The spin: from quantum tops to elementary particles

Xavier Bekaert

Laboratoire de Mathématiques et Physique Théorique
Université François Rabelais de Tours

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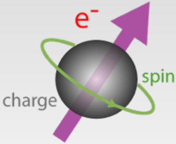
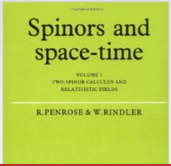
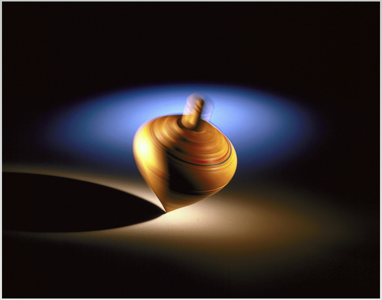




WHAT IS SPIN ?

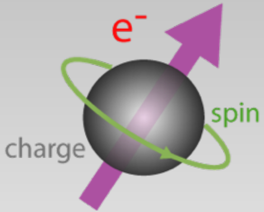
Quantum tops

1. Classical spin as intrinsic angular momentum



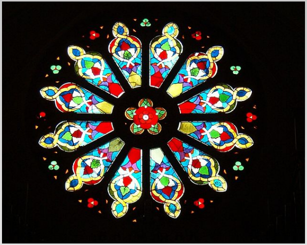
Quantum tops

1. Classical spin as intrinsic angular momentum
2. Quantum spin as intrinsic angular momentum



Quantum tops

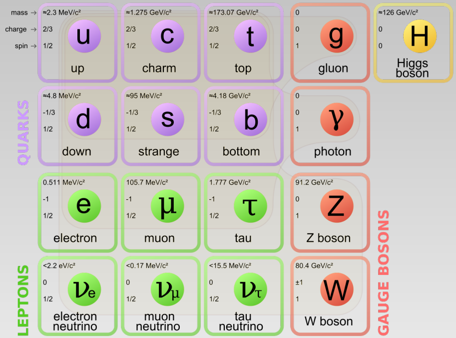
1. Classical spin as intrinsic angular momentum
2. Quantum spin as intrinsic angular momentum
3. Spin as rotational symmetry



IS SPIN BOUNDED ?

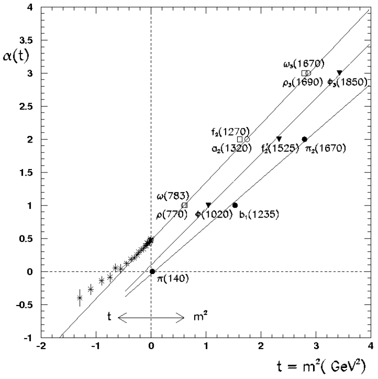
Elementary particles

1. Elementary particules & symmetries



Elementary particles

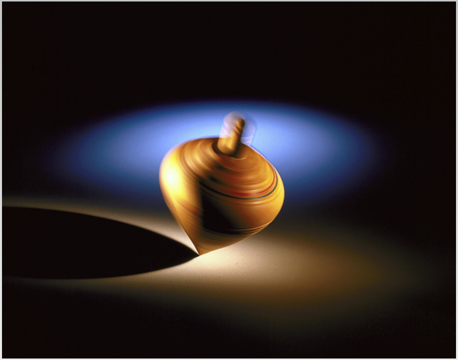
1. Elementary particules & symmetries
2. History of higher-spin particles



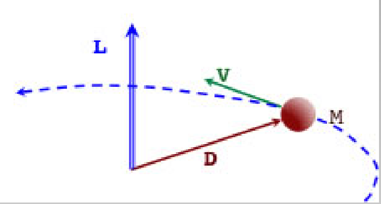
QUANTUM TOPS

Wolfgang Pauli (1900-1958) and Niels Bohr (1885-1962)

1. Classical spin as intrinsic angular momentum



Classical spin as intrinsic angular momentum



Angular momentum

$$\mathbf{L} = \mathbf{D} \wedge \mathbf{P}$$

where

- D = Distance from the axis
- $\mathbf{P} = M\,\mathbf{V}$ = "Linear" momentum

Conservation of angular momentum in daily life

Examples:

- Spinning tops

Classical spin as intrinsic angular momentum

Conservation of angular momentum in daily life

Examples:

- Spinning tops
- Biking

Classical spin as intrinsic angular momentum

Conservation of angular momentum in daily life

Examples:

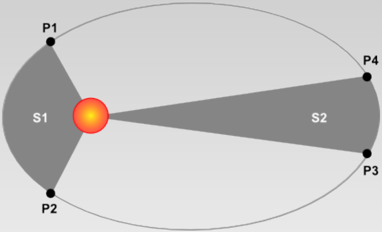
- Spinning tops
- Biking
- Figure skating spins

Classical spin as intrinsic angular momentum

Conservation of angular momentum in daily life

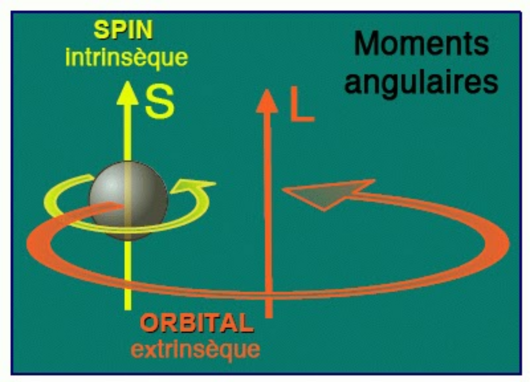
Examples:

- Spinning tops
- Biking
- Figure skating spins
- Solar system



Kepler's 2nd law: the law of equal areas

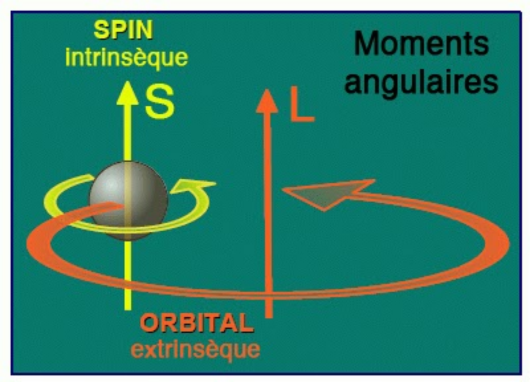
Classical spin as intrinsic angular momentum



Angular momenta
Total = Orbital + Intrinsic

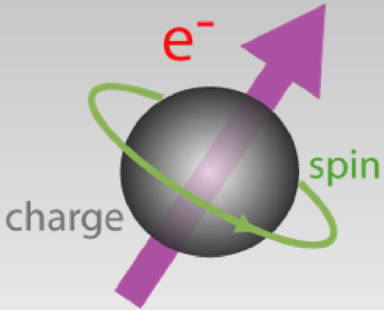
$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

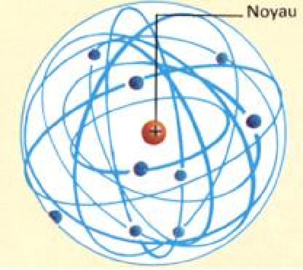
Classical spin as intrinsic angular momentum



Example (solar system):
Earth turns around the Sun (orbital angular momentum) but it also turns on itself (intrinsic angular momentum).

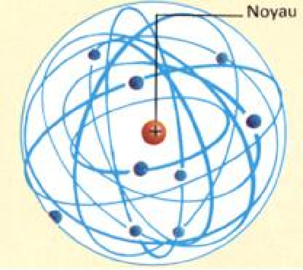
2. Quantum spin as intrinsic angular momentum





Metaphor (model of Rutherford):
An electron "turns" around the nucleus (orbital angular momentum) but it also "turns" on itself (intrinsic angular momentum).

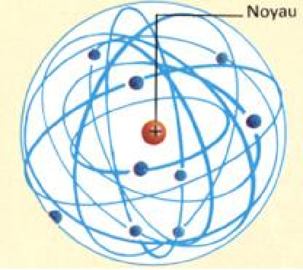
Quantum spin as intrinsic angular momentum



Metaphor (model of Rutherford):
An electron "turns" around the nucleus (orbital angular momentum) but it also "turns" on itself (intrinsic angular momentum).

Question: How to measure such angular momenta of atomic scale?

Quantum spin as intrinsic angular momentum



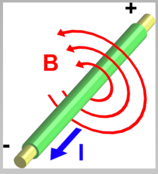
Metaphor (model of Rutherford):
An electron "turns" around the nucleus (orbital angular momentum) but it also "turns" on itself (intrinsic angular momentum).

Question: How to measure such angular momenta of atomic scale?

Answer: By measuring magnetic moment.

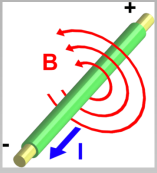
Quantum spin as intrinsic angular momentum

Electric charges in motion create a magnetic field.

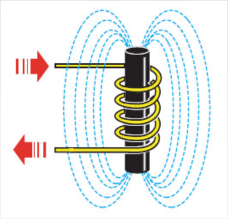


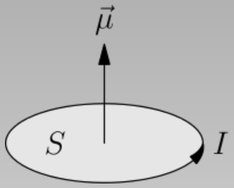
Quantum spin as intrinsic angular momentum

Electric charges in motion create a magnetic field.



In particular, electric charges in circular motion create a magnetic dipole analogous to the one of a magnet.





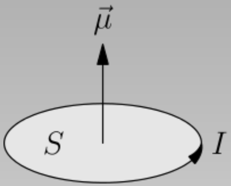
Orbital magnetic moment
of a circular electrical wire

$$\vec{\mu}_{L} = I\,\vec{S}$$

where

- I = electric current
- S = area of the disk

Magnetic moment in classical mechanics



Orbital magnetic moment
of an electric charge in circular motion

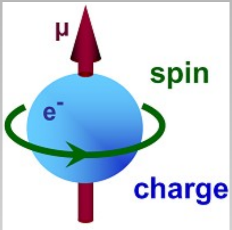
$$\vec{\mu}_L = \frac{Q}{2M}\,\vec{L}$$

where the particle is characterized by

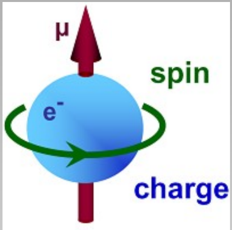
- Q = electric charge
- M = mass
- L = orbital angular momentum

Magnetic moment of the electron

If the electron was an electrically charged rotating ball, then he would possess a magnetic moment.



Magnetic moment of the electron



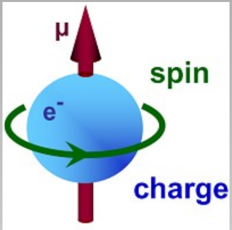
Intrinsic magnetic moment
of a charged rotating ball

$$\vec{\mu}_S = \frac{Q}{2M}\,\vec{S}$$

where the ball is characterized by

- Q = electric charge
- M = mass
- S = intrinsic angular momentum

Magnetic moment of the electron



Intrinsic magnetic moment
of a particle

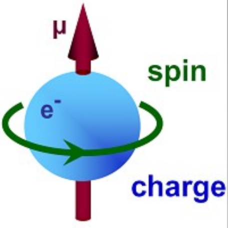
$$\vec{\mu}_S = g \, \frac{Q}{2M} \, \vec{S}$$

where g = Landé factor.

In quantum mechanics,

- g is generically $\neq 1$
- $Q = 0 \not\Rightarrow \mu_S = 0$ (example: neutron)

Magnetic moment of the electron



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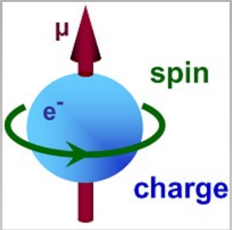
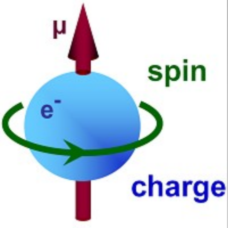


Conclusion: The classical model of an electron as a charged rotating ball is at best a metaphor.

Magnetic moment of the electron



Conclusion: The classical model of an electron as charged rotating ball is at best a metaphor.

This is also evidenced by

- the Stern & Gerlach experiment (1922)
- its explanation in terms of quantised intrinsic angular momentum by Goudsmit & Uhlenbeck (1925)

WHAT SPIN IS NOT

What spin is not

In quantum mechanics an elementary particle must be assigned a certain "intrinsic" angular momentum unconnected with its motion in space. (...) It would be wholly meaningless to imagine the "intrinsic" angular momentum of an elementary particle as being the result of its rotation "about its own axis".

Evgeny Lifshitz (1915-1985) and Lev Landau (1908-1968)

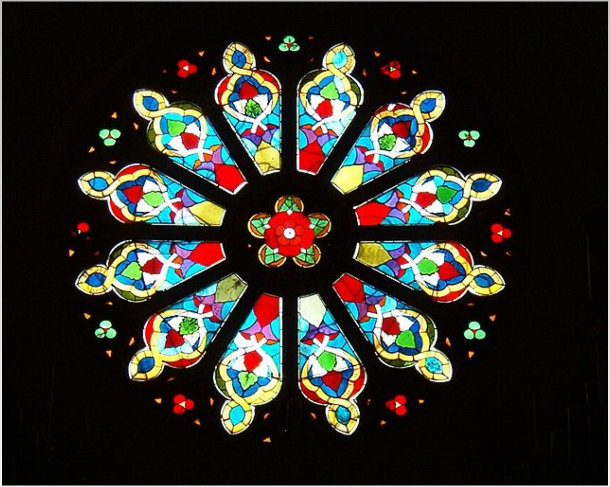
WHAT IS SPIN ?

What is spin?

In both classical and quantum mechanics, the law of conservation of angular momentum is a consequence of the isotropy of space. (...) This already demonstrates the relation between the angular momentum and the symmetry properties under rotation. In quantum mechanics however, the relation in question is a particularly far-reaching one, and essentially constitutes the basic content of the concept of angular momentum.

Evgeny Lifshitz (1915-1985) and Lev Landau (1908-1968)

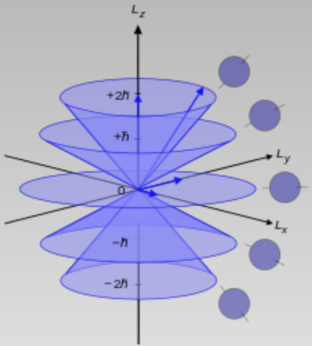
3. Spin as rotational symmetry



Spin as rotational symmetry

In quantum mechanics, angular momentum is "quantised":

- **Orbital angular momentum** (multiple of $\hbar$)



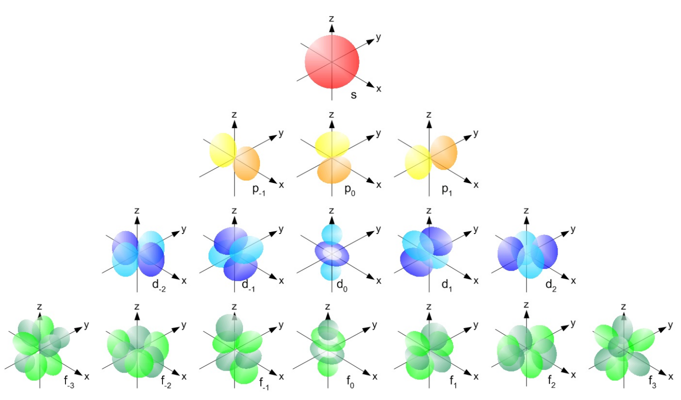
Metaphor of a rotating point

$$L_z = m\,\hbar \qquad m \in \{-\ell, -\ell+1, \cdots, -1, 0, 1, \cdots, \ell-1, \ell\}$$

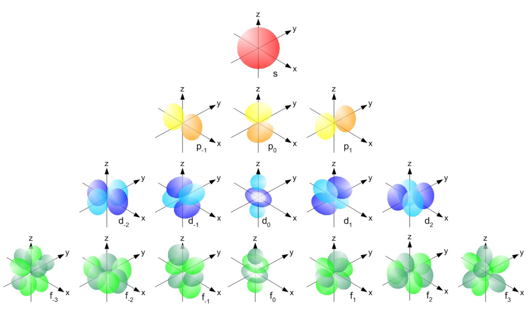
$$\mathbf{L}^2 = \ell(\ell+1)\,\hbar^2 \qquad \ell = 0, 1, 2, 3, \cdots$$

Spin as rotational symmetry

The metaphor of an electron as a point rotating around the nucleus is inconsistent with quantum mechanics since description in terms of trajectories must be replaced by atomic orbitals.



Spin as rotational symmetry



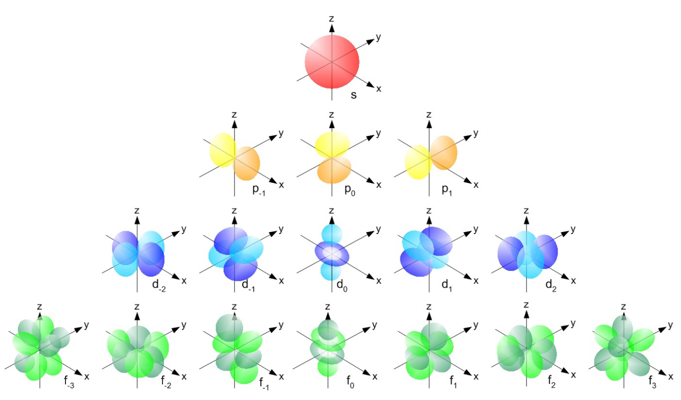
Atomic orbital of a single electron

Labelled by two **quantum numbers** (integers $\in \mathbb{Z}$)

- azimuthal quantum number $\ell \in \mathbb{N}$
- magnetic quantum number m: $2\ell + 1$ possible values $-\ell \leqslant m \leqslant \ell$

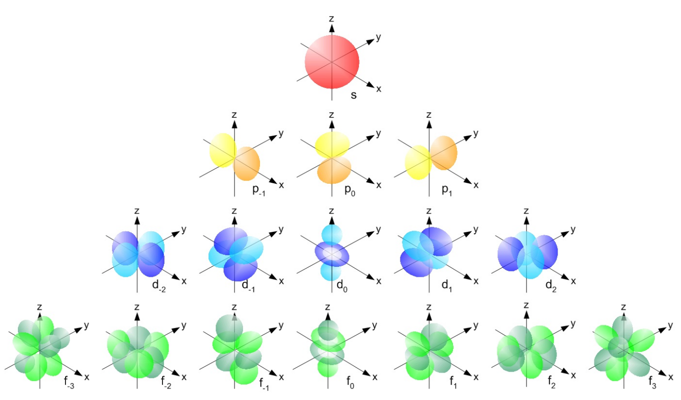
$\Leftrightarrow$ **Spherical harmonic** $Y^{\ell}_{m}(\theta, \phi)$

Spin as rotational symmetry



Orbitals and rotational symmetry around the measurement axis

- s ("simple") $\Leftrightarrow \ell = 0$: Arbitrary rotation ($m = 0$)
- p ("principal") $\Leftrightarrow \ell = 1$: Complete rotation ($m = \pm 1$)
- d ("diffuse") $\Leftrightarrow \ell = 2$: Half-turn ($m = \pm 2$)
- f ("fundamental") $\Leftrightarrow \ell = 3$: Rotation of $120°$ ($m = \pm 3$)

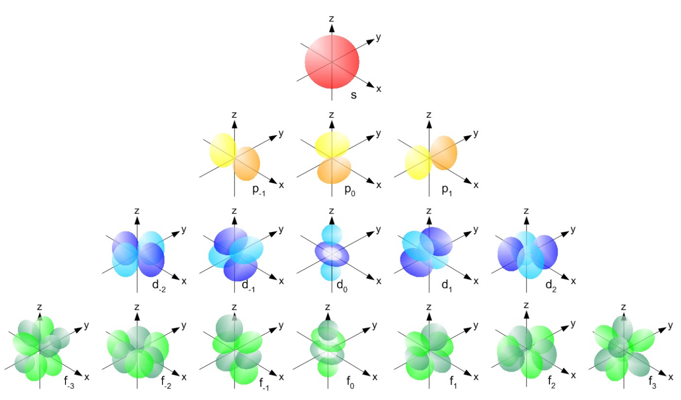


Orbitals and rotational symmetry around the measurement axis

Rotations around different axes permute the orbitals of identical azimuthal quantum number ℓ:

The orbitals, and all their superpositions, carry a representation of the rotation group, which is irreducible for fixed ℓ.

Spin as rotational symmetry



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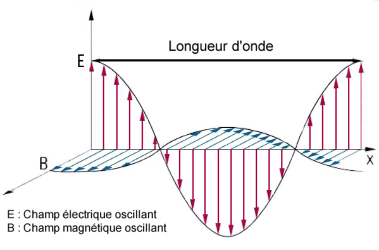
Spin as rotational symmetry

Conclusion:
In quantum mechanics, angular momentum is

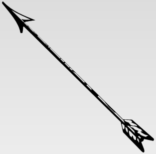
- quantised (thus the metaphor of the rotating point must be abandoned)
- associated to
  - the number $2\ell + 1$ of possible values of the quantum number m
  - symmetry under rotations of angle $\frac{360^\circ}{m}$

Spin as rotational symmetry

Photon: spin 1 particle described by the electromagnetic field.



The electric and magnetic fields are vector fields and belong to the transverse planes. A vector is left invariant under complete rotations.

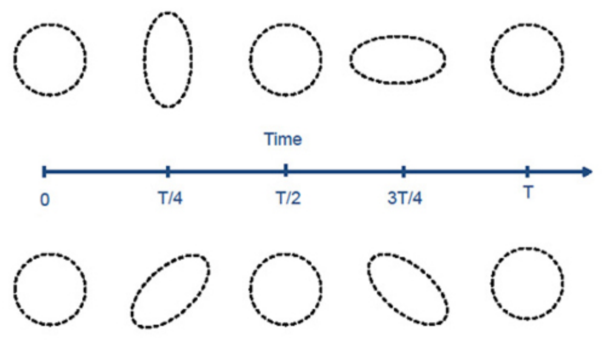


Spin as rotational symmetry

Scalar boson: The Brout-Englert-Higgs boson is of spin 0 and described by a scalar field.

A scalar is left invariant by arbitrary rotations (like a point).

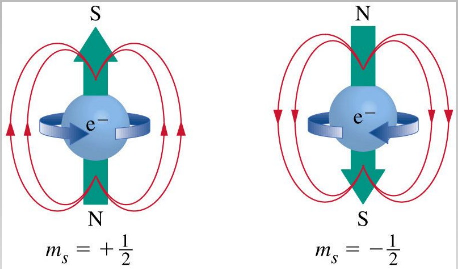
Graviton: (hypothetical) mediator of the gravitational interaction, spin 2 particle described by a tensor field of rank 2, the metric.



Its polarisation modes are left invariant under half-turns.

Spin as rotational symmetry

Electron: The Stern & Gerlach experiment shows that an electron only admits two possible values of the *intrinsic magnetic momentum* along an axis.



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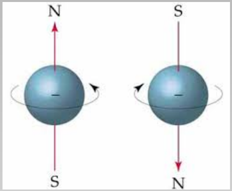

Spin $s = \frac{1}{2} \leftrightarrow$ two possible values of $s_z = \pm\frac{1}{2}$ ("up" or "down")

$\Rightarrow$ The rotational symmetry of an electron are rotations of 720°!

Spin as rotational symmetry

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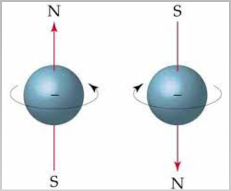

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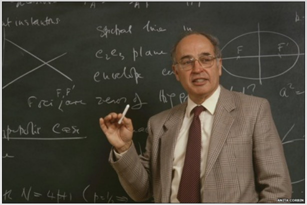
Spin as rotational symmetry

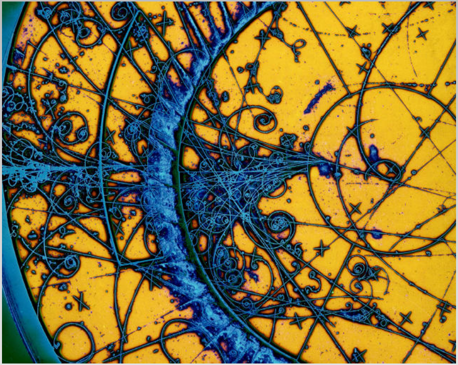
Leptons and Quarks: spin 1/2 particle are described by spinor fields

> *No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the "square root" of geometry and, just as understanding the square root of -1 took centuries, the same might be true of spinors.*

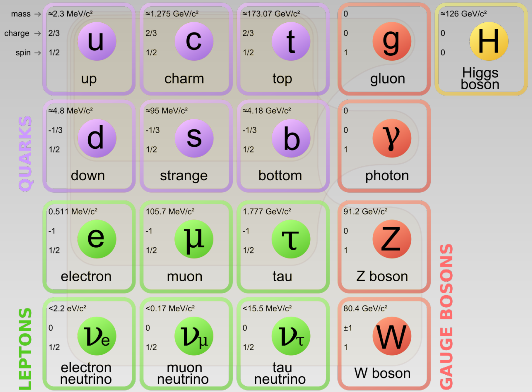
Michael Atiyah, 2007



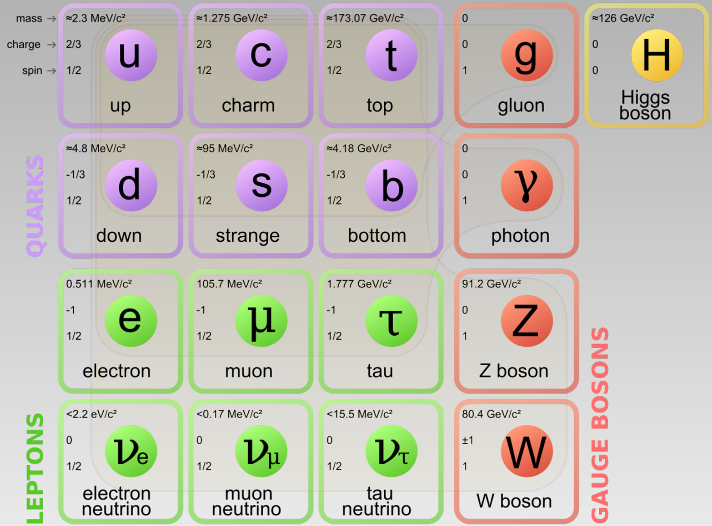
ELEMENTARY PARTICLES



1. Elementary particles & symmetries



Elementary particles



Elementary particles & symmetries

- **Bosons:** integer spin, mediators of known fundamental interactions.
- **Fermions:** half-integer spin, constituents of ordinary matter.

Standard model

Particle	Field	Spin
Higgs	Scalar	0
Leptons & Quarks	Spinor	1/2
Gauge bosons	Vector	1

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Standard model + Gravity

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Graviton (?)	Tensor	2

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Standard model + (Super)gravity

Particle	Field	Spin
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Leptons & Quarks	Spinor	1/2
Gauge bosons	Vector	1
Gravitino (??)	Spin-vector	3/2
Graviton (?)	Tensor	2

The spin-two barrier

At quantum level, consistent interactions including the graviton are not renormalizable by power-counting.

For this reason, for a long time spin 1 has been considered by theoreticians as the maximal possible spin of an elementary particle in an interacting QFT and "higher spin" meant spin greater or equal to $\frac{3}{2}$.

Since the surge of research in quantum gravity (in particular string theory) and the new perspective on power-counting renormalizability (e.g. Weinberg and asymptotic safety) nowadays "higher spin" refers to spin greater or equal to $\frac{5}{2}$.

Elementary particles & symmetries

The modern theoretical description of elementary particles is as quantum fields.

More precisely, a free elementary particle is described by (or defined as) a *unitary irreducible representation of the maximal isometry group*, either Poincaré for $\Lambda = 0$ or (anti) de Sitter for $\Lambda \neq 0$.

The latter representations are labelled by two numbers: the mass m and the spin s.

This fact provides a final answer to "what is spin?"

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Elementary particles & symmetries

The modern (group-theoretical) point of view suggests various lessons:

- Rather than thinking about spin as the (dimensionful) intrinsic angular momentum $\hbar\, s$, spin can also be thought as the (dimensionless) quantum number $s \in \frac{1}{2}\mathbb{N}$ labelling the irreducible representation of the "little group" (a suitable subgroup of the isometry group).
  $\Rightarrow$ Rather than saying that spin is an intrinsically *quantum* property without classical conterpart, it might be better to say that spin is an intrinsically *field-theoretical* property without analogue in point particle classical mechanics.
  (In fact even classical fields, such as the electromagnetic field at tree level, possess spin in the above mathematical sense.)

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- Rather than thinking about spin as the (dimensionful) intrinsic angular momentum $\hbar\, s$, spin can also be thought as the (dimensionless) quantum number $s \in \frac{1}{2}\mathbb{N}$ labelling the irreducible representation of the "little group" (a suitable subgroup of the isometry group).
  $\Rightarrow$ Rather than saying that spin is an intrinsically *quantum* property without classical conterpart, it might be better to say that spin is an intrinsically *field-theoretical* property without analogue in point particle classical mechanics.
  (In fact even classical fields, such as the electromagnetic field at tree level, possess spin in the above mathematical sense.)

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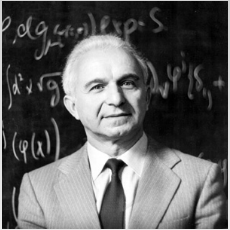
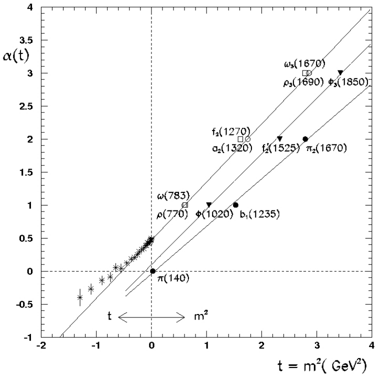
Elementary particles & symmetries

"There is one lesson which theoretical physicists learnt (...) All that is consistent is possible, and all that is possible happens. They recall this lesson each time when a new barrier bars the way to their ideal. At such a barrier we stand now. This is the upper bound two on the spin of gauge fields, imposed by the existing theory. (...) We claimed that there is nothing that forbids the existence of such particles with any spin except our present inability to describe their complete interactions."

E.S. Fradkin
(Speach for his Dirac medal,
received in 1989)



2. History of higher-spin particles



Major challenge

The question of the consistency of interactions including massless higher-spin fields remains a tantalising open problem in field theory, physically well motivatived and mathematically well posed.

$\Longrightarrow$ One should try to answer it, in case Nature would make use of such exotic representation of the isometry group.

But even if the answer would turn out to be negative (i.e. if such fields do not admit consistent interactions or only under unphysical hypotheses), this result would provide a theoretical explanation for the experimental fact that no *elementary* particles of higher-spin have yet been observed.

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Well posed problems

Theoretical works on higher-spins can be structured around 4 main questions, formulated here in terms of mathematical classification research programs (ordered in logical progression).

- **Wigner's programme (1939):**
  Unitary representations of isometry groups
- **Bargmann-Wigner's programme (1948):**
  Relativistic wave equations
- **Fierz-Pauli's programme (1939):**
  Variational principles
  (inverse problem of variational calculus)
- **Frønsdal's programme (1978):**
  Consistent interactions
  (Noether method)

Slicing history

- **1932-1939:** “Birth: generalise Dirac”
- **1939 et 40’s:** “Foundations: elementary particles as unitary irreducible representations”
- **decades 50-60:** “The demographic explosion: the hadronic boom”
- **70’s:** “Prolongations: the Lagrangian quest”
- **1978:** “A well posed problem: the Frønsdal programme”
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Chronology (retrospective view) of linear relativistic wave equations describing a free quantum particle:

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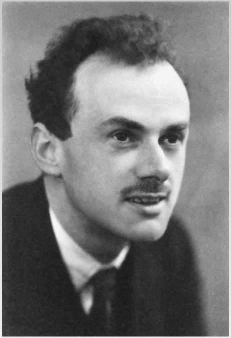

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$\Rightarrow$ Large zoo of relativistic equations

How to put some order (i.e. classify inequivalent ones) ?

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1939 and 40's: "Foundations: elementary particles as unitary irreducible representations"

In 1939, Wigner offered an extremely profound and modern (since it is still valid) view on linear relativistic waves equations.

Combining the axioms of quantum mechanics and the principles of special relativity necessarily leads to the following identifications:

Wave equation describing a free quantum relativistic particle

$\Updownarrow$

Unitary representation of the spacetime isometry group

Space of states (rays) of a free quantum relativistic particle

$\Updownarrow$

Unitary module of the spacetime isometry group

1939 and 40's: "Foundations: elementary particles as unitary irreducible representations"

This identification between free particles and linear representation is perfect on maximally-symmetric spacetimes (= homogeneous Riemannian manifolds with Lorentz signature):

- **Minkowski** $\mathbb{R}^{D-1,1}$: zero scalar curvature

  Poincaré group $\quad ISO(D-1,1) := \mathbb{R}^{D-1,1} \rtimes SO(D-1,1)$

- **de Sitter** dS_D: constant positive scalar curvature

  Pseudo-orthogonal group $\quad SO(D,1) \quad$ *of Lorentzian signature*

- **anti de Sitter** AdS_D: constant negative scalar curvature

  Pseudo-orthogonal group $\quad SO(D-1,2) \quad$ *of conformal signature*

1939 and 40's: "Foundations: elementary particles as unitary irreducible representations"

Let $\mathcal{M}$ be a maximally-symmetric spacetime (background) and $\mathcal{H}$ a unitary module (= space of representation) of the isometry group.

- *Special relativity*: Isometry ⇔ Symmetry
- *Quantum mechanics*: Symmetry ⇔ Unitary operator
- Temporal translations ⇔ 1-parameter subgroup
- Temporal evolution ⇔ Evolution operator ⇔ Linear wave equation
- Wave function of the particle ⇔ Field on $\mathcal{M}$
- Unitary module of inequivalent solutions ⇔ Hilbert space $\mathcal{H}$ of physical states
- *Irreducible* module ⇔ *Elementary* particle

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These identifications motivate the following mathematical problem

Wigner's programme: *Classify all unitary irreducible representations of the isometry groups of maximally-symmetric spacetimes*

This programme somewhat gave birth to the modern theory of representations by the subsequent works of mathematical physicists such as Bargmann, Gel'fand, Harish-Chandra, ...

1939 and 40's: "Foundations: elementary particles as unitary irreducible representations"

From a mathematical point of view, Wigner's programme was a "quickly" solved problem, but from a physical point of view, it was only a first step towards the description of free elementary particles.

Indeed, the next step consist in going from the abstract representation (classified by mathematicians) to a more concrete realisation (as solution space). This step is not trivial because it is not algorithmic: writing relativistic equations is some sort of art.

***Bargmann-Wigner's programme**: Associate a linear covariant differential equation to each unitary irreducible representation of the isometry group of maximally-symmetric spacetimes, such that the space of inequivalent solutions carries the corresponding representation.*

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Let us fix ideas with a "concrete" example: the modules of the Poincaré group describing the propagation of free massive particles of integer spin on Minkowski spacetime can be realised as spaces of tensor fields $\varphi_{\mu_1\dots\mu_r}(x)$ on $\mathbb{R}^{D-1,1}$ which are

- solutions of Klein-Gordon equation

$$(\Box - m^2)\,\varphi_{\mu_1\dots\mu_r}(x) = 0\,,$$

  where $\Box$ is the d'Alembertian and $m > 0$ is the mass,
- divergenceless

$$\partial^\nu \varphi_{\mu_1\dots\nu\dots\mu_r}(x) = 0\,.$$

Moreover, in order to have irreducibility under the Poincaré group $ISO(D-1,1)$, these fields must take value in an irreducible module of the Lorentz subgroup $SO(D-1,1)$.

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Decades 50-60:
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Evolution of the number of distinct particles observed experimentally:

- **Middle of 40's:** can count on the fingers of one hand (electron, photon, proton, neutron, muon)
- **Beginning of 70's:** more than 50
- **Today:** much more than 150

Decades 50-60:
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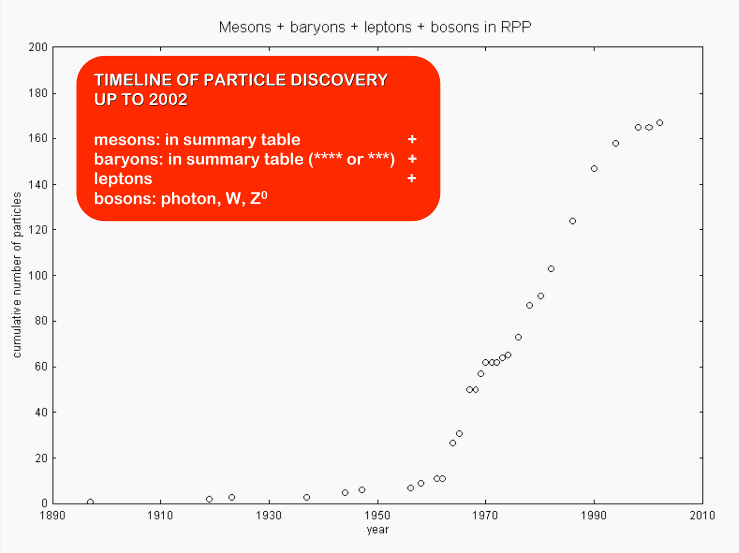


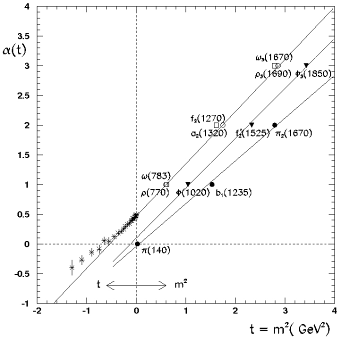
Decades 50-60: “The demographic explosion: the hadronic boom”

However, most of these particuls are hadrons. Such particles are not elementary but composite (pairs or triplets of quarks).

At the beginning of the 60’s, the proliferation of hadrons with “high” ($\geqslant 3/2$) spin was one of the main mystery of the strong nuclear interaction.

Decades 50-60: "The demographic explosion: the hadronic boom"

In 1961, Chew and Frautschi noticed that the mass (squared) spectrum could be approximately described by a growing linear Regge trajectory. The extension of their plot to higher values of the spin suggest the existence of an infinite tower of hadrons, with unbounded spin.



HERA Collider Physics (2008)

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Remark:
In 2010, the observed hadron with highest spin was the baryon $\Delta(2950)$ of spin $15/2$.

70's: "Prolongations: the Lagrangian quest"

The attempts to model scattering cross sections of higher-spin hadrons required the knowledge of the propagators for fields of arbitrary spin. This provided a new motivation for

***Fierz-Pauli's programme**: Associate a quadratic local covariant Lagrangian to each unitary irreducible representation of the isometry group of maximally-symmetric spacetimes, such that the space of inequivalent solutions to Euler-Lagrange equations carries the corresponding representation.*

70's: "Prolongations: the Lagrangian quest"

Completion of Fierz-Pauli's programme in Minkowski spacetime of dimension $D = 4$ for the representations

- **massive (arbitrary spin):** Singh & Hagen (1974)
- **massless (arbitrary helicity):** Fang & Frønsdal (1978)

Massless particles

The propagation of free massless integer-spin particles on Minkowski spacetime are described by tensor fields $\varphi_{\mu_1\ldots\mu_r}(x)$ on $\mathbb{R}^{D-1,1}$ which are harmonic, $\Box\,\varphi_{\mu_1\ldots\mu_r}(x)=0$ and obey to various supplementary conditions.

Gauge symmetries

With respect to the massive case, another novelty is the existence of "gauge symmetries" (= equivalence relations)

$$\varphi_{\mu_1\ldots\mu_r}(x) \sim \varphi_{\mu_1\ldots\mu_r}(x) + \partial_{\mu_1}\varepsilon_{\mu_2\ldots\mu_r}(x) + \cdots$$

The non-Abelian deformations for "low" spins are well known:

Spin	Theory	Geometry	Field	Gauge symmetries
1	Yang-Mills	Principal bundles	Connection	Transitions
2	Gravitation	Riemannian	Metric	Diffeomorphisms

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1978: "A well posed problem: the Frønsdal programme"

Two recent achievements for massless particles

- **1976:** generalisation of Gupta's programme for all "low" spins ($s \leqslant 2$) via supergravity
- **1978:** completion of Fierz-Pauli's programme for all "higher" spins

lead Frønsdal to further generalise Gupta's programme for arbitrary spins.

1978: "A well posed problem: the Frønsdal programme"

Frønsdal programme: *List of all interactions*

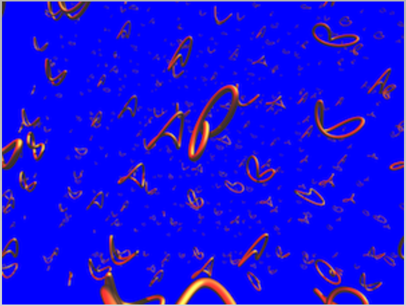
- ***perturbative***,
- ***consistent***,
- ***covariant***,
- ***local***,
- ***deforming*** *a positive sum (finite or not) of quadratic (local covariant) Lagrangians associated with unitary irreducible representations of the isometry group of a maximally symmetric spacetime,*
- ***non Abelian***, *i.e. such that the algebra of gauge symmetries is non commutative already at first order in the deformation parameter(s).*

1978: "A well posed problem: the Frønsdal programme"

"It seems that massless fields of spin $3/2$ cannot couple without the cooperation of gravity. Perhaps massless fields of spin 3 can couple only with the help of other massless fields of integer spin. The analogy with supergravity suggests a higher symmetry that unites massless particles of all spins."

Christian Frønsdal

Strings: the ultraviolet lightning



Strings: the ultraviolet lightning

Incidentally, the theoretical study of hadronic physics gave birth to string theory, the spectrum of which is made of an infinite pyramid of particles with unbounded spin.

All particules in exotic representations (higher spin, mixed symmetries) have a mass above (or of the order of) Planck mass ($\approx 10^{19}$ proton mass).

This infinite pyramid of extremely massive higher-spin particles is responsible for the very good ultraviolet behaviour (UV finiteness) of string theory.

Strings: the ultraviolet lightning

$\Rightarrow$ From the point of view of Frønsdal's programme, string field theory is a highly nontrivial example of consistent interacting theory of *massive* higher-spin particles.

$\Leftarrow$ Conversely, the development of Frønsdal's programme could shed new light on

- string theory:
  ultraviolet behaviour, underlying symmetry principle, ...
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Decades 80-90: "The hurdle race: bypassing no-go theorems"

Bypassing the spin-two barrier is similar to a hurdle race because many obstacles (no-go theorems) are present on the road.

Decades 80-90: "The hurdle race: bypassing no-go theorems"

Coleman-Mandula theorem: *Under very general hypotheses (e.g. $D \geqslant 3$), nontrivial scattering in Minkowski spacetime precludes conserved quantities which, besides Poincaré generators, would form a Lie algebra and belong to a nontrivial representation of the Lorentz algebra.*

As any theorem, the weakness of no-go theorem lies in the strength of its hypotheses, c.f.

- **Spinorial (spin $1/2$):** Supersymmetry ("square root")
- **Tensorial (spin $\geqslant 2$):** Higher-spin symmetry ("powers")

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By now a very large list of positive results have been obtained in the last three decades:

Theory	Higher-spins	Background	Approx.	Lagrangian
Strings	Massive	(Flat)	No	Yes
Vertices	Massless	Flat & (A)dS	Cubic	Yes
Fradkin-Vasiliev	Massless	(Anti) de Sitter	Cubic	Yes
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The necessity of a non-vanishing cosmological constant for the consistency of higher-spin interactions indeed was a curious feature in the early 90's but it has now found a perfectly natural explanation in holographic duality.

2001-now: "The holographic lightning: spacetime reconstruction"

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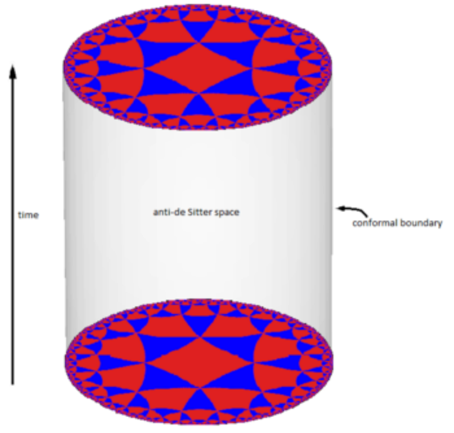
At the beginning of this century, a major conceptual change of perspective on higher-spin gravity and its potential relevance for high-energy physics was brought by holographic duality (aka the "AdS/CFT correspondence").

2001-now: "The holographic lightning: spacetime reconstruction"

A **holographic duality** is an equivalence between a theory of quantum gravity in the bulk of (an asymptotically) AdS spacetime $\mathcal{M}$ and a CFT (without gravity) living on the conformal boundary $\partial\mathcal{M}$.



2001-now: "The holographic lightning: spacetime reconstruction"

The semiclassical limit (tree approximation) in the bulk gravitational theory corresponds to the limit of a large number of fields on the boundary.

Usually the bulk is weakly curved and corresponds to a strongly coupled CFT on the boundary, but the converse limit is also of interest (as argued by Witten, Sundborg, Sezgin, Sundell, etc) in the sense that in principle one should be able to reconstruct the gravitational theory in the strongly curved bulk.

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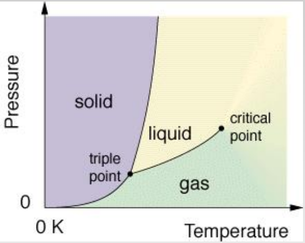
2001-now: "The holographic lightning: spacetime reconstruction"

In particular, *integrable CFTs should be dual to unbroken higher-spin gravity theories in the bulk*.

The most inspiring example of such holographic duality was pushed forward by Klebanov and Polyakov who conjectured in 2002 that: *Vasiliev higher-spin gravity around AdS_4 is dual to the Wilson-Fisher fixed point of the $O(N)$ model*, of which various checks have been performed.



Conclusion

- Spin remains one of the most important – though elusive – manifestations of the field-theoretical nature of elementary particles.
- Spin is one of those physical properties whose proper understanding requires a quite abstract approach (based on representation theory).
- The spin two barrier constitutes a natural frontier between the territory where traditional quantum field theoretical descriptions of massless interacting particles (gauge bosons) are successful and the still largely uncharted lands of higher-spin theories.
- The history of higher-spin particles provides a suggestive example of oscillations between waves of theoretical developments based on the transformation of physically motivated problems into well posed problems of mathematical physics (e.g. the early days of relativistic wave equations and their group-theoretical description in the 30's-40's) and waves of experimental collection of vast data (e.g. the discovery of a plethora of higher-spin hadrons in the 50's-60's).
- In a sense this remains true for its more recent developments linked with string theory and holographic duality due to their potential applications in QCD and condensed matter.

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