[1511.05220], [1511.05975] with Seungho Gwak, Karapet Mkrtchyan, SooJong Rey

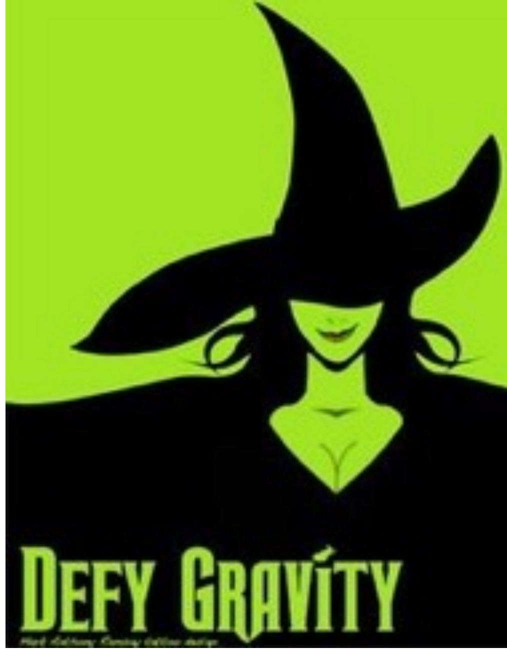
# RAINBOW Valley of Colored (HS) Gravity

**Euihun JOUNG**

Seoul National University, Korea

100 years old

# Defy Einstein Gravity?




- Super Gravity
- Higher-derivative Gravity
- Massive Gravity
- **Higher Spin Gravity**
- **Colored Gravity**

Einstein Gravity $\longleftrightarrow$ **a single** massless spin 2

Einstein Gravity $\longleftrightarrow$ a single massless spin 2

## Colored Gravity:

**Multiple** massless spin 2

with color decoration

Einstein Gravity $\longleftrightarrow$ a single massless spin 2

**Colored Gravity:**

**Multiple** massless spin 2
with color decoration

Q1 Isn't it wrong? No, it works in certain cases

Q2 Isn't it straightforward and boring?

There are **surprising** features

# No-Go for colored gravity

- Multiple massless spin 2 $h^{I}_{\mu\nu}$

☛ cubic interactions $g_{IJK}\Big(h^{I}_{\mu\rho}\,\partial^{\rho}\,h^{J}_{\nu\lambda}\partial^{\lambda}h^{K\,\mu\nu}+\cdots\Big)$

**symmetric**

☛ global symmetries $[\,M^{I}_{\mu\nu}\,,\,M^{J}_{\rho\lambda}\,]=4\,g^{IJ}{}_{K}\,\eta_{[\nu[\rho}\,M^{K}_{\lambda]\mu]}$

**associative**

➠ **only trivial solution**

# No-Go for colored gravity

- Multiple massless spin 2 $\quad h^{I}_{\mu\nu}$

☛ cubic interactions $\qquad g_{IJK}\left(h^{I}_{\mu\rho}\,\partial^{\rho}\,h^{J}_{\nu\lambda}\partial^{\lambda}h^{K\,\mu\nu}+\cdots\right)$

**symmetric**

☛ global symmetries $\qquad [\,M^{I}_{\mu\nu}\,,\,M^{J}_{\rho\lambda}\,]=4\,g^{IJ}{}_{K}\,\eta_{[\nu[\rho}\,M^{K}_{\lambda]\mu]}$

**associative**

➠ **only trivial solution**

- Color-charged massless spin 2 in flat space

☛ minimal interaction to gauge field

➠ **violation of color gauge inv.**

# No-Go for colored gravity

- Multiple massless spin 2 $h^I_{\mu\nu}$

☛ cubic interactions $g_{IJK}\big(h^I_{\mu\rho}\,\partial^\rho h^J_{\nu\lambda}\partial^\lambda h^{K\,\mu\nu}+\cdots\big)$

**symmetric**

☛ global symmetries $[\,M^I_{\mu\nu}\,,\,M^J_{\rho\lambda}\,] = 4\,g^{IJ}{}_K\,\eta_{[\nu[\rho}\,M^K_{\lambda]\mu]}$

**associative**

➠ **only trivial solution**

- Color-charged massless spin 2 in ~~flat space~~ **(A)dS**

☛ minimal interaction to gauge field

➠ ~~**violation of color gauge inv.**~~

# Yes-Go for colored gravity

## Global Symmetries

isometry $\mathfrak{g}_i \otimes \mathfrak{g}_c$ color sym.

$M_X$ $\boldsymbol{T}_I$

# Yes-Go for colored gravity

## Global Symmetries

isometry $\mathfrak{g}_i \otimes \mathfrak{g}_c$ color sym.

$M_X$ $\boldsymbol{T}_I$

$$[\, M_X \otimes \boldsymbol{T}_I, M_Y \otimes \boldsymbol{T}_J \,] = \frac{1}{2}[\, M_X, M_Y \,] \otimes \boxed{\{\, \boldsymbol{T}_I, \boldsymbol{T}_J \,\}} + \frac{1}{2}\boxed{\{\, M_X, M_Y \,\}} \otimes [\, \boldsymbol{T}_I, \boldsymbol{T}_J \,]$$

not defined for Lie algebra

# Yes-Go for colored gravity

## Global Symmetries

Associative | | Associative
isometry $\mathfrak{g}_i \otimes \mathfrak{g}_c$ color sym.
$M_X$ | | $\boldsymbol{T}_I$

$$[\, M_X \otimes \boldsymbol{T}_I , M_Y \otimes \boldsymbol{T}_J \,] = \frac{1}{2}\,[\, M_X , M_Y \,] \otimes \boxed{\{\, \boldsymbol{T}_I , \boldsymbol{T}_J \,\}} + \frac{1}{2} \boxed{\{\, M_X , M_Y \,\}} \otimes [\, \boldsymbol{T}_I , \boldsymbol{T}_J \,]$$

**not defined for Lie algebra**

- $\mathfrak{g}_c$ associative color algebra: $\mathfrak{u}(N)$
- $\mathfrak{g}_i$ associative algebra containing isometry

# Yes-Go for colored gravity

## Global Symmetries

| Associative | | Associative |
|---|---|---|
| isometry | $\mathfrak{g}_i \otimes \mathfrak{g}_c$ | color sym. |
| $M_X$ | | $\boldsymbol{T}_I$ |

$$[\,M_X \otimes \boldsymbol{T}_I, M_Y \otimes \boldsymbol{T}_J\,] = \frac{1}{2}[\,M_X, M_Y\,] \otimes \boxed{\{\,\boldsymbol{T}_I, \boldsymbol{T}_J\,\}} + \frac{1}{2}\boxed{\{\,M_X, M_Y\,\}} \otimes [\,\boldsymbol{T}_I, \boldsymbol{T}_J\,]$$

not defined for Lie algebra

- $\mathfrak{g}_c$ associative color algebra: $\mathfrak{u}(N)$
- $\mathfrak{g}_i$ associative algebra containing isometry

➠ **contains more spectrum (e.g. HS)**

# 3D Chern-Simons

# Colored Gravity

- CS action: $S[\mathcal{A}] = \frac{\kappa}{4\pi}\int \mathrm{Tr}\Big(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\Big)$

- Gauge Algebra: $\mathfrak{g} = (\mathfrak{gl}_2 \oplus \mathfrak{gl}_2) \otimes \mathfrak{u}(N) \;\ominus\; \mathrm{id} \otimes \boldsymbol{I}$

# 3D Chern-Simons

# Colored Gravity

- CS action: $S[\mathcal{A}] = \frac{\kappa}{4\pi} \int \mathrm{Tr}\Big(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\Big)$

- Gauge Algebra: $\mathfrak{g} = (\mathfrak{gl}_2 \oplus \mathfrak{gl}_2) \otimes \mathfrak{u}(N) \ \ominus \ \mathrm{id} \otimes \boldsymbol{I}$

two additional gauge fields

# 3D Chern-Simons Colored Gravity

- CS action: $$S[\mathcal{A}] = \frac{\kappa}{4\pi}\int \mathrm{Tr}\left(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right)$$

subtract Abelian CS

- Gauge Algebra: $$\mathfrak{g} = (\mathfrak{gl}_2 \oplus \mathfrak{gl}_2)\otimes \mathfrak{u}(N) \ominus \mathrm{id}\otimes \boldsymbol{I}$$

two additional gauge fields

# 3D Chern-Simons Colored Gravity

- CS action: $S[\mathcal{A}] = \frac{\kappa}{4\pi} \int \mathrm{Tr}\Big(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3}\,\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\Big)$

subtract Abelian CS

- Gauge Algebra: $\mathfrak{g} = (\mathfrak{gl}_2 \oplus \mathfrak{gl}_2) \otimes \mathfrak{u}(N) \ominus \ \mathrm{id} \otimes \boldsymbol{I}$

two additional gauge fields

$$[\,M_{ab}, M_{cd}\,] = 2\left(\eta_{d[a}\,M_{b]c} + \eta_{c[b}\,M_{a]d}\right), \quad [\,M_{ab}, P_c\,] = 2\,\eta_{c[b}\,P_{a]}\,, \quad [\,P_a, P_b\,] = \sigma\,M_{ab}$$

$$M_{ab} = \frac{1}{2}\,\epsilon_{ab}{}^{c}\,(J_c + \tilde{J}_c)\,, \qquad P_a = \frac{1}{2\sqrt{\sigma}}\,(J_a - \tilde{J}_a)$$

$\sigma = +1$ for $\mathrm{AdS}_3$

$\sigma = -1$ for $\mathrm{dS}_3$

$$\mathrm{Tr}(J_a\,J_b) = 2\sqrt{\sigma}\,\eta_{ab}\,, \qquad \mathrm{Tr}(\tilde{J}_a\,\tilde{J}_b) = -2\sqrt{\sigma}\,\eta_{ab}\,, \qquad \mathrm{Tr}(\boldsymbol{T}_I\,\boldsymbol{T}_J) = \delta_{IJ}$$

# 3D Chern-Simons
# Colored Gravity

- CS action: $S[\mathcal{A}] = \frac{\kappa}{4\pi} \int \mathrm{Tr}\Big(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\Big)$

☹ not tangible

☹ is it over?

☹ so what?

# 3D Chern-Simons Colored Gravity

- CS action: $$S[\mathcal{A}] = \frac{\kappa}{4\pi} \int \mathrm{Tr}\Big(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\Big)$$

☹ not tangible

☹ is it over?

☹ so what?

**Let's rewrite this in METRIC form!**

solve torsion condition only for the genuine graviton (singlet spin two)

# 3D Colored Gravity

☟ **solve torsion condition** ☟

# 3D Colored Gravity

## solve torsion condition

$$S = S_{\rm CS} + \frac{1}{16\pi\, G} \int \mathrm{d}^3 x \sqrt{|g|} \left[ R - V(\boldsymbol{\varphi}, \tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell}\, \epsilon^{\mu\nu\rho}\, \mathrm{Tr} \left( \boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda} \right) \right]$$

# 3D Colored Gravity

## solve torsion condition

$$S = S_{\rm CS} + \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{|g|}\left[R - V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell}\,\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda}\right)\right]$$

- SU(N) CS: $S_{\rm CS} = \frac{\kappa\sqrt{\sigma}}{2\pi}\int\left[\mathrm{Tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A} + \frac{3}{2}\,\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}\right) - \mathrm{Tr}\left(\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}} + \frac{3}{2}\,\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right)\right]$

must be quantized!

# 3D Colored Gravity

## solve torsion condition

$$S = S_{\text{CS}} + \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{|g|}\left[R - V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell}\,\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda}\right)\right]$$

- SU(N) CS: $S_{\text{CS}} = \frac{\kappa\sqrt{\sigma}}{2\pi}\int\left[\mathrm{Tr}\left(\boldsymbol{A}\wedge d\boldsymbol{A} + \frac{3}{2}\,\boldsymbol{A}\wedge\boldsymbol{A}\wedge\boldsymbol{A}\right) - \mathrm{Tr}\left(\tilde{\boldsymbol{A}}\wedge d\tilde{\boldsymbol{A}} + \frac{3}{2}\,\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\wedge\tilde{\boldsymbol{A}}\right)\right]$

must be quantized!

- Newton's constant: $\kappa = \frac{\ell}{4\,N\,G}$

**semi-classical gravity: compatible with small CS level for large N!**

# 3D Colored Gravity

## solve torsion condition

$$S = S_{\rm CS} + \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{|g|}\left[R - V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell}\,\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda}\right)\right]$$

- Colored spinning matter: $D_\mu \boldsymbol{\varphi}_{\nu\rho} = \nabla_\mu \boldsymbol{\varphi}_{\nu\rho} + [\boldsymbol{A}_\mu, \boldsymbol{\varphi}_{\nu\rho}]$

**covariant couplings!**

Potential

$$\begin{aligned} V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) \;=\; & -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\left[2\,\sigma\,\boldsymbol{I} + 4\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} + \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right) + 8\sqrt{\sigma}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\right] \\ & -\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\nu}{}^{\mu}\,\boldsymbol{\varphi}_{\lambda]}{}^{\lambda} - \tilde{\boldsymbol{\varphi}}_{[\nu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\lambda]}{}^{\lambda}\right) + \frac{6\,\sigma}{N^2\,\ell^2}\left[\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)\right]^2 \end{aligned}$$

# 3D Colored Gravity

## ☟ solve torsion condition ☟

$$S = S_{\mathrm{CS}} + \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{|g|}\left[R - V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell}\,\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda}\right)\right]$$

- Colored spinning matter: $D_\mu \boldsymbol{\varphi}_{\nu\rho} = \nabla_\mu \boldsymbol{\varphi}_{\nu\rho} + [\boldsymbol{A}_\mu, \boldsymbol{\varphi}_{\nu\rho}]$

**covariant couplings!**

**Potential**

$$\begin{aligned} V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) \;=\; & -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\left[2\,\sigma\,\boldsymbol{I} + 4\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} + \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right) + 8\sqrt{\sigma}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\right] \\ & -\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\nu}{}^{\mu}\,\boldsymbol{\varphi}_{\lambda]}{}^{\lambda} - \tilde{\boldsymbol{\varphi}}_{[\nu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\lambda]}{}^{\lambda}\right) + \frac{6\,\sigma}{N^2\,\ell^2}\left[\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)\right]^2 \end{aligned}$$

**Matter self-interaction:** $\sqrt{N}$ times ***STRONGER*** than gravity!

# Closer look on Potential

**Potential**

$$
\begin{aligned}
V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) \;=\;& -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\left[2\,\sigma\,\boldsymbol{I}+4\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu}+\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)+8\sqrt{\sigma}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\right] \\
& -\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\mu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\Big)\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\nu}{}^{\mu}\,\boldsymbol{\varphi}_{\lambda]}{}^{\lambda}-\tilde{\boldsymbol{\varphi}}_{[\nu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\lambda]}{}^{\lambda}\Big)+\frac{6\,\sigma}{N^2\,\ell^2}\left[\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)\right]^2
\end{aligned}
$$

# Closer look on Potential

**Potential**

$$V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) = -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\left[2\,\sigma\,\boldsymbol{I}+4\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu}+\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)+8\sqrt{\sigma}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\right)\right]$$
$$-\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\mu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\Big)\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\nu}{}^{\mu}\,\boldsymbol{\varphi}_{\lambda]}{}^{\lambda}-\tilde{\boldsymbol{\varphi}}_{[\nu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\lambda]}{}^{\lambda}\Big)+\frac{6\,\sigma}{N^2\,\ell^2}\left[\mathrm{Tr}\left(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu}-\tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\right)\right]^2$$

- reduction to parity-invariant sector

$$\boldsymbol{\chi}_{\mu\nu} = \sqrt{\sigma}\,(\boldsymbol{\varphi}_{\mu\nu}-\tilde{\boldsymbol{\varphi}}_{\mu\nu})$$
$$\boldsymbol{\tau}_{\mu\nu} = \boldsymbol{\varphi}_{\mu\nu}+\tilde{\boldsymbol{\varphi}}_{\mu\nu}$$

$$V(\boldsymbol{\chi}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{I}+\boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu]}{}^{\nu}+\boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu}{}^{\nu}\,\boldsymbol{\chi}_{\rho]}{}^{\rho}\Big)$$

# Closer look on Potential

**Potential**

$$
\begin{aligned}
V(\boldsymbol{\varphi},\tilde{\boldsymbol{\varphi}}) \;=\; & -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\Big[2\,\sigma\,\boldsymbol{I} + 4\,\big(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} + \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\big) + 8\sqrt{\sigma}\,\big(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\big)\Big] \\
& -\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\mu}{}^{\nu}\,\boldsymbol{\varphi}_{\rho]}{}^{\rho} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\nu}\,\tilde{\boldsymbol{\varphi}}_{\rho]}{}^{\rho}\Big)\,\mathrm{Tr}\Big(\boldsymbol{\varphi}_{[\nu}{}^{\mu}\,\boldsymbol{\varphi}_{\lambda]}{}^{\lambda} - \tilde{\boldsymbol{\varphi}}_{[\nu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\lambda]}{}^{\lambda}\Big) + \frac{6\,\sigma}{N^2\,\ell^2}\,\Big[\mathrm{Tr}\,\big(\boldsymbol{\varphi}_{[\mu}{}^{\mu}\,\boldsymbol{\varphi}_{\nu]}{}^{\nu} - \tilde{\boldsymbol{\varphi}}_{[\mu}{}^{\mu}\,\tilde{\boldsymbol{\varphi}}_{\nu]}{}^{\nu}\big)\Big]^2
\end{aligned}
$$

- reduction to parity-invariant sector

$$
\boldsymbol{\chi}_{\mu\nu} = \sqrt{\sigma}\,(\boldsymbol{\varphi}_{\mu\nu} - \tilde{\boldsymbol{\varphi}}_{\mu\nu})
$$
$$
\boldsymbol{\tau}_{\mu\nu} = \boldsymbol{\varphi}_{\mu\nu} + \tilde{\boldsymbol{\varphi}}_{\mu\nu}
$$

$$
V(\boldsymbol{\chi}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{I} + \boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu]}{}^{\nu} + \boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu}{}^{\nu}\,\boldsymbol{\chi}_{\rho]}{}^{\rho}\Big)
$$

- reduction to diffeo-invariant sector

$$
\boldsymbol{\chi}_{\mu\nu} = g_{\mu\nu}\,\boldsymbol{X}
$$

$$
V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\,\big(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\big)
$$

# Closer look on Potential

Potential

$$V(\varphi,\tilde{\varphi}) = -\frac{1}{N\,\ell^2}\,\mathrm{Tr}\left[2\,\sigma\,\boldsymbol{I} + 4\left(\varphi_{[\mu}{}^{\mu}\,\varphi_{\nu]}{}^{\nu} + \tilde{\varphi}_{[\mu}{}^{\mu}\,\tilde{\varphi}_{\nu]}{}^{\nu}\right) + 8\sqrt{\sigma}\left(\varphi_{[\mu}{}^{\mu}\,\varphi_{\nu}{}^{\nu}\,\varphi_{\rho]}{}^{\rho} - \tilde{\varphi}_{[\mu}{}^{\mu}\,\tilde{\varphi}_{\nu}{}^{\nu}\,\tilde{\varphi}_{\rho]}{}^{\rho}\right)\right]$$
$$-\frac{16\,\sigma}{N^2\,\ell^2}\,\mathrm{Tr}\Big(\varphi_{[\mu}{}^{\nu}\,\varphi_{\rho]}{}^{\rho} - \tilde{\varphi}_{[\mu}{}^{\nu}\,\tilde{\varphi}_{\rho]}{}^{\rho}\Big)\,\mathrm{Tr}\Big(\varphi_{[\nu}{}^{\mu}\,\varphi_{\lambda]}{}^{\lambda} - \tilde{\varphi}_{[\nu}{}^{\mu}\,\tilde{\varphi}_{\lambda]}{}^{\lambda}\Big) + \frac{6\,\sigma}{N^2\,\ell^2}\left[\mathrm{Tr}\left(\varphi_{[\mu}{}^{\mu}\,\varphi_{\nu]}{}^{\nu} - \tilde{\varphi}_{[\mu}{}^{\mu}\,\tilde{\varphi}_{\nu]}{}^{\nu}\right)\right]^2$$

- reduction to parity-invariant sector

$$\boldsymbol{\chi}_{\mu\nu} = \sqrt{\sigma}\,(\boldsymbol{\varphi}_{\mu\nu} - \tilde{\boldsymbol{\varphi}}_{\mu\nu})$$
$$\boldsymbol{\tau}_{\mu\nu} = \boldsymbol{\varphi}_{\mu\nu} + \tilde{\boldsymbol{\varphi}}_{\mu\nu}$$

$$V(\boldsymbol{\chi}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\Big(\boldsymbol{I} + \boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu]}{}^{\nu} + \boldsymbol{\chi}_{[\mu}{}^{\mu}\,\boldsymbol{\chi}_{\nu}{}^{\nu}\,\boldsymbol{\chi}_{\rho]}{}^{\rho}\Big)$$

- reduction to diffeo-invariant sector

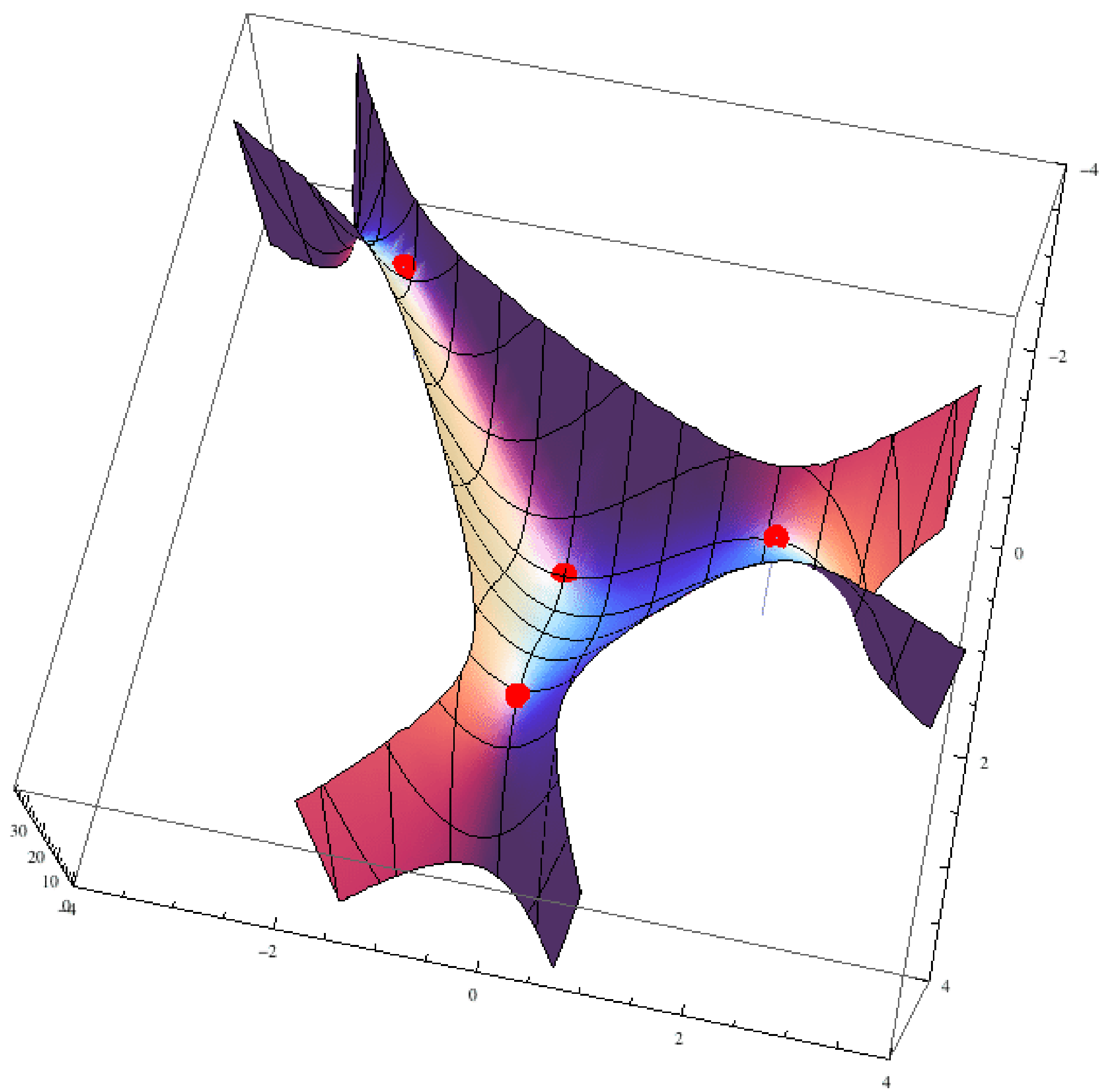
$$\boldsymbol{\chi}_{\mu\nu} = g_{\mu\nu}\,\boldsymbol{X}$$

$$\boxed{V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)}$$

**Non-trivial Potential with Many Extrema!**

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$



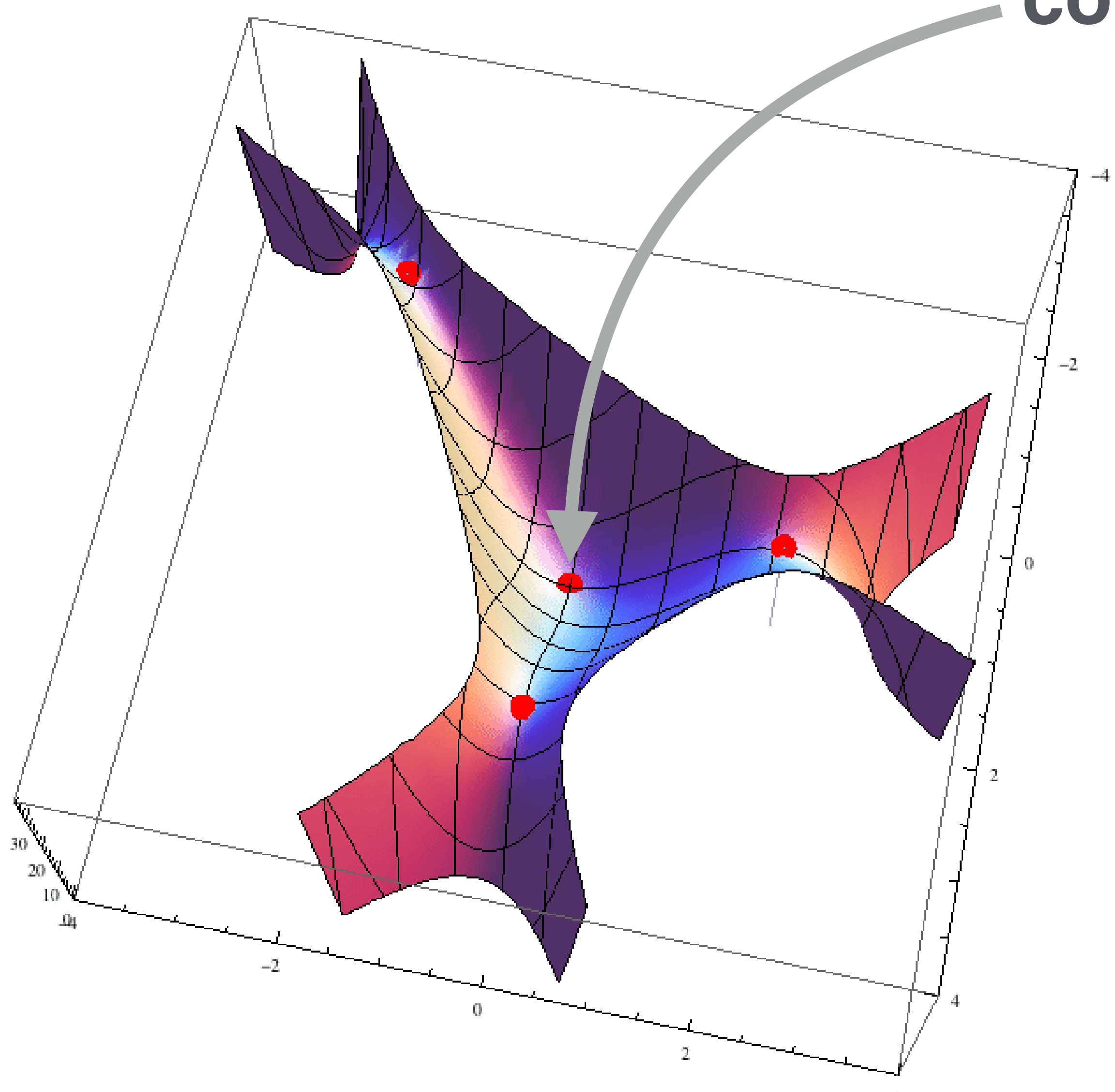
$\mathfrak{su}(3)$ case

$$\mathbf{X} = a\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix} + b\begin{bmatrix}-2&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

**color singlet vacuum**



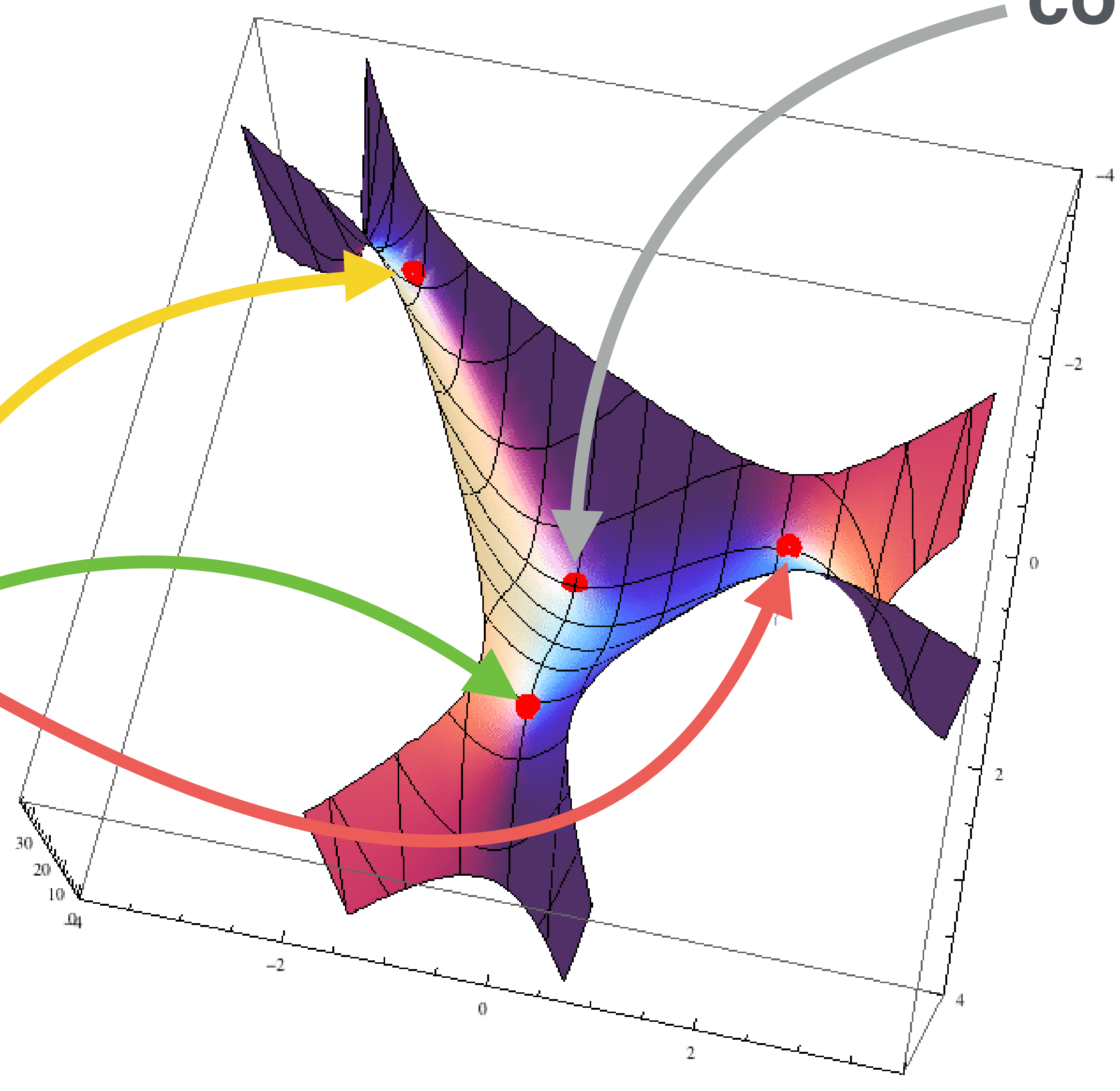
$\mathfrak{su}(3)$ case

$$\mathbf{X} = a\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{bmatrix} + b\begin{bmatrix}-2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

color singlet vacuum



colored vacua

$\mathfrak{su}(3)$ case

$$\mathbf{X} = a\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{bmatrix} + b\begin{bmatrix}-2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

# Rainbow Vacua

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

- extremum condition: $2\,\boldsymbol{X} + \boldsymbol{X}^2 = \dfrac{1}{N}\,\mathrm{Tr}\left(2\,\boldsymbol{X} + \boldsymbol{X}^2\right)\boldsymbol{I}$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

- extremum condition: $2\,\boldsymbol{X} + \boldsymbol{X}^2 = \frac{1}{N}\,\mathrm{Tr}\left(2\,\boldsymbol{X} + \boldsymbol{X}^2\right)\,\boldsymbol{I}$
- solutions up to SU(N) rotation:

$$\boldsymbol{X} = \frac{N}{\mathrm{Tr}(\boldsymbol{Z})}\,\boldsymbol{Z} - \boldsymbol{I} \qquad \boldsymbol{Z}_k = \begin{bmatrix} \boldsymbol{I}_{(N-k)\times(N-k)} & 0 \\ 0 & -\boldsymbol{I}_{k\times k} \end{bmatrix}$$

**Parameter** $\quad k = 0, 1, \ldots, \left[\frac{N-1}{2}\right]$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

- extremum condition: $2\,\boldsymbol{X} + \boldsymbol{X}^2 = \frac{1}{N}\,\mathrm{Tr}\left(2\,\boldsymbol{X} + \boldsymbol{X}^2\right)\boldsymbol{I}$
- solutions up to SU(N) rotation:

$$\boldsymbol{X} = \frac{N}{\mathrm{Tr}(\boldsymbol{Z})}\,\boldsymbol{Z} - \boldsymbol{I} \qquad \boldsymbol{Z}_k = \begin{bmatrix} \boldsymbol{I}_{(N-k)\times(N-k)} & 0 \\ 0 & -\boldsymbol{I}_{k\times k} \end{bmatrix}$$

**Parameter** $k = 0, 1, \ldots, \left[\frac{N-1}{2}\right]$

**Symmetry Breaking** $SU(N-k) \times SU(k) \times U(1)$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

- extremum condition: $2\,\boldsymbol{X} + \boldsymbol{X}^2 = \frac{1}{N}\,\mathrm{Tr}\left(2\,\boldsymbol{X} + \boldsymbol{X}^2\right)\,\boldsymbol{I}$
- solutions up to SU(N) rotation:

$$\boldsymbol{X} = \frac{N}{\mathrm{Tr}(\boldsymbol{Z})}\,\boldsymbol{Z} - \boldsymbol{I} \qquad \boldsymbol{Z}_k = \begin{bmatrix} \boldsymbol{I}_{(N-k)\times(N-k)} & 0 \\ 0 & -\boldsymbol{I}_{k\times k} \end{bmatrix}$$

**Parameter** $k = 0, 1, \ldots, \left[\frac{N-1}{2}\right]$

**Symmetry Breaking** $SU(N-k) \times SU(k) \times U(1)$

- extremum values: $V(\boldsymbol{X}_k) = -\frac{2\,\sigma}{\ell^2}\left(\frac{N}{N-2k}\right)^2$

# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

- extremum condition: $2\,\boldsymbol{X} + \boldsymbol{X}^2 = \frac{1}{N}\,\mathrm{Tr}\left(2\,\boldsymbol{X} + \boldsymbol{X}^2\right)\boldsymbol{I}$
- solutions up to SU(N) rotation:

$$\boldsymbol{X} = \frac{N}{\mathrm{Tr}(\boldsymbol{Z})}\,\boldsymbol{Z} - \boldsymbol{I} \qquad\qquad \boldsymbol{Z}_k = \begin{bmatrix} \boldsymbol{I}_{(N-k)\times(N-k)} & 0 \\ 0 & -\boldsymbol{I}_{k\times k} \end{bmatrix}$$

**Parameter** $k = 0, 1, \ldots, \left[\frac{N-1}{2}\right]$

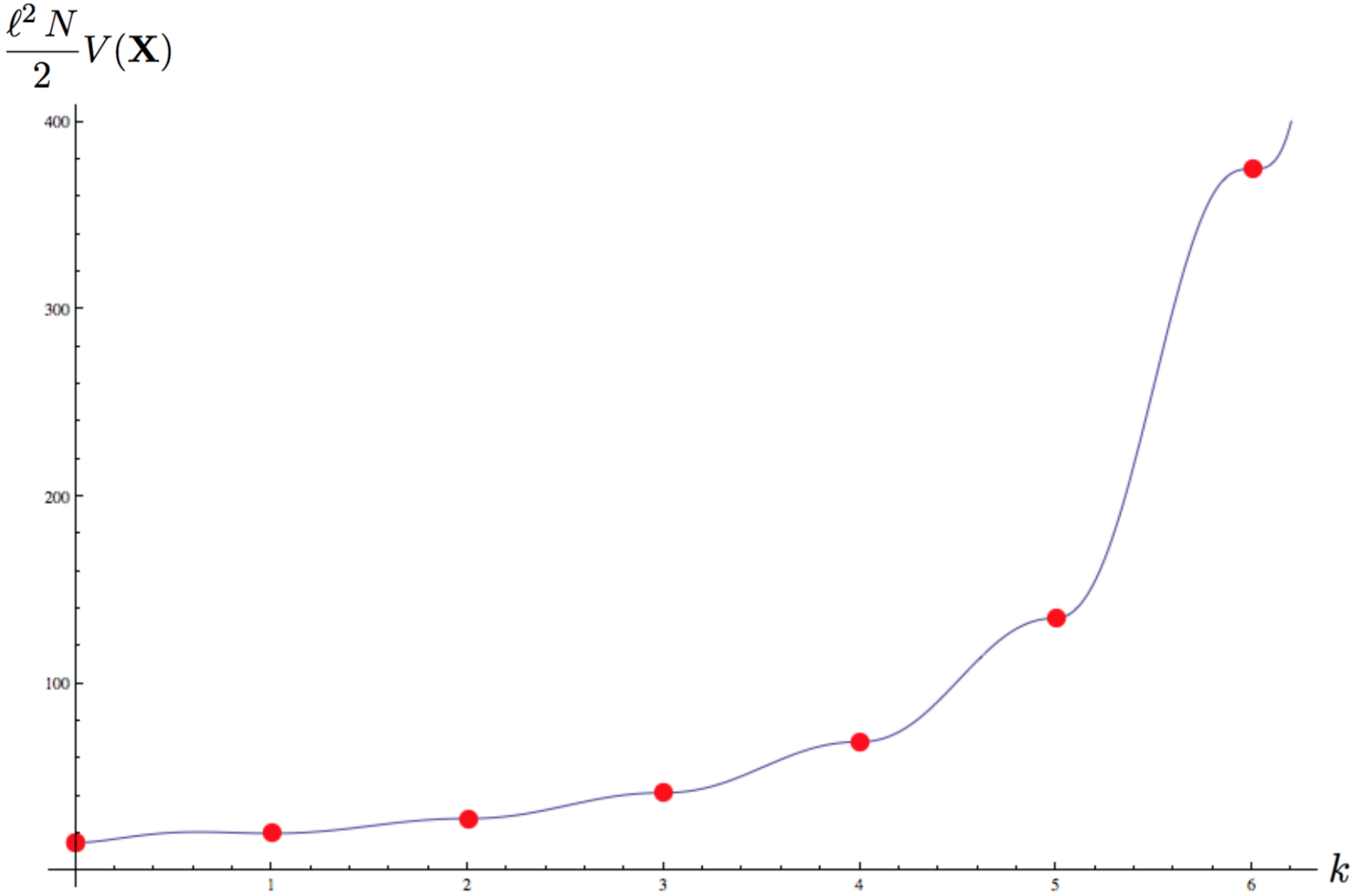
**Symmetry Breaking** $SU(N-k)\times SU(k)\times U(1)$

- extremum values: $V(\boldsymbol{X}_k) = -\frac{2\,\sigma}{\ell^2}\left(\frac{N}{N-2k}\right)^2 = 2\,\Lambda_k$

**Vacuum dependent Cosmological Constant!**

# Rainbow Vacua

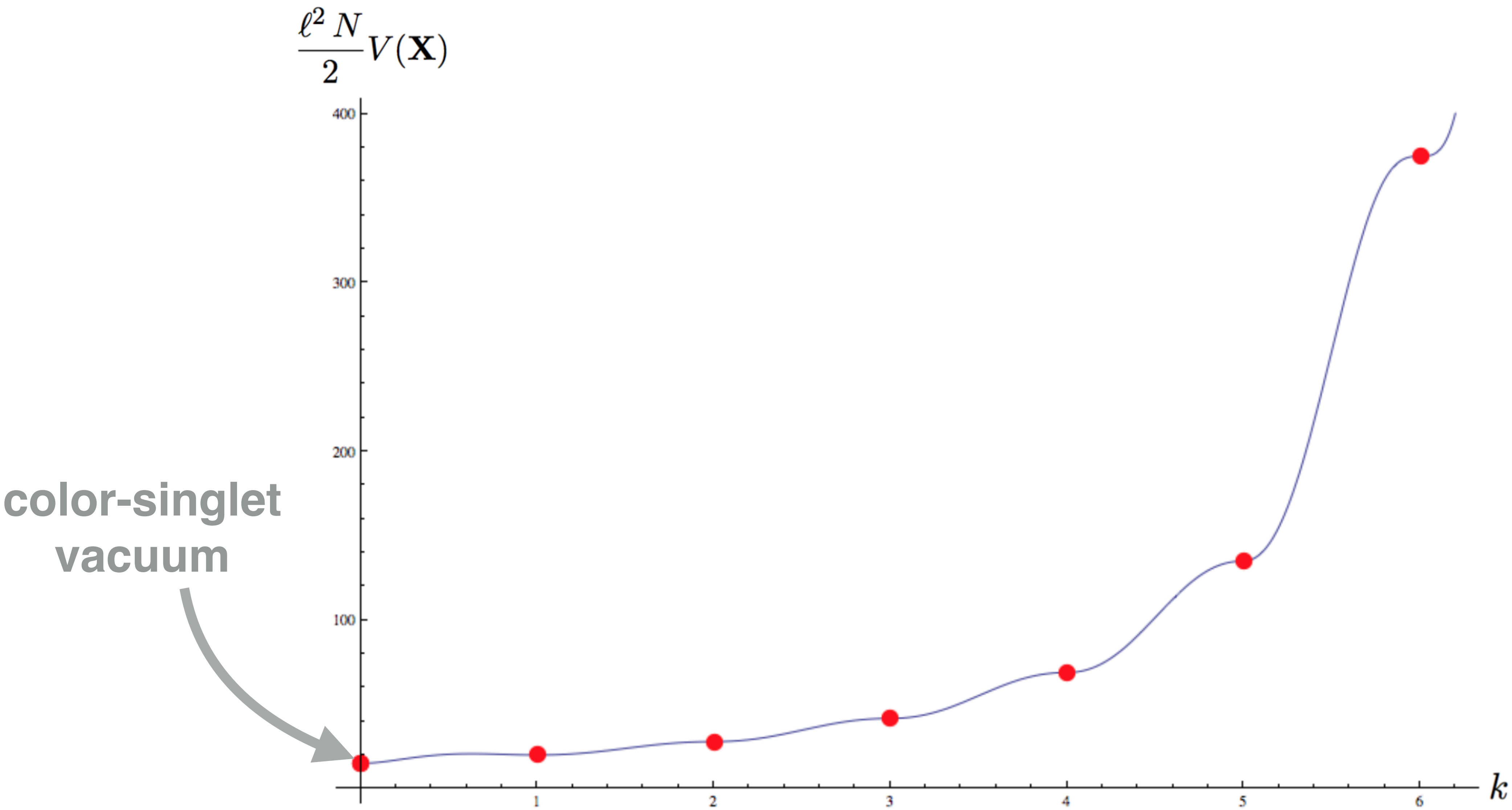
$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$




$N = 15$

# Rainbow Vacua

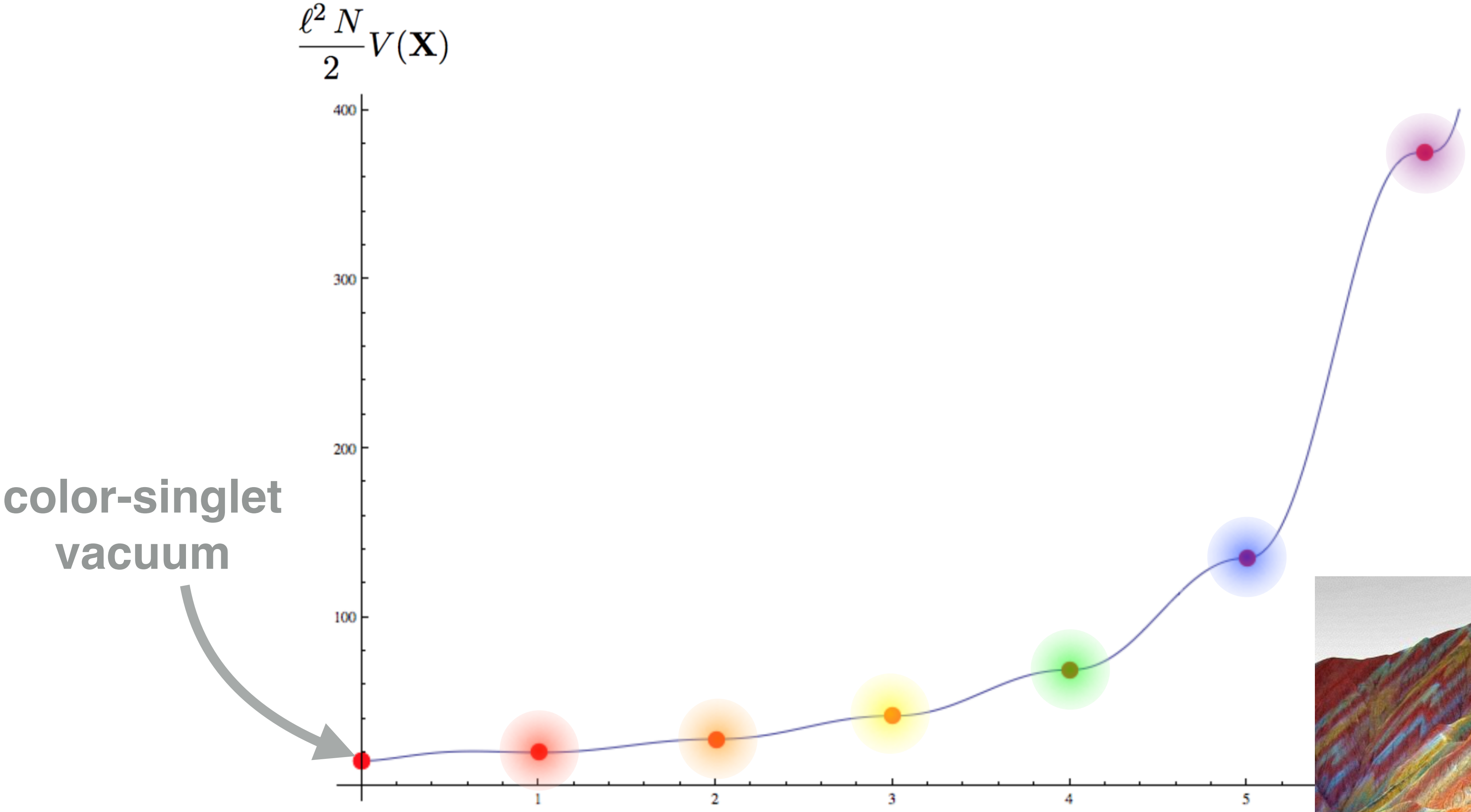
$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$




$N = 15$

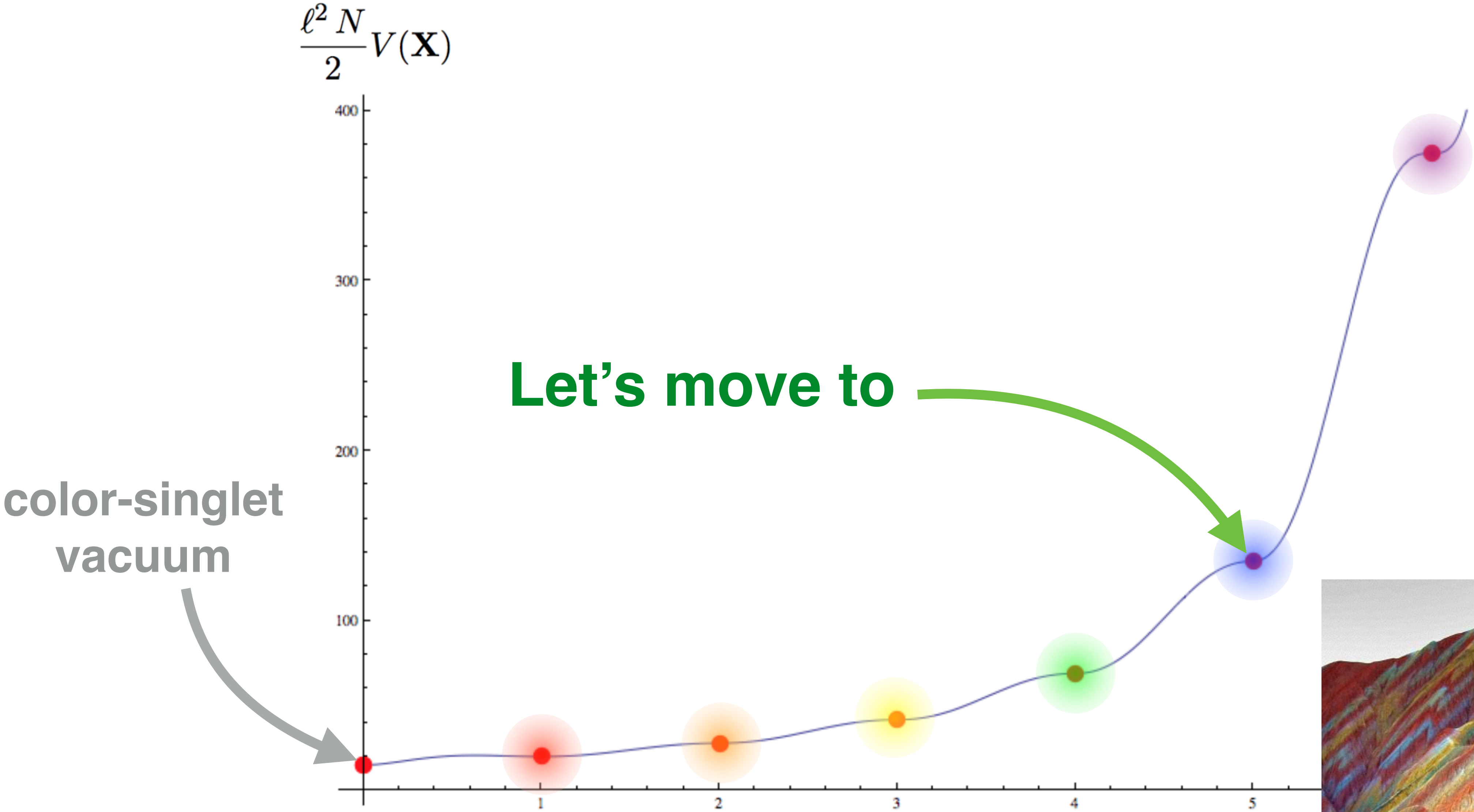
# Rainbow Vacua

$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

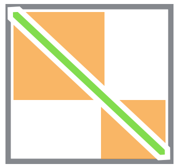


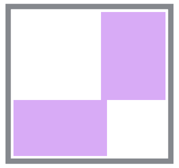

# Rainbow Vacua

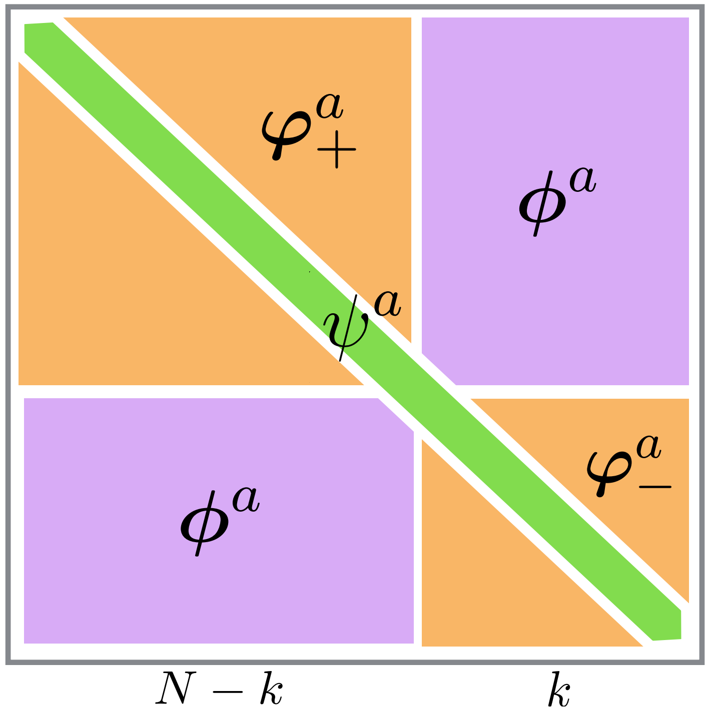
$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$



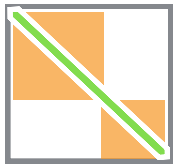
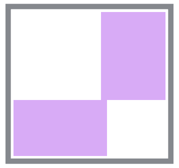

# Symmetry Breaking

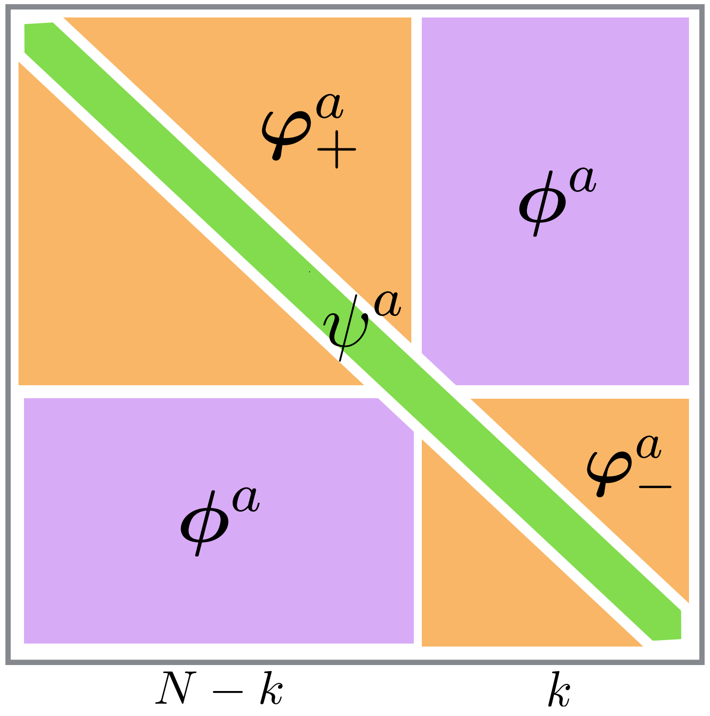
- Diagonal parts: 

- Broken-sym. part: 

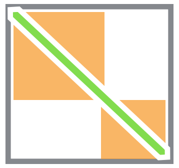
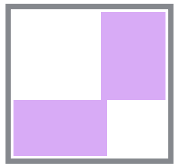


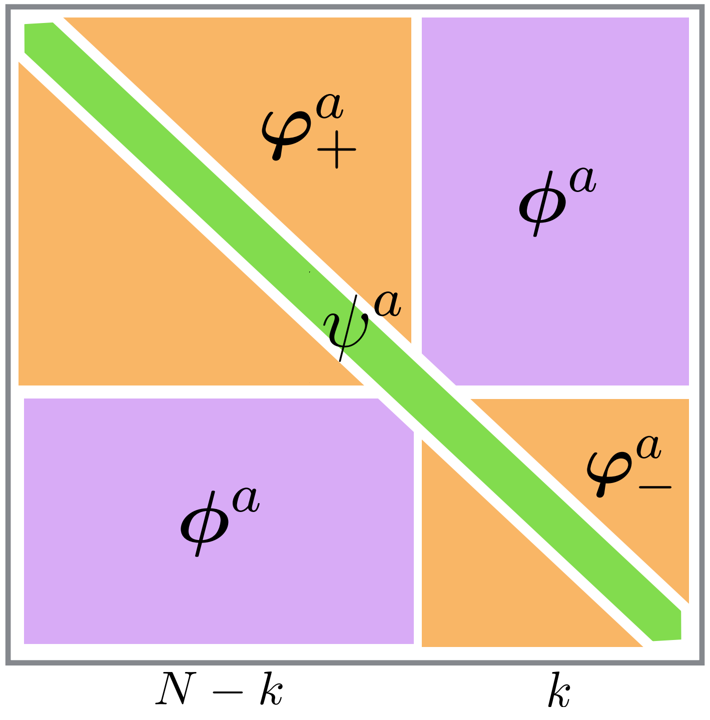
# Symmetry Breaking

- Diagonal parts: 
  - adjoint in $SU(N-k)\times SU(k)\times U(1)$
  - still describe massless spin-two
- Broken-sym. part: 

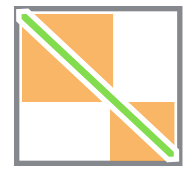
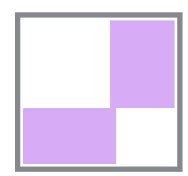


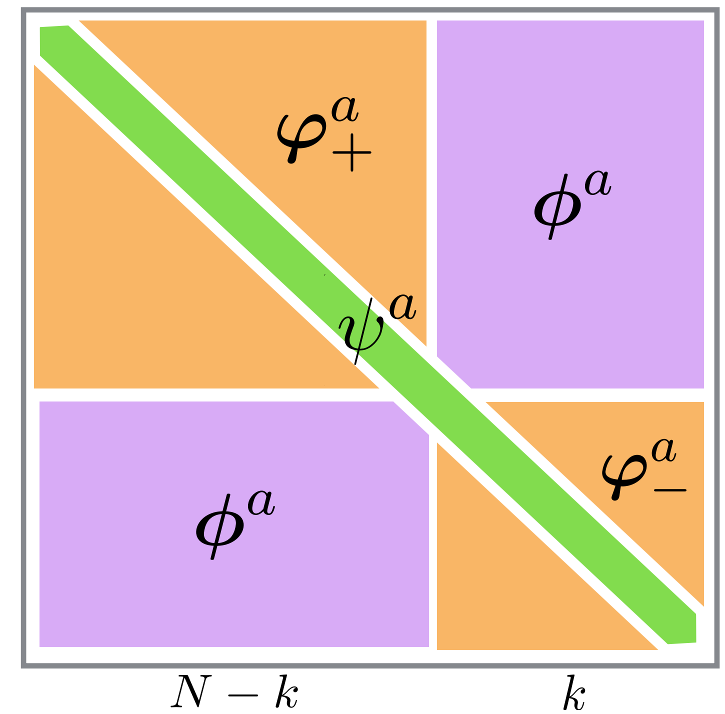
# Symmetry Breaking

- Diagonal parts: 
  - adjoint in $SU(N-k)\times SU(k)\times U(1)$
  - still describe massless spin-two
- Broken-sym. part: 
  - bi-fundamental



# Symmetry Breaking

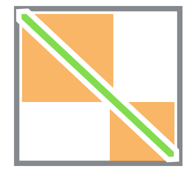
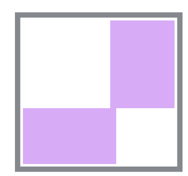
- Diagonal parts: 
  - adjoint in $SU(N-k) \times SU(k) \times U(1)$
  - still describe massless spin-two
- Broken-sym. part: 
  - bi-fundamental
  - combines with (or eaten by) spin-one field

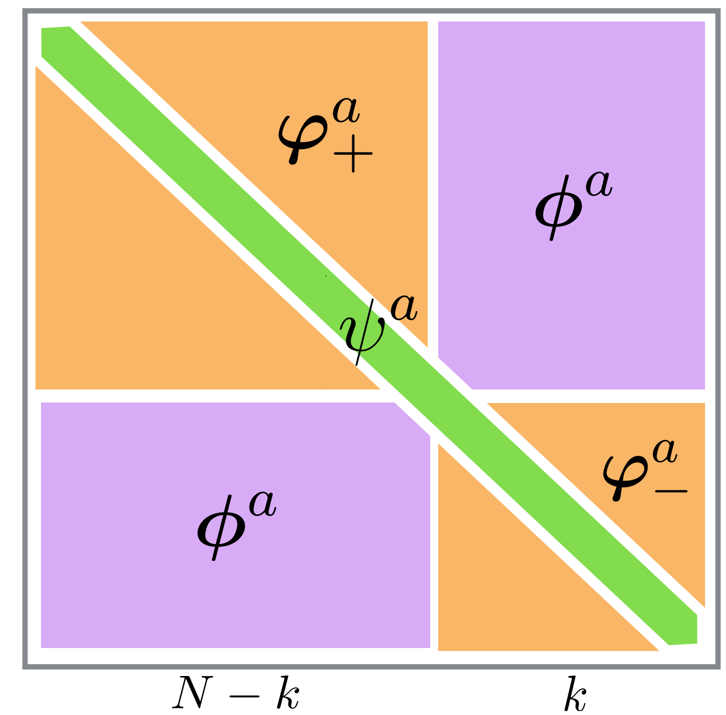



$$S_{\rm BS}[\phi, \phi^a] = \int \phi \wedge \left(d\phi - \frac{1}{\ell}\, e_a \wedge \phi^a\right) - \phi_a \wedge \left(D\phi^a - \frac{1}{\ell}\, e^a \wedge \phi\right)$$

**Higgs-like Mechanism!**

# Symmetry Breaking

- Diagonal parts: 
  - adjoint in $SU(N-k)\times SU(k)\times U(1)$
  - still describe massless spin-two
- Broken-sym. part: 
  - bi-fundamental
  - combines with (or eaten by) spin-one field

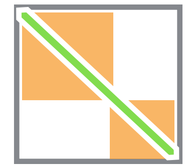
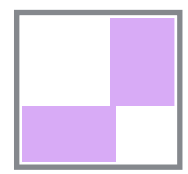


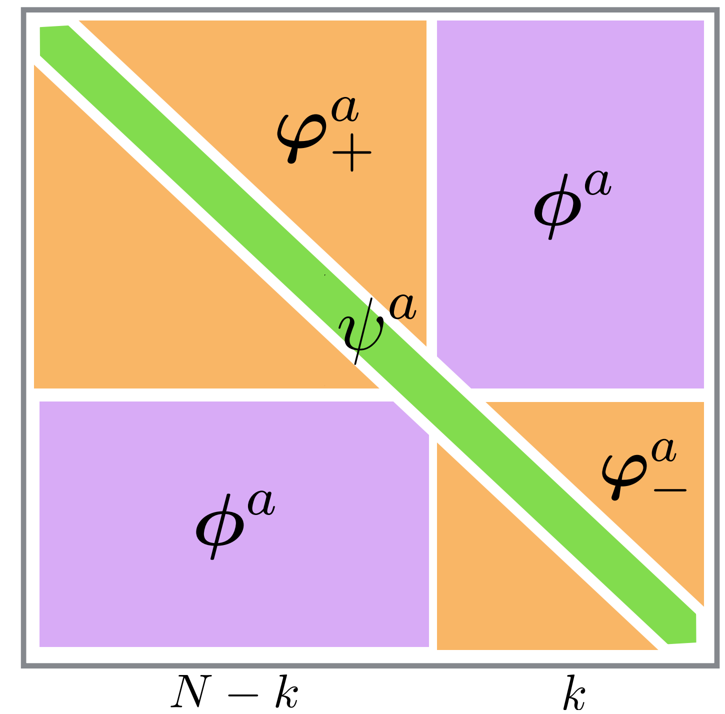

$$S_{\rm BS}[\phi,\phi^a]=\int \phi\wedge\left(d\phi-\frac{1}{\ell}\,e_a\wedge\phi^a\right)-\phi_a\wedge\left(D\phi^a-\frac{1}{\ell}\,e^a\wedge\phi\right)$$

**Higgs-like Mechanism!**

  - describe **partially-massless** spin-two

# Symmetry Breaking

- Diagonal parts: 
  - adjoint in $SU(N-k)\times SU(k)\times U(1)$
  - still describe massless spin-two
- Broken-sym. part: 
  - bi-fundamental
  - combines with (or eaten by) spin-one field




$$S_{\rm BS}[\phi,\phi^a]=\int \phi\wedge\left(d\phi-\frac{1}{\ell}\,e_a\wedge\phi^a\right)-\phi_a\wedge\left(D\phi^a-\frac{1}{\ell}\,e^a\wedge\phi\right)$$

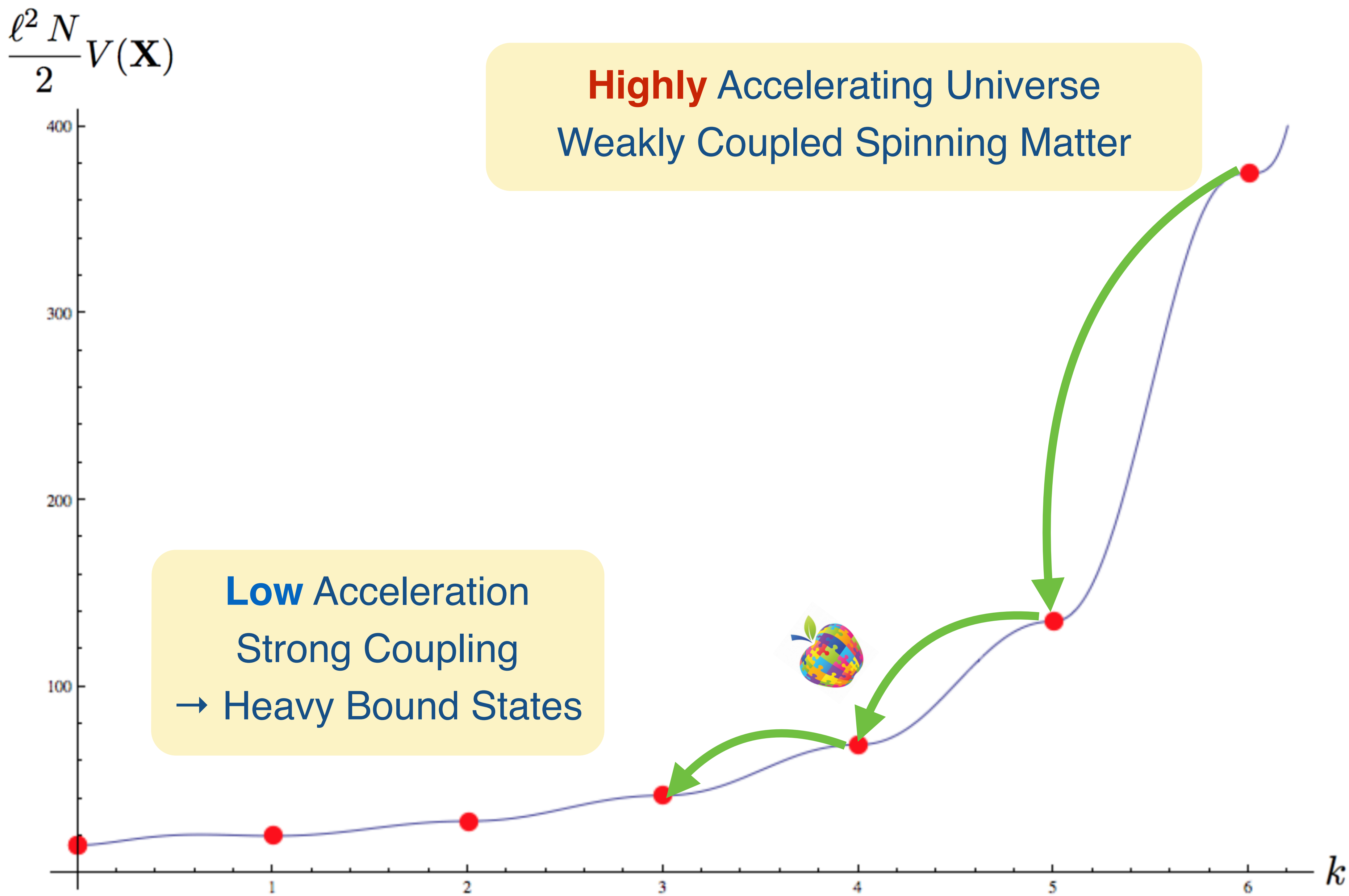
**Higgs-like Mechanism!**

  - describe **partially-massless** spin-two

- **All Weakly Interacting for Large k~N/2**

Speculation 1

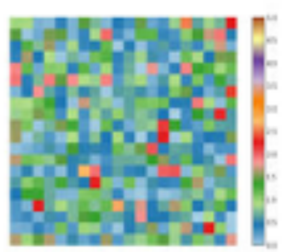
# Cosmological Scenario



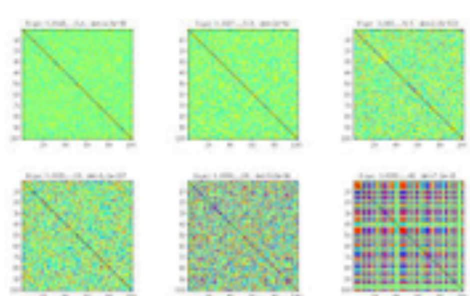
# Quantum Colored Gravity

- Rainbow vacua contribution in the path integral:

**Random Matrix Model**



$$\mathcal{Z}_{\mathrm{MM}} = \int d\boldsymbol{X}\ \exp\left[i\, c\, V(\boldsymbol{X})\right]$$



$$V(\boldsymbol{X}) = -\frac{2\,\sigma}{N\,\ell^2}\,\mathrm{Tr}\left(\boldsymbol{I} + 3\,\boldsymbol{X}^2 + \boldsymbol{X}^3\right)$$

# HS extension

- Color-decoration & Rainbow vacua extend to
  - ✓ 3D CS formulation of HS Gravity
  - ✓ Vasiliev Equations [to appear]

# HS extension

- Color-decoration & Rainbow vacua extend to

  ✓ 3D CS formulation of HS Gravity

  ✓ Vasiliev Equations [to appear]

- Resulting spectrum after symmetry breaking

  ▸ all the spins glue together to form an exotic one

# Thank you

BRITISH LION
Presents
MONTE HALE
ADRIAN BOOTH
with
THE SAGEBRUSH SERENADERS
and featuring
OUTLAW-THE WILD HORSE
THE First OF A NEW SERIES
OF COLOUR WESTERNS
FROM REPUBLIC
IN
MAGNACOLOR
Greater THRILLS!
Greater ACTION!
Greater SPECTACLE!
GRAVITY from RAINBOW VALLEY
TRADE SHOW: AUG. 29
STUDIO ONE
A MAGNACOLOR PRODUCTION