Conserved Higher-Spin Charges in AdS_4

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Introduction

Gauge invariant (Generalized Bell-Robinson) conserved currents of different spins in $4d$ Minkowski space in terms of generalized HS Weyl curvatures were constructed by Berends, Burgers, van Dam (1985)
Later conserved currents were considered by Anco, Pohjanpelto (2002)
in particular, within the unfolded approach OG, Skvortsov, Vasiliev (2006)
Non-zero charges were identified by Kaparulin, Lyakhovich,Sharapov (2011)

We analyze AdS_4 currents, built in terms of generalized Weyl tensors, treating them as on-shell closed 3-forms, characterizing nontrivial conserved charges.

Since exact forms do not contribute to the charges,
our goal is to find cohomology of currents .

The resulting list of charges matches the space of parameters of the conformal HS symmetry algebra.

Currents in AdS_4. Oscillator realization

Two sets of two-component spinor oscillators

$$Y = (y^{\alpha}_{j}, \bar{y}^{\alpha'}_{k}), \;\; Z = (z^{j}_{\alpha}, \bar{z}^{k}_{\alpha'})$$

Commutation relations $[\, z^{k}_{\beta}\,, y^{\alpha}_{j}] = \delta^{\alpha}_{\beta}\delta^{k}_{j}\,, \qquad [\bar{z}^{k}_{\alpha'}\,, \bar{y}^{\beta'}_{j}] = \delta^{\beta'}_{\alpha'}\delta^{k}_{j}$

Current $\big|J(Y|x)\big\rangle$ takes value in the Fock module generated

from the Fock vacua $|0\rangle$ that defined to obey $\;z^{j}_{\alpha}|0\rangle = 0\,, \quad \bar{z}^{j}_{\alpha'}|0\rangle = 0$

Current equations (= rank two eq.) in AdS_4 OG, Vasiliev,2012

$$D\big|J(Y|x)\big\rangle := \Big(D^L_2 + W(Y,Z|x)\Big)\big|J(Y|x)\big\rangle = 0\;,$$

D^L_2 is the rank two Lorentz covariant derivative

$$D^L_2 = \mathsf{d} + \omega^{L\alpha\beta}(y_{1\alpha}z^1_{\beta} + y_{2\alpha}z^2_{\beta}) + \bar{\omega}^{L\alpha'\beta'}(\bar{y}_{1\alpha'}\bar{z}^1_{\beta'} + \bar{y}_{2\alpha'}\bar{z}^2_{\beta'})\,,$$

$$W(Y,Z|x) \;\; = \;\; \lambda e^{\alpha\alpha'}\Big(y_{1\alpha}\bar{y}_{1\alpha'} + z^1_{\alpha}\bar{z}^1_{\alpha'} + y_{2\alpha}\bar{y}_{2\alpha'} + z^2_{\alpha}\bar{z}^2_{\alpha'}\Big)\,.$$

$\mathsf{d} = dx^n\frac{\partial}{\partial x^n}$ is the De Rham derivative, $e^{\alpha\alpha'}$ is the frame field

Covariantly constant oscillators

Eight independent linear in Y, Z covariantly constant oscillators $\mathcal{A}_j^{\underline{A}}(Y,Z|x)$ and $\mathcal{B}_{\underline{B}}^j(Y,Z|x)$ $(\underline{A}=(a,\bar{a}),\ a=1,2,\ \bar{a}=1,2\,, j=1,2)$ satisfying

$$D_2^L \mathcal{A}_j^{\underline{A}} + [W, \mathcal{A}_j^{\underline{A}}] = 0, \qquad D_2^L \mathcal{B}_{\underline{A}}^i + [W, \mathcal{B}_{\underline{B}}^j] = 0$$

form Weyl algebra A_4.

$\mathcal{A}_j^{\underline{A}}$ and $\mathcal{B}_{\underline{B}}^j$ are normalized so that

$$\mathcal{A}_j^{\alpha}(Y,Z|0) = y_j^{\alpha}\,, \quad \mathcal{A}_j^{\alpha'}(Y,Z|0) = \bar{y}_j^{\alpha'}\,, \quad \mathcal{B}_{\alpha'}^i(Y,Z|0) = \bar{z}_{\alpha'}^i\,, \quad \mathcal{B}_{\alpha}^i(Y,Z|0) = z_{\alpha}^i\,.$$

This normalization guarantees for arbitrary x

$$[\mathcal{A}_i^{\underline{A}}, \mathcal{A}_j^{\underline{B}}] = 0\,, \qquad [\mathcal{B}_{\underline{A}}^i, \mathcal{B}_{\underline{B}}^j] = 0\,, \qquad [\mathcal{B}_{\underline{A}}^j, \mathcal{A}_i^{\underline{B}}] = \delta_i^j \delta_{\underline{A}}^{\underline{B}}\,.$$

If $|J(Y|x)\rangle$ is a solution of the current equations then $\eta(\mathcal{A},\mathcal{B})|J(Y|x)\rangle$ solves the current equations for arbitrary polynomial $\eta(\mathcal{A},\mathcal{B}) \Rightarrow$ $\eta(\mathcal{A},\mathcal{B})$ are parameters of the global conformal HS symmetry.

$(\mathfrak{sl}_2)^4$

Packing the oscillators y_i^α, $\bar{y}_i^{\alpha'}$, z_α^i, $\bar{z}_{\alpha'}^i$ into $\kappa_\alpha^{n\,\widehat{n}}$, $\chi_{\alpha'}^{n\,\widehat{n}}$

with $n=-,+$ and $\widehat{n}=\widehat{-},\widehat{+}$

$$\kappa_\alpha^{+\widehat{+}}=iy_{2\alpha}\,,\qquad \kappa_\alpha^{+\widehat{-}}=-y_{1\alpha}\,,\qquad \kappa_\alpha^{-\widehat{+}}=z_\alpha^1\,,\qquad \kappa_\alpha^{-\widehat{-}}=-iz_\alpha^2\,,$$
$$\chi_{\alpha'}^{+\widehat{+}}=i\bar{z}_{\alpha'}^2\,,\qquad \chi_{\alpha'}^{+\widehat{-}}=-\bar{z}_{\alpha'}^1\,,\qquad \chi_{\alpha'}^{-\widehat{+}}=\bar{y}_{1\alpha'}\,,\qquad \chi_{\alpha'}^{-\widehat{-}}=-i\bar{y}_{2\alpha'}\,,$$

nonzero commutators acquire the form

$$[\kappa_\beta^{n\,\widehat{k}}\,,\,\kappa_\alpha^{m\,\widehat{n}}]=\varepsilon^{n\,m}\varepsilon^{\widehat{k}\,\widehat{n}}\varepsilon_{\beta\alpha}\,,\qquad [\chi_{\beta'}^{n\,\widehat{k}}\,,\,\chi_{\alpha'}^{m\,\widehat{n}}]=\varepsilon^{n\,m}\varepsilon^{\widehat{k}\,\widehat{n}}\varepsilon_{\beta'\alpha'}$$

In these variables

$$W(Y,Z|x)=\lambda e^{\alpha\beta'}\,\kappa_\alpha^{m\,\widehat{n}}\,\chi_{\beta'\,m\,\widehat{n}}$$

Analogously, the covariantly constant oscillators are packed into

$\tau_a^{n\,\widehat{n}}(\kappa,\chi|x)$ and $\upsilon_{a'}^{n\,\widehat{n}}(\kappa,\chi|x)$, so that

$\tau_a^{m\,\widehat{n}}(\kappa,\chi|0)=\kappa_\alpha^{m\,\widehat{n}}\delta_a^\alpha$ and $\upsilon_{b'}^{m\,\widehat{n}}(\kappa,\chi|0)=\chi_{\beta'}^{m\,\widehat{n}}\delta_{b'}^{\beta'}$

Nonzero commutation relations hold at any x

$[\upsilon_{b'}^{n\,\widehat{k}}(x)\,,\,\upsilon_{a'}^{m\,\widehat{n}}(x)]=\varepsilon^{n\,m}\varepsilon^{\widehat{k}\,\widehat{n}}\varepsilon_{b'a'}$ and $[\tau_b^{n\,\widehat{k}}(x)\,,\,\tau_a^{m\,\widehat{n}}(x)]=\varepsilon^{n\,m}\varepsilon^{\widehat{k}\,\widehat{n}}\varepsilon_{ba}$

Howe dual algebra

$$
\begin{aligned}
f^{\,n\,m} &= \frac{1}{4}\{\kappa_{\beta}{}^{n}{}_{\widehat{m}}\,,\kappa^{\beta\,m\,\widehat{m}}\} + \frac{1}{4}\{\chi_{\beta'}{}^{n}{}_{\widehat{m}}\,,\chi^{\beta'\,m\,\widehat{m}}\}\,,\\
g^{\,\widehat{n}\,\widehat{m}} &= \frac{1}{4}\{\kappa_{\beta\,k}{}^{\widehat{n}}\,,\kappa^{\beta\,k\,\widehat{m}}\} + \frac{1}{4}\{\chi_{\beta'\,k}{}^{\widehat{n}}\,,\chi^{\beta'\,k\,\widehat{m}}\}
\end{aligned}
$$

form two mutually commutative vertical ${}^{v}\mathfrak{sl}_2$ and horizontal ${}^{h}\mathfrak{sl}_2$ algebras which are dual to the covariant derivative of the current equations

Being covariantly constant, $f \in {}^{v}\mathfrak{sl}_2$ and $g \in {}^{h}\mathfrak{sl}_2$ keep the same form in terms of the covariantly constant oscillators $\tau_a^{\,m\,\widehat{m}}(\kappa,\chi|x)$ and $\upsilon_{a'}^{\,m\,\widehat{k}}(\kappa,\chi|x)$

$$
f^{\,n\,m} = \frac{1}{4}\{\tau_b{}^{n}{}_{\widehat{m}}\,,\tau^{b\,m\,\widehat{m}}\} + \frac{1}{4}\{\upsilon_{a'}{}^{n}{}_{\widehat{m}}\,,\upsilon^{a'\,m\,\widehat{m}}\}\,,
$$

$$
g^{\,\widehat{n}\,\widehat{m}} = \frac{1}{4}\{\tau_{b\,k}{}^{\widehat{n}}\,,\tau^{b\,k\,\widehat{m}}\} + \frac{1}{4}\{\upsilon_{a'\,k}{}^{\widehat{n}}\,,\upsilon^{a'\,k\,\widehat{m}}\}\,.
$$

Conserved charges

Consider

$$\left\langle \Omega^{\widehat{m}\,\widehat{k}} \right| := \left\langle \mathcal{H}^{\alpha\alpha'} \{ \kappa_{\alpha}^{-\,\widehat{m}} \chi_{\alpha'-}{}^{\widehat{k}} + \kappa_{\alpha}^{-\,\widehat{k}} \chi_{\alpha'-}{}^{\widehat{m}} \} \right|$$

where $\mathcal{H}^{\alpha\delta'} = -\frac{1}{3} e^{\alpha}{}_{\alpha'} e^{\beta\alpha'} e_{\beta}{}^{\delta'}$ is the differential tree-form. Then

$$d\langle \Omega^{\widehat{m}\,\widehat{n}}| + \langle \Omega^{\widehat{m}\,\widehat{n}}|(D^L + W) = 0$$

For any solution $|J\rangle$ of the current equations

differential form $\omega^{\widehat{m}\,\widehat{n}}(\eta J) = \langle \Omega^{\widehat{m}\,\widehat{n}}|\eta(\tau, \upsilon)|J\rangle$ is a closed form

=current form

ω generates conserved charges Q independent of local variations of Σ^3

$$Q(\omega(\eta J)) = \int_{\Sigma^3} \omega(\eta J)$$

Exact ω does not contribute to Q $\Rightarrow$ nontrivial charge Q are generated

by closed non-exact forms *i.e.*, by the current cohomology

Exact current forms

The following generators

$$\mathcal{G}^{m\,k}_{a\,b} = \frac{1}{2}\{\tau_a{}^m{}_{\widehat{k}}, \tau_b{}^{k\,\widehat{k}}\}\,, \qquad \mathcal{G}^{m\,k}_{a\,b'} = \frac{1}{2}\{\tau_a{}^m{}_{\widehat{k}}, \upsilon_{b'}{}^{k\,\widehat{k}}\}\,, \qquad \mathcal{G}^{m\,k}_{a'\,b'} = \frac{1}{2}\{\upsilon_{a'}{}^m{}_{\widehat{k}}, \upsilon_{b'}{}^{k\,\widehat{k}}\}$$

form a Lie algebra $\mathfrak{o}(8)$ that commutes with ${}^h\mathfrak{sl}_2$ acting on the hatted indices.

The central fact of the analysis of the current cohomology

$$\Big\langle \Omega^{\,\widehat{m}\,\widehat{k}} \Big| \tau_a{}^m{}_{\widehat{k}}\,\eta(\tau,\upsilon) \Big| \mathcal{J}(Y|x) \Big\rangle\,, \qquad \Big\langle \Omega^{\,\widehat{m}\,\widehat{k}} \Big| \upsilon_{b'}{}^m{}_{\widehat{k}}\,\eta(\tau,\upsilon) \Big| \mathcal{J}(Y|x) \Big\rangle$$

are exact. As a consequence, the following forms are exact

$$\Big\langle \Omega^{\,\widehat{m}\,\widehat{n}} \mathcal{G}^{m\,k}_{a\,b}\eta(\tau,\upsilon) \Big| \mathcal{J} \rangle, \quad \langle \Omega^{\,\widehat{m}\,\widehat{n}} \Big| \mathcal{G}^{m\,k}_{a'\,b'}\eta(\tau,\upsilon) \Big| \mathcal{J} \rangle, \quad \langle \Omega^{\,\widehat{m}\,\widehat{n}} \Big| \mathcal{G}^{m\,k}_{a\,b'}\eta(\tau,\upsilon) \Big| \mathcal{J} \rangle$$

Factoring out any polynomials in oscillators containing antisymmetrization of a pair of the hatted Latin indices $\Rightarrow$ Remaining differential forms $\langle \Omega^{\,\widehat{n},\,\widehat{m}} | \eta(\tau^{m\,\widehat{m}}_{\beta}, \upsilon^{n\,\widehat{n}}_{a'}) J \rangle$ have totally symmetrized hatted indices.

HS current cohomology in AdS_4

A space $\mathbf{P}_{AdS}$ of preforms $\Omega^{\widehat{n},\widehat{m}}\eta(\tau_\beta^{m\,\widehat{m}},\upsilon_{a'}^{n\,\widehat{n}})$ with totally symmetrized hatted indices is ${}^h\mathfrak{sl}_2$-invariant and can be analyzed in terms of highest weight reps.

The raising ${}^h\mathfrak{sl}_2$-operator $g^{\widehat{+}\widehat{+}}$ acts on preforms as "$\widehat{+}\frac{\partial}{\partial\widehat{-}}$" , while the lowering one acts vise versa. For instance

$$[g^{\widehat{-}\widehat{-}},\eta(\tau_\beta^{m\,\widehat{m}},\upsilon_{a'}^{n\,\widehat{n}})] = \left[\tau_\gamma^{m\widehat{-}}\frac{\partial}{\partial\tau_\gamma^{m\widehat{+}}} + \upsilon_\gamma^{m\widehat{-}}\frac{\partial}{\partial\upsilon_\gamma^{m\widehat{+}}}\,,\eta(\tau_\beta^{m\,\widehat{m}},\upsilon_{a'}^{n\,\widehat{n}})\right]$$

$$[g^{\widehat{+}\widehat{+}},\eta(\tau_\beta^{m\,\widehat{m}},\upsilon_{a'}^{n\,\widehat{n}})] = \left[\tau_\gamma^{m\widehat{+}}\frac{\partial}{\partial\tau_\gamma^{m\widehat{-}}} + \upsilon_\gamma^{m\widehat{+}}\frac{\partial}{\partial\upsilon_\gamma^{m\widehat{-}}}\,,\eta(\tau_\beta^{m\,\widehat{m}},\upsilon_{a'}^{n\,\widehat{n}})\right]$$

Then the highest vectors in $\mathbf{P}_{AdS}$ are $\Omega^{\widehat{+}\widehat{+}}\eta(\tau_\beta^{m\widehat{+}},\upsilon_{a'}^{n\widehat{+}})$

Then preforms of $\mathbf{P}_{AdS}$ can be represented as

$$\sum_N \left(\mathrm{ad}_{g^{\widehat{-}\widehat{-}}}\right)^N \left(\Omega^{\widehat{+}\widehat{+}}\eta(\tau_a^{m\widehat{+}},\upsilon_{a'}^{n\widehat{+}})\right) c_N(g^{\widehat{+}\widehat{-}})\,,$$

with some coefficients depended on the Cartan element $g^{\widehat{+}\widehat{-}}\in {}^h\mathfrak{sl}_2$, encoding the freedom in normalization of charges.

Generating functions

The simplest choices of coefficients yield the generating functions

$$\Lambda^{\pm}_{gen} = \exp \pm \left(\mathrm{ad}_{g^{\widehat{-}\widehat{-}}}\right)\left(\Omega^{\widehat{+}\widehat{+}}\eta(\tau_a^{m\widehat{+}}, \upsilon_{a'}^{n\widehat{+}})\right)\alpha^{\pm}(g^{\widehat{+}\widehat{-}})$$

with some $\alpha^{\pm}(g^{\widehat{+}\widehat{-}})$. The preforms $\Lambda^{\pm}_{gen}$ generate nontrivial closed forms

$$\begin{aligned}
\omega_{+} &= \left\langle \widetilde{\Omega}^{+} \Big| \eta_{+}(\tilde{\tau}_a^{m}{}_{+}, \tilde{\upsilon}_{a'}^{n}{}_{+} | g^{\widehat{+}\widehat{-}}) \Big| \mathcal{J}(Y|x) \right\rangle , \\
\omega_{-} &= \left\langle \widetilde{\Omega}^{-} \Big| \eta_{-}(\tilde{\tau}_a^{m}{}_{-}, \tilde{\upsilon}_{a'}^{n}{}_{-} | g^{\widehat{+}\widehat{-}}) \Big| \mathcal{J}(Y|x) \right\rangle
\end{aligned}$$

$$\tilde{\tau}_a^{m}{}_{\pm} = \tau_a^{m\widehat{+}} \pm \tau_a^{m\widehat{-}}, \qquad \tilde{\upsilon}_{a'}^{n}{}_{\pm} = \upsilon_{a'}^{n\widehat{+}} \pm \upsilon_{a'}^{n\widehat{-}}, \qquad \widetilde{\Omega}^{\pm} = \Omega^{\widehat{+}\widehat{+}} + \Omega^{\widehat{-}\widehat{-}} \pm 2\Omega^{\widehat{-}\widehat{+}}.$$

Current helicity operator $\mathcal{H} = \frac{1}{4}\left(g^{\widehat{+}\widehat{+}} + g^{\widehat{-}\widehat{-}}\right)$

Since $[\mathcal{H}, \tilde{\tau}_a^{m}{}_{\pm}] = \pm\frac{1}{2}\tilde{\tau}_a^{m}{}_{\pm}\,, \qquad [\mathcal{H}, \tilde{\upsilon}_{a'}^{n}{}_{\pm}] = \pm\frac{1}{2}\tilde{\upsilon}_{a'}^{n}{}_{\pm}\,, \qquad [\mathcal{H}, \widetilde{\Omega}^{\pm}] = \pm\widetilde{\Omega}^{\pm} \quad \Rightarrow$

ω_{+} and ω_{-} depend on the parameters carrying non-negative and non-positive current helicities, respectively

In the usual notations $\quad z^j{}_\alpha \to \partial^j{}_\alpha = \frac{\partial}{\partial y_j^\alpha}\,,\quad \bar z^j{}_{\alpha'} \to \partial^j{}_\alpha = \frac{\partial}{\partial \bar y_j^{\alpha'}}$

$$\partial_{\pm\alpha}\sim\partial^1{}_\alpha \pm \partial^2{}_\alpha,\quad y_{\pm\alpha}\sim y_{1\alpha}\pm y_{2\alpha},\ etc$$

Nontrivial charges are represented by the closed three-forms

$$\begin{aligned}\omega(\eta\mathcal{J}) &= \mathcal{H}^{\alpha\alpha'}\partial_{-\alpha}\partial_{-\alpha'}\eta(\varrho,\bar\varrho|g^{\widehat{+}\widehat{-}})\mathcal{J}(y^\pm,\bar y^\pm|x)\Big|_{y^\pm=\bar y^\pm=0},\\ \omega(\tilde\eta\mathcal{J}) &= \mathcal{H}^{\alpha\alpha'}\partial_{+\alpha}\partial_{+\alpha'}\tilde\eta(\epsilon,\bar\epsilon|g^{\widehat{+}\widehat{-}})\mathcal{J}(y^\pm,\bar y^\pm|x)\Big|_{y^\pm=\bar y^\pm=0},\end{aligned}$$

$$\varrho_{-a} = \mathsf{c}_a{}^\alpha(x)\partial_{-\alpha} + \mathsf{s}_a{}^{\alpha'}(x)\bar y^+{}_{\alpha'}\,,\qquad \varrho^+{}_a = \mathsf{c}_a{}^\alpha(x)y^+_\alpha + \mathsf{s}_a{}^{\alpha'}(x)\bar\partial_{-\alpha'}\,,$$
$$\epsilon^-{}_a = \mathsf{c}_a{}^\alpha(x)y^-_\alpha + \mathsf{s}_a{}^{\alpha'}(x)\bar\partial_{+\alpha'}\,,\qquad \epsilon_{+a} = \mathsf{c}_a{}^\alpha(x)\partial_{+\alpha} + \mathsf{s}_a{}^{\alpha'}(x)\bar y^-{}_{\alpha'}$$

$\mathsf{c}^\beta(x)$ **and** $\mathsf{s}^{\beta'}(x)$ **are Killing spinors that obey**

$$D^L\mathsf{c}^\alpha(x) + \lambda e^{\alpha\beta'}\mathsf{s}_{\beta'}(x) = 0\,,\quad D^L\mathsf{s}^{\beta'}(x) + \lambda e^{\alpha\beta'}\mathsf{c}_\alpha(x) = 0\,.$$

A basis of the space of solutions of this system is formed by four independent pairs of spinors $(\mathsf{c}_a{}^\beta(x),\mathsf{s}_a{}^{\beta'}(x))$ **and** $(\mathsf{c}_{a'}{}^\beta(x),\mathsf{s}_{a'}{}^{\beta'}(x))$ **labeled by** $a=1,2$ and $a'=1,2$, **obeying**

$$\mathsf{c}_a{}^\beta(0) = \delta_a{}^\beta\,,\qquad \mathsf{s}_{a'}{}^{\beta'}(0) = \delta_{a'}{}^{\beta'}$$

Parameters of the $4d$ conformal HS symmetry algebra

Let fields $C^{\pm}_{h_\pm}(y\,,\bar{y}|x)$ carry helicities $h_\pm$ and solve rank–one equations

For bilinear currents $\mathcal{J} = C^{+}_{h_+}(y^+ + y^-, \bar{y}^+ + \bar{y}^- |x) C^{-}_{h_-}(y^+ - y^-, \bar{y}^+ - \bar{y}^- |x)$

$$\mathcal{H}^{\alpha\alpha'} \frac{\partial}{\partial y^{-\alpha}} \frac{\partial}{\partial \bar{y}^{-\alpha'}} \eta(\varrho, \bar{\varrho}|h_+ - h_-) C^{+}_{h_+}(y^+ + y^-, \bar{y}^+ + \bar{y}^- |x) C^{-}_{h_-}(y^+ - y^-, \bar{y}^+ - \bar{y}^- |x)\Big|_{y=\bar{y}=0},$$

$$\mathcal{H}^{\alpha\alpha'} \frac{\partial}{\partial y^{+\alpha}} \frac{\partial}{\partial \bar{y}^{+\alpha'}} \widetilde{\eta}(\epsilon, \bar{\epsilon}|h_+ - h_-) C^{+}_{h_+}(y^+ + y^-, \bar{y}^+ + \bar{y}^- |x) C^{-}_{h_-}(y^+ - y^-, \bar{y}^+ - \bar{y}^- |x)\Big|_{y=\bar{y}=0}$$

represent two generating functions for gauge invariant conformal HS current cohomology in AdS_4. The charges

$$Q^{+}{}_{\eta} = \int \mathcal{H}^{\alpha\alpha'} \partial_{-\alpha} \partial_{-\alpha'} \eta(\varrho, \bar{\varrho}||h_+ - h_-) C^{+}_{h_+} C^{-}_{h_-}\Big|_{y=\bar{y}=0},$$

$$Q^{-}{}_{\widetilde{\eta}} = \int \mathcal{H}^{\alpha\alpha'} \partial_{+\alpha} \partial_{+\alpha'} \widetilde{\eta}(\epsilon, \bar{\epsilon}||h_+ - h_-) C^{+}_{h_+} C^{-}_{h_-}\Big|_{y=\bar{y}=0}$$

are supported by the parameters of non-negative and non-positive current helicities, respectively. This list of charges matches the space of parameters of the $4d$ conformal HS symmetry algebra

Summary

The exact currents forms in AdS are described manifestly, as those containing antisymmetrization with respect to a dual $sl(2)$ algebra acting on hatted indices.

The nonexact currents forms in AdS are associated with the parameters totally symmetrized with respect the hatted indices.

Two pairs of oscillators carrying three sets of two-component indices are introduced, providing a convenient basis for a dual $\mathfrak{o}(8)$ algebra allowing to factor out exact current forms. The role of this algebra remains to be understood.