalar + **Weyl Gravity**
- Re-introduce **Coupling Const.** fixed by **CHS action**

Conformal HS Exchanges

$$\sum_s$$



$$= g^2 \, \frac{1}{(-\mathsf{t})^{\frac{d-4}{2}}} \, F_d\left(-\frac{\mathsf{s}-\mathsf{u}}{\mathsf{s}+\mathsf{u}}\right)$$

$$\boxed{F_d(z) = \sum_{s=0}^{\infty} (s+\alpha_d)\, C_s^{(\alpha_d)}(z)\,, \qquad \alpha_d = \frac{d-3}{2}}$$

- For a fixed z, the series is **divergent**
- We need to **regularize** it somehow

$$\boxed{F_d(z) = \sum_{s=0}^{\infty} (s+\alpha_d)\, C_s^{(\alpha_d)}(z)\,, \qquad \alpha_d = \frac{d-3}{2}}$$

regul. parameter

$$F_d(z,w) = \sum_{s=0}^{\infty} (s+\alpha_d)\ w^s\, C_s^{(\alpha_d)}(z)$$

$$= w^{1-\alpha_d}\frac{d}{dw}\left(w^{\alpha_d}\sum_{s=0}^{\infty} w^s\, C_s^{(\alpha_d)}(z)\right) = \alpha_d\,\frac{1-w^2}{(1-2z\,w+w^2)^{\alpha_d+1}}$$

Regularized Sum

$$\boxed{F_d^{\text{reg}}(z) = \frac{(-1)^{d-4}}{(d-4)!}\,\delta^{[d-4]}(z-1)}$$

t-channel

$$z = \frac{u-s}{u+s}$$

- same result in a few other regularizations

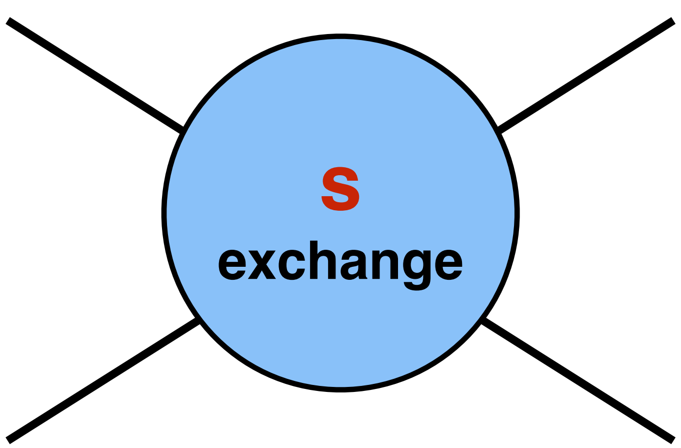
CHS Exchange Amplitude

$$A_{\phi\phi\to\phi\phi} = \frac{g^2}{2}\left[\delta\left(\frac{\mathsf{s}}{\mathsf{t}}\right) + \delta\left(\frac{\mathsf{s}}{\mathsf{u}}\right)\right] = 0$$

$$A_{\phi\bar{\phi}\to\phi\bar{\phi}} = \frac{g^2}{2}\left[\delta\left(\frac{\mathsf{u}}{\mathsf{t}}\right) + \delta\left(\frac{\mathsf{u}}{\mathsf{s}}\right)\right] = 0$$

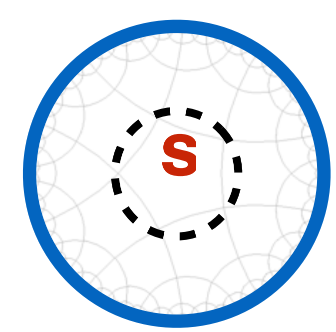
$$A_{\mathbb{R}} = \frac{g^2}{4}\left[\delta\left(\frac{\mathsf{s}}{\mathsf{t}}\right) + \delta\left(\frac{\mathsf{u}}{\mathsf{t}}\right) + \delta\left(\frac{\mathsf{u}}{\mathsf{s}}\right) + \delta\left(\frac{\mathsf{t}}{\mathsf{s}}\right) + \delta\left(\frac{\mathsf{t}}{\mathsf{u}}\right) + \delta\left(\frac{\mathsf{s}}{\mathsf{u}}\right)\right] = 0$$

vanishing amplitude

$\sum_s$ 

$= 0$

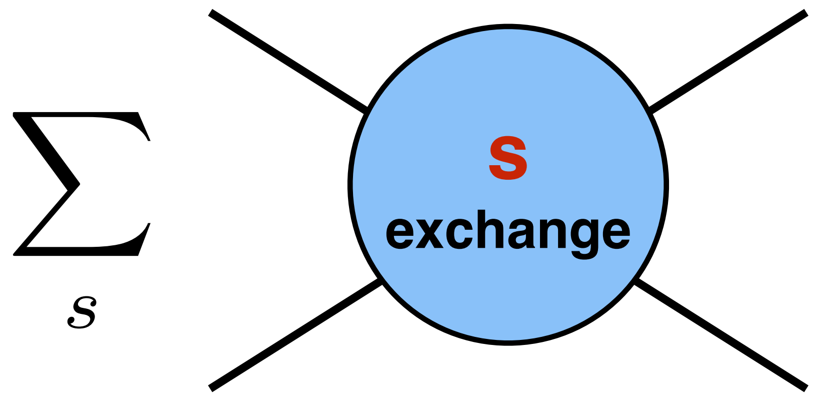
this reminds us

$\sum_s$ 

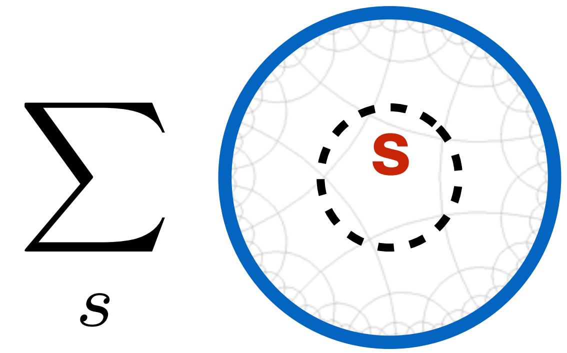
$= 0$

CHS Symmetry and Amplitude

Conf. Scalar + CHS



Massless HS in AdS

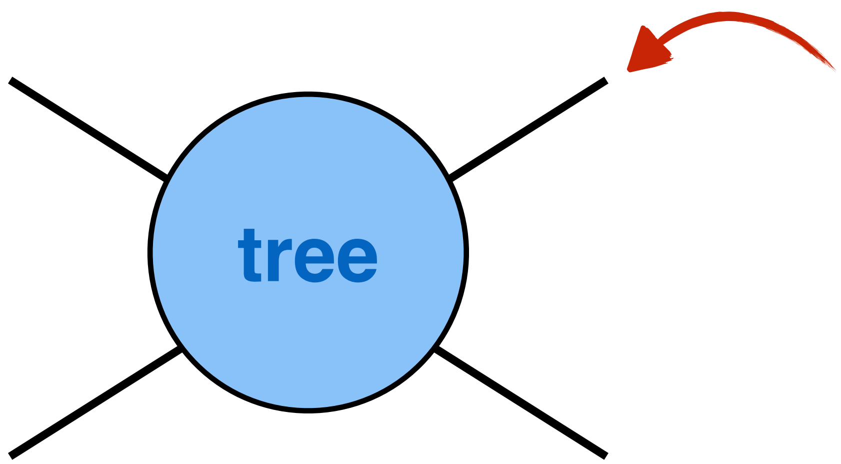


- They have the **same symmetry** : Vasiliev HS Algebra

$$\mathrm{CHS}_d = hs(so(2,d)) = VA_{d+1}$$

- Regularizations compatible with the symmetries?

CHS Symmetry and Amplitude



- we know how **CHS sym.** act
- Tree-level **amplitude** should be **invariant** under this CHS sym.

$$\delta\phi^a(x) = \varepsilon^{\mu_1 \cdots \mu_r}\, \partial_{\mu_1} \cdots \partial_{\mu_r}\, \phi^a(x)$$ (hyper-translations)

$$\delta\phi^a(x) = (e^{y\cdot\partial_x} - e^{-y\cdot\partial_x})\phi^a(x) = \phi^a(x+y) - \phi^a(x-y)$$

$$\boxed{\sin(p_{12}\cdot y)\,\sin(p_{13}\cdot y)\,\sin(p_{14}\cdot y)\,\langle \tilde{\phi}^{a_1}(p_1)\,\tilde{\phi}^{a_2}(p_2)\,\tilde{\phi}^{a_3}(p_3)\,\tilde{\phi}^{a_4}(p_4)\rangle = 0}$$

CHS Symmetry and Amplitude

$$\boxed{\sin(p_{12}\cdot y)\,\sin(p_{13}\cdot y)\,\sin(p_{14}\cdot y)\,\langle\tilde\phi^{a_1}(p_1)\,\tilde\phi^{a_2}(p_2)\,\tilde\phi^{a_3}(p_3)\,\tilde\phi^{a_4}(p_4)\rangle = 0}$$

with $y = 4\,(\sigma\, p_{12} + \tau\, p_{13} + \upsilon\, p_{14})$

$$\sin(\sigma\,\mathsf{s})\,\sin(\tau\,\mathsf{t})\,\sin(\upsilon\,\mathsf{u})\,A_{\mathbb{R}}(\mathsf{s},\mathsf{t},\mathsf{u}) = 0$$

since σ, τ, υ are arbitrary

$$A_{\mathbb{R}}(\mathsf{s},\mathsf{t},\mathsf{u}) = c\,\delta(\mathsf{s}\,\mathsf{t}\,\mathsf{u})$$

combining with dilatation sym.

$$\boxed{A_{\mathbb{R}}(\mathsf{s},\mathsf{t},\mathsf{u}) = 0}$$

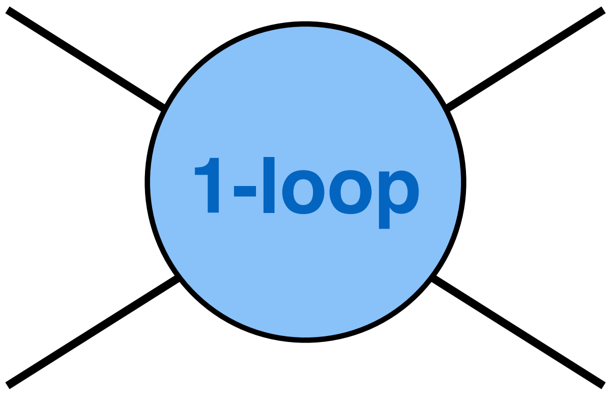
What we have shown

- Scattering of conf. scalars by exchanges of CHS (or induced HS) fields
- The system has **CHS symmetry** at tree level
- CHS symmetry requires the amplitude to vanish
- However, for a given spin s exchange, the amplitude is non-trivial
- we need a **regularization** of the spin sum to recover vanishing amplitude **even at classical level !!**

we need to (properly) throw away ∞

Next

- Understand spin-sum regularizations
  - ✦ if no idea about the correct result ?
  - ✦ Study compatibility between HS sym and Regularization
- Include loop corrections (in progress)



THANK YOU