Higher-Spin symmetries: exact and broken

HSTH: Friends and Higher Spins

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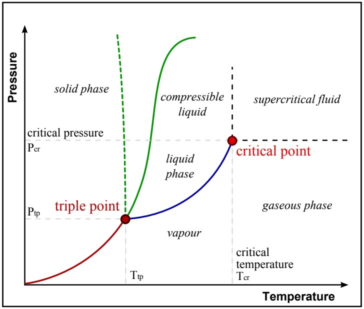
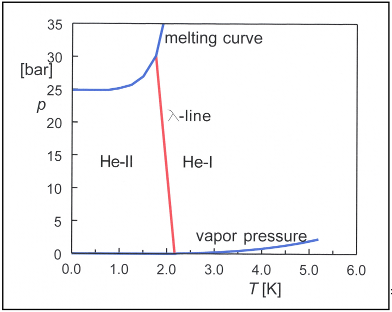
HS algebra is ∞-dim. extension of conformal symmetry and it is a crucial ingredient of the HS theory. By the AdS/CFT reasoning it should play a fundamental role on the CFT side as well, the fact that remains obscure.

HS symmetry, when unbroken, is a signature of a free CFT. It becomes useful when it is broken (Anselmi). The breaking of HS symmetry is also important for the consistency of the HS theories as Quantum Gravities.

The plan is to discuss HS breaking and relate it to the critical indices.

4/3-duality





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Wilson-Fischer CFT's are known to describe many of the second-order phase transitions in $3d$ for small $N \leq 5$. Klebanov and Polyakov at large-N told us these are AdS/CFT dual to some QG, which is $4d$ HS theory (no SUSY and extra dimensions required, but there are examples also with SUSY and in $d > 4$). For another b.c. HS theory is dual to free CFT's (Sundborg; Konstein, Vasiliev, Zaikin; Sezgin-Sundell).

*The pictures by Matthieumarechal and SliteWrite are taken from Wikipedia

Nice features of WF CFT

- one of the few that describe real physics;
- free of any free parameters, e.g. coupling constants etc.;
- provide infinitely many of measurable (calculable) observables — critical indices

$$\langle OO \rangle \sim \frac{1}{x^{2\Delta_O}} \qquad \Delta_O = \text{free} + \text{anomalous}$$

- are well-defined QFT's according to mathematical standards of rigour;
- can be used to define some QG — HS theories;
- almost have ∞-dim. symmetry — HS symmetry, which is realized by

$$\partial^m J_{ma(s-1)} = \text{something}$$

Simplest implications of unbroken HS symmetry

$2d$ CFT's are governed by Virasoro, e.g. decoupling of $L_{-2} + \alpha L_{-1}^2$ imposed on $\langle O_\Delta O_{\Delta_1} O_{\Delta_2} \rangle$ relates Δ, Δ_1 and Δ_2

With HS symmetry $L_{-2} + \alpha L_{-1}^2$ gets replaced by ∂^ν, so the signature of HS symmetry is the presence of HS currents

$$\boxed{\partial^b J_{ba(s-1)} = 0}$$

Simplest three-point functions already tell something:

$$\langle j_s O_{\Delta_1} O_{\Delta_2} \rangle \qquad \Delta_1 = \Delta_2,\ s = 1, 2, \ldots$$

$$\langle j_s j_{s'} O_\Delta \rangle \qquad \Delta = 2\frac{d-2}{2}$$

The latter suggests $O = \phi^2$ for free ϕ. More nontrivial info is in OPE and Ward identities.

Not HS Ward identities

The conformal symmetry Ward identity reads:

$$-\int dS^{\underline{m}}\,\xi^a\langle J_{\underline{m}a}OO\rangle=\partial_a\langle OO\rangle$$

where ξ^a is a conformal Killing vector. In the differential form it is clear that the distribution is not well-defined and has to be regularized.

The Ward identity allows to recover the three-point functions from the two-point functions. Similar statement is true for the global $O(N)$ symmetry Ward identity.

The central charges can be read off from the two-point functions:

$$\langle TT\rangle=\frac{C_T}{S_d^2}\times\text{standard structure}$$

HS Ward identities

HS current can be used to construct the actual current by contracting it with a Killing tensor (applies for $s = 2$ as well)

$$j_{\underline{m}}(v) = J_{\underline{m}a(s-1)}v^{a(s-1)} \qquad \partial^a v^{a(s-1)} - \text{traces} = 0$$

The simplest Killing tensor is a constant that induces

$$\delta\phi = v^{a(s-1)}\partial_a ... \partial_a \phi$$

The simplest Ward identity for these 'hyper-translations' is

$$-\int dS^{\underline{m}}\langle J_{\underline{m}a(s-1)} OO\rangle = \partial_a ... \partial_a \langle OO\rangle$$

and, as in $s = 1$ and $s = 2$ cases, fixes the coupling constant in

$$\langle J_{a(s)} OO\rangle \sim g_{s00} \times \text{standard structure}$$

Step into AdS

The normalization of correlators is ambiguous, the only invariant for $00s$ is (HS algebra):

$$I_{s00} = \frac{\langle J_s O O\rangle^2}{\langle J_s J_s\rangle\langle O O\rangle^2}$$

Using the known result from AdS/CFT (Penedones et al)

$$g_{s00}\int \Phi^{a(s)}\phi\nabla_{a(s)}\phi \sim \langle J_s O O\rangle$$

the four (Bekaert et al) found the coupling constant

$$g_{s00}\sim\frac{1}{\Gamma[s]}$$

The general result was foreseen back in 1990 by Ruslan

$$g_{s_1 s_2 s_3}\sim\frac{1}{(s_1+s_2+s_3-1)!}$$

OPE—representation—algebra

The OPE of two HS currents is difficult to study

$$J_{s_1} J_{s_2} = O_2 + \sum_s J_s + \delta_{s_1,s_2} \langle JJ \rangle$$

If we integrate it once we get the action of charges on J

$$[Q_{s_1}, J_{s_2}] = \sum_s J_s$$

If we integrate one more time, we get the algebra

$$[Q_{s_1}, Q_{s_2}] = \sum_s Q_s$$

We assume stress-tensor J_2 and at least one HS current $J_{s>2}$.

Implications of unbroken HS symmetry

It can be proved via $[Q, J] = J$ or the Jacobi identity $[Q, [Q, Q]] = 0$, (Fradkin, Vasiliev; Anselmi; Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S, Taronna; Alba, Diab; Stanev), that

(i) the presence of at least one HS charge requires all even at least, $s = 2, 4, 6, ...$;
(ii) the algebra is unique* in $d > 2$ except for $4d$;
(iii) in $4d$ there is a one-parameter family (Fradkin, Linetsky; Gunaydin; Boulanger, E.S.; Mkrtchyan^2, Manvelyan, Theisen);
(iv) the algebra can always be associated with a free field (not necessary scalar, e.g. $4d$ Maxwell)

$$\boxed{\text{unbroken HS} = \text{free CFT}}$$

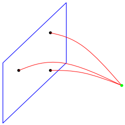
The algebra of HS charges is what is gauged in HS theory (global symmetry of CFT = gauge symmetry in AdS).

From exact to broken

Free CFT's are still nontrivial and should be dual to HS theories with b.c. preserving full HS symmetry. Still useful for AdS/CFT as an example of QG without extra dims and SUSY, but without post-Newtonian limit.

The correlators of single-trace operators are nontrivial and the dual theory is supposed to be highly nonlinear

$$\langle J ... J \rangle \neq 0 \quad \Longleftrightarrow \quad \int_{AdS} \Phi ... \Phi \neq 0$$



if quartic is needed (Bekaert et al).

The HS symmetry is broken in Wilson-Fischer CFT's. The HS currents have small anomalous dimensions. It was argued (Giombi, Yin; Hartman, Rastelli) that the duality to critical model follows from the free case order by order in $1/N$.

General comments

Physics tells us that HS currents are anomalous. Usually people study the anom. dim. of lowest weight operators ϕ^i (4,5,6 loops) or leading twist (QCD).

In $2d$ HS currents are conserved. In $d > 2$ more interesting CFT's have to break HS symmetry in a controllable way Anselmi. Slightly broken HS symmetry still constrains correlators (Maldacena, Zhiboedov) and is useful for small N as well (Alday, Zhiboedov). It is crucial to understand what is the small parameter.

Thanks to the renessaince of the bootstrap program there has been some analytical progress as well, (Maldacena, Zhiboedov; Alday, Zhiboedov; Rychkov, Tan; Manashov, Strohmaier; ...)

HS Breaking

One can think of two mechanisms, each can be considered on either CFT or AdS side.

3-names	CFT side	AdS side
BEH	via other single-trace operators, $\partial J = gO_1$, (Sundborg; Bianchi)	massless absorbs massive, $(s,m)\|_{m\to 0} = (s,0) \oplus (s\text{-}1, M)$ (Zinoviev; Bianchi)
GPZ	via double-, triple-trace operators, $\partial \cdot J = gJJ$	radiative corrections?

In the former case breaking is impossible without appending pure HS theory with some matter like fields. In the latter case the fate of gauge invariance is unclear.

The e.o.m. are still useful at the quantum level. Given

$$\langle \phi^i \phi^j \rangle = \delta^{ij} \frac{C_{\phi\phi}}{(x_{12}^2)^{\Delta}_{\phi}}$$

we can apply box twice to get, $\Delta = (d-2)/2 + \gamma$,

$$\Box_1 \Box_2 \langle \phi\phi \rangle = 4\gamma(\gamma+1)(2\gamma+d-2)(2\gamma+d) \frac{C_{\phi\phi}}{(x_{12}^2)^{\Delta_\phi+2}}$$

For the large-N vector model with $\Box\phi = \sigma\phi$:

$$C_{\sigma\sigma} g_\star^2 = 4d(d-2)\gamma_\phi$$

For the Wilson-Fisher $4-2\epsilon$ with $\Box\phi = g_\star \phi^2 \phi$:

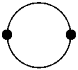
$$\gamma_\phi = \frac{(N+2)g_\star^2}{256\pi^4}$$

For the large-N vector model

$$S = \int \frac{1}{2}\partial\phi^2 + \frac{1}{2}(\phi^2)\sigma + \frac{3N}{2\lambda}\sigma^2 \qquad \rightarrow \qquad \Box\phi^i = \phi^i\sigma$$

we have $\Box\phi = \sigma\phi$ and

$$\langle\phi\phi\rangle = \frac{\Gamma[d/2-1]}{2\pi^{d/2}}\frac{1}{x^{d-2}}$$

$\langle\sigma\sigma\rangle = \left[-\frac{1}{2}\right.$  $\left.\right]^{-1} = \dfrac{2^{d+2}\Gamma\left[\frac{d-1}{2}\right]\mathrm{Sin}\left[\pi\frac{d}{2}\right]}{\pi^{3/2}\Gamma[d/2-2]x^4}$

$\gamma_\phi =$  $= \dfrac{2\mathrm{Sin}[\pi\frac{d}{2}]\Gamma[d-2]}{\pi\Gamma[d/2+1]\Gamma[d/2-2]}$

Combining the two quantities we see that

$$C_{\sigma\sigma} = 4d(d-2)\gamma_\phi$$

Anomalous dimensions: Anselmi's trick

Trivial identity — check of non-conservation

$$\langle \mathbb{D}_1 J_s(x_1) \mathbb{D}_2 J_s(x_2)\rangle = \mathbb{D}_1\mathbb{D}_2 \langle J_s(x_1) J_s(x_2)\rangle$$

can give important information provided the two sides can be computed independently. Let J be anomalous

$$\langle J_s(x_1,\eta_1) J_s(x_2,\eta_2)\rangle = \frac{C_s}{\mu^{2\gamma}(x_{12}^2)^{d+s-2+\gamma}}(P_{12})^s$$

and the non-conservation be via double-trace operators

$$K = \mathbb{D} J = g_\star JJ \qquad\qquad g_\star \sim \frac{1}{N}\,,\epsilon$$

The ratio gains $g_\star^2$ on the left and γ on the right

$$g_\star^2 \frac{\langle KK\rangle}{\langle JJ\rangle} \sim \gamma$$

Non-conservation

The conserved currents are produced by Gegenbauer polynomials. More precisely $\phi\partial^s\phi$ is (Todorov et al)

$$J = (\hat{\partial}_1 + \hat{\partial}_2)^s C_s^{\frac{d-3}{2}} \left(\frac{\hat{\partial}_1 - \hat{\partial}_2}{\hat{\partial}_1 + \hat{\partial}_2}\right) \phi(x_1)\phi(x_2)\Big|_{x_1=x_2=x}$$

where $\hat{\partial}_i = \xi \cdot \partial_i$. If $\Box\phi = g_\star\phi\phi^2$ then we get instead

$$\partial \cdot J = g_\star(\hat{\partial}_1 + \hat{\partial}_2 + \hat{\partial}_3 + \hat{\partial}_4)^{s-1} K(u, v)\, \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\Big|_{x_i=x}$$

where the point-splitting arguments are

$$u = \frac{\hat{\partial}_1 - \hat{\partial} + \hat{\partial}_3 + \hat{\partial}_4}{\hat{\partial}_1 + \hat{\partial}_2 + \hat{\partial}_3 + \hat{\partial}_4} \qquad v = \frac{\partial_1 - \hat{\partial}_2 - \hat{\partial}_3 - \hat{\partial}_4}{\hat{\partial}_1 + \hat{\partial}_2 + \hat{\partial}_3 + \hat{\partial}_4}$$

and K is a certain combination of Gegenbauer polynomials.

Wilson-Fisher in $4-2\epsilon$

For the Wilson-Fischer CFT defined by

$$S=\int \frac{1}{2}\partial\phi^2+\frac{g}{4}\mu^{2\epsilon}(\phi^2)^2 \qquad \rightarrow \qquad \Box\phi^i=g_\star\phi^i\phi^2$$

anomalous dim. of non-singlet currents $\phi^i\partial^s\phi^j$ are

$$\gamma^{\square\square}=2\gamma_\phi\left(1-\frac{2(N+6)}{(N+2)s(s+1)}\right) \quad \gamma^{\begin{smallmatrix}\square\\\square\end{smallmatrix}}=2\gamma_\phi\left(1-\frac{2}{s(s+1)}\right)$$

and anomalous dimensions the singlet currents $\phi^i\partial^s\phi_i$ are

$$\gamma=2\gamma_\phi\left(1-\frac{6}{s(s+1)}\right)$$

which vanishes for the stress-tensor $s=2$. This requires one-loop, see Wilson, Kogut

Scalar QCD or $6d$ ϕ^3

Toy model of QCD: asymptotically free theory in $6d$

$$S = \int \frac{1}{2}\partial\phi^2 + \partial\bar{\psi}\partial\psi + g\bar{\psi}\phi\psi$$

The anomalous dimensions of the non-singlet currents are

$$\gamma_1 = \frac{(s-1)(s+4)}{24(s+1)(s+2)} = \frac{1}{4}\left(\frac{1}{6} - \frac{1}{(s+1)(s+2)}\right)$$

which agrees with (Kubota; Radyshkin; Belitsky et al) and vanishes for $s=1$ since it is a protected current that generates global $O(N)$-symmetry.

Large-N Vector-Model

Klebanov-Polyakov duality is based on the large-N of

$$S = \int \frac{1}{2}\partial\phi^2 + \frac{1}{2}(\phi^2)\sigma + \frac{3N}{2\lambda}\sigma^2 \qquad \rightarrow \qquad \Box\phi^i = \phi^i\sigma$$

The anomalous dimensions of the non-singlet currents are

$$\gamma_s = \frac{8(s-1)(d+s-2)}{(d+2s-4)(d+2s-2)}$$

which agrees with (Lang, Ruhl) and vanishes for $s=1$ since it is a protected current generating global $O(N)$-symmetry. That of the singlet currents is a bit trickier and in $3d$ is:

$$\gamma_s = 4\gamma_\phi \frac{(s-2)}{(2s-1)}$$

HS algebra interpretation

Anomalous dimensions to the one-loop order are ratios:

$$c_s\gamma_s = \frac{\langle K|K\rangle}{\langle J|J\rangle} = \frac{\langle J\otimes J|J\otimes J\rangle}{\langle J|J\rangle}$$

The two-point functions are related to the trace on the HS algebra. The non-conservation operator K belongs to the tensor square of the HS algebra representation $J\sim\phi\otimes\phi$. Therefore, to find K we need to implicitly know the Clebsh-Gordon coefficients of the HS algebra.

The kinematical factor c_s comes from the HS algebra of generalized free fields (Eastwood; Alkalaev, Grigoriev, E.S.) at $\Delta=(d-2)/2+\epsilon$ when the ideal is formed:

$$U\left(\ydiagram{1,1}\right) \Big/ \left[\ydiagram{1,1,1,1} \oplus \left(C_2 - C_2(\Delta)\right)\right] \qquad\qquad I\sim\ydiagram{2}$$

HS theory in $5-\epsilon$

Wilson-Fisher in $4-\epsilon$ should be dual to HS theory in $AdS_{5-\epsilon}$.

The mass shift of the HS fields in $AdS_{5-\epsilon}$ HS theory has to be

$$\boxed{\delta m^2 = -2(s-2)\epsilon + 2\epsilon^2\gamma_\phi\left(1-\frac{6}{s(s+1)}\right)}$$

Since the massless fields should acquire a mass, there can be a problem with gauge invariance. It is not the case when the massless spin-s field recombines with a massive spin-$(s-1)$ field to form a massive spin-s field, (Bianchi). At the free level the action proposed by Zinoviev does the job

$$S = S_{s,m=0} + \delta m^2 \int \Phi_s \partial \Phi_{s-1} + S_{s-1,M}$$

In the $1/N$-expansion the situation is analogous, with $\delta m^2 \sim (s-2)/N$, Ruhl.

Adding HS quarks to HS gluons

Scalar, Maxwell, Weyl tensor and HS analogs thereof can be described all at once by a simple equation (Vasiliev):

$$\widetilde{\mathrm{D}}C = 0 \qquad\qquad C(Y) = C + C_{\alpha\alpha} + C_{\alpha(4)} + \dots$$

where Y^A are two pairs of canonical operators

$$[Y^A, Y^B] = C^{AB} \qquad\qquad \{Y^A, Y^B\} \sim sp(4)$$

By extending $Y^A \to Y^A_i$ (Gelfond, Vasiliev) one can describe the duals of the double-, triple-, etc. operators via the same type of equations. Moreover, the correlations functions can be computed to the lowest order by the same trick of traces

$$\langle O_n \dots O_n \rangle = tr[C_n \star \dots \star C_n]$$

This makes the spectrum of HS theory a bit more stringy and should solve the mass shift problem provided a non-linear completion exists.

Conclusions

Unbroken HS symmetry is very rigid, unique and corresponds to free fields. Everything is just HS algebra rep. theory.

There are several ways to break HS symmetry as to get access to interesting CFT's. The most promising is the quantum breaking via double-trace operators.

The study of almost classical equations of motion allows to determine all one-loop anomalous dimensions. The method works nicely for all $CFT's$ in diverse dimensions that were treated by distinct methods. One-loop again can be explained by HS algebras. What is the right structure?

To make HS theory more stringy we can add matter fields (quarks) to HS gauge fields (gluons) that correspond to double-, triple-, etc. operators. They can supply additional modes as to account for the mass shift.