Pseudo-local Theories on AdS: A Functional Class Proposal

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HS-holography: Status & Motivations

Singlet Sector of O(N)/U(n) Vector Model $\Longleftrightarrow$ Bosonic Vasiliev's theory

Klebanov-Polyakov, Sezgin-Sundell (2002)

All known HS theories are pseudo-local also at cubic order(!)

- String Theory
- Vasiliev's equations

$$e^{\Box}\,,\quad\star$$

Cubic couplings have been classified by Metsaev and they are local

- Metsaev results extends to AdS via ambient space: AdS couplings have still bounded number of derivatives

$$\#\partial \leq s_1 + s_2 + s_3$$

Joung & M.T. 2012

Goals & Plan

The Goal:

$$\Box\Phi_{\underline{m}(s)} + \ldots = J_{\underline{m}(s)}(\Phi, \Phi)$$

- Study current interactions in HS theories
- Propose a functional class criterion that allows to deal with pseudo-local currents arising in string theory and Vasiliev's equations

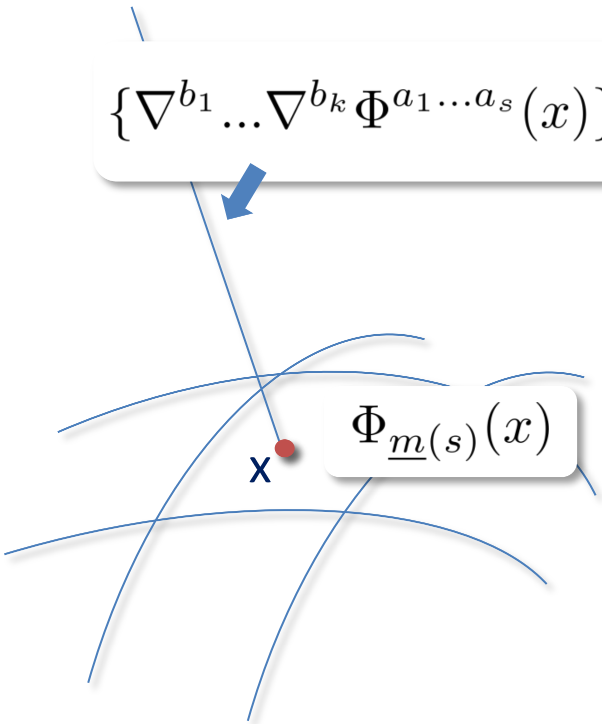
$$\Box\Phi_{\underline{m}(s)} + \ldots = \sum_{k,l} a_{k,l}\Lambda^{-l}\nabla_{\underline{m}(s-k)\underline{n}(l)}\Phi\,\nabla_{\underline{m}(k)}{}^{\underline{n}(l)}\Phi + \ldots$$

We want to understand when there exists a "Metsaev" field frame for a pseudo-local theory at the cubic order

Plan of the talk:

- Unfolding and Jet space
- Pseudo-local currents and the functional class proposal
- Some checks of the proposal

Unfolding and Jet space



Jet Bundle: each field and all of its derivatives as independent coordinates

It can be decomposed as:

- Gauge covariant components
- Gauge dependent components
- Components set to zero on the on-shell surface

- Gauge dependent components encoded into 1-forms: $\delta\omega = d\Lambda + \ldots$
- Gauge covariant components encoded into 0-forms

$e^a\,,\omega^{ab}$ (frame-like)

$W_{\mu\nu\rho\sigma}\,,\nabla_\rho W_{\mu\nu\rho\sigma}\,,\ldots$

(Weyl and derivatives)

1-forms & 0-forms (4d)

$$so(3,2) \sim sp(4,\mathbb{R}) \quad \longrightarrow \quad [\hat{y}^\alpha, \hat{y}^\beta] = 2i\epsilon^{\alpha\beta} \qquad [\hat{\bar{y}}^{\dot\alpha}, \hat{\bar{y}}^{\dot\beta}] = 2i\epsilon^{\dot\alpha\dot\beta}$$

- Gauge dependent component are represented by 1-forms (Vasiliev's 1980):

$$\omega^{(s)}(y,\bar{y}|x) = \sum_{k=0}^{s-1} \frac{1}{(s-1+k)!(s-1-k)!}\,\omega_{\alpha(s-1+k)\dot\alpha(s-1-k)}(x)\,y^{\alpha(s-1+k)}\bar{y}^{\dot\alpha(s-1-k)}$$

$$\omega_{\alpha(s-1+k)\dot\alpha(s-1-k)} \sim \nabla^{k<s}\Phi_{\alpha(s)}$$

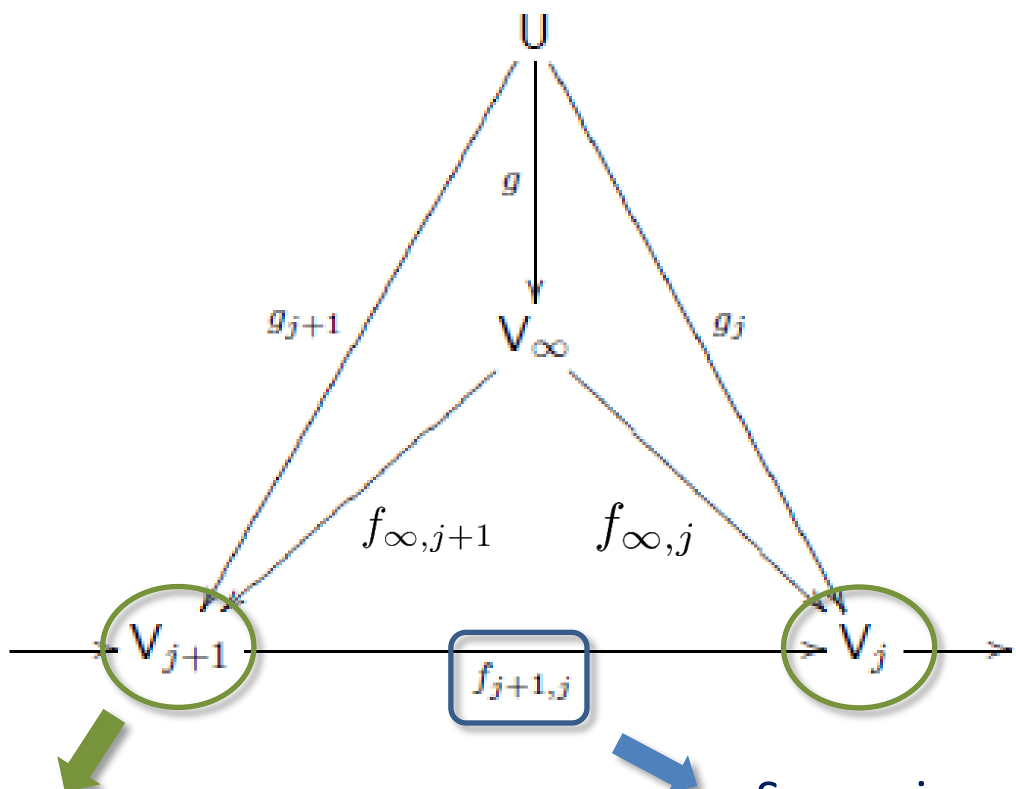
- The gauge covariant components are (anti-)selfdual HS Weyl tensors + the scalar (Weyl module)

$$C^{(s)}(y,\bar{y}|x) = \sum_{k=0}^{\infty} \frac{1}{(s+k)!k!}\,C_{\alpha(s+k)\dot\alpha(k)}(x)\,y^{\alpha(s+k)}\bar{y}^{\dot\alpha(k)} \qquad C_{\alpha(s+k)\dot\alpha(k)} \sim \nabla^k C_{\alpha(s)}$$

$$C(y,\bar{y}) \sim J^\infty$$

Pseudo-Locality & Inverse Limit

Pseudo-locality can be controlled by a formal construction in Category theory which is *“Inverse Limit”*



Space of “local functionals” with at most j+1/j derivatives

Some given linear continuous maps to be specified.

Pseudo-Locality & Inverse Limit

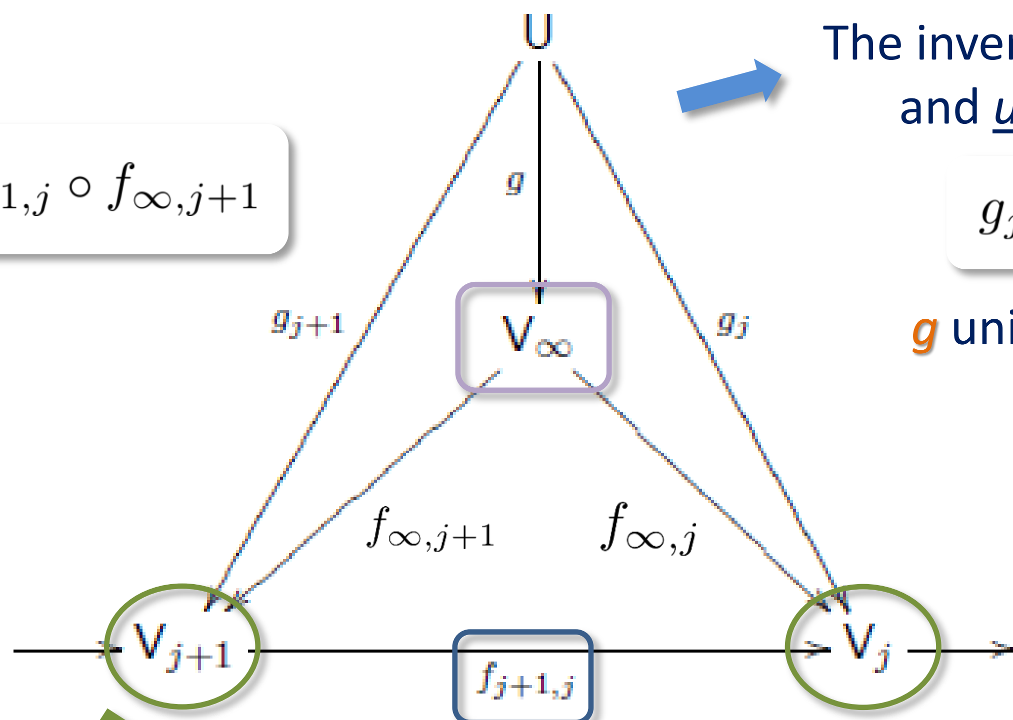
Pseudo-locality can be controlled by a formal construction in Category theory which is *"Inverse Limit"*

The inverse limit is *universal* and *unique* if existing

$$\forall j \quad f_{\infty,j} = f_{j+1,j} \circ f_{\infty,j+1}$$

$$g_j = f_{\infty,j} \circ g$$

g unique linear map



Space of "local functionals" with at most j+1/j derivatives

Some given linear continuous maps to be specified.

0-0-s Currents & unfolding on AdS

- Current interactions are the simplest possible interactions:

$$J^{\underline{m}(s)} \sim \Phi^* \nabla^{\underline{m}(s)} \Phi + \dots \qquad \nabla_{\underline{n}} J^{\underline{nm}(s-1)} \approx 0$$

- In the unfolded language classifying currents is a cohomology problem

$$D\omega + \dots = J(C,C) \qquad DJ(C,C) \approx 0$$

$$D : V_\infty \to V_\infty \qquad D^2 = 0$$

- Improvements in this language are exact currents

$$J = D\xi(C,C)$$

Some facts we can prove:

- ***Local cohomology:*** s ≤ k < ∞ and $J \in V_k(C,C)$ – 1-dim cohomology for each *s*, representative with *s* derivatives $\tilde{J} \in V_s(C,C)$ (*Metsaev coupling* – *canonical primary current*)
- ***Non-local cohomology:*** $J \in V_\infty(C,C)$ – the cohomology is empty.

Kessel, Lucena-Gómez, Skvortsov & M.T. '15; Skvortsov & M.T. '15

0-0-s Currents & unfolding on AdS

What do we learn?

- Without locality conserved currents are *quasi-locally* exact

$$DJ = 0 \quad \Longrightarrow \quad J = D\left(\sum_{l=0}^{\infty} g_l \Box^l \xi\right) = \sum_{l=0}^{\infty} g_l \underbrace{D\left(\Box^l \xi\right)}$$

Each of this is a
local Improvement

- Any pseudo-local conserved current is an (infinite) sum of conserved local Improvements

Conserved local currents span all conserved currents (!)

- In 4d such a basis for canonical traceless currents is schematically:

$$\left\{C_{\ldots\alpha(l)\dot{\alpha}(l)} C_{\ldots}{}^{\alpha(l)\dot{\alpha}(l)}\right\}_{l=0,\ldots,\infty} \sim \left\{\nabla_{\ldots\alpha(l)\dot{\alpha}(l)} \Phi^* \nabla_{\ldots}{}^{\alpha(l)\dot{\alpha}(l)} \Phi\right\}_{l=0,\ldots,\infty}$$

- There *exist* basis for conserved currents compatible with the inverse limit

$$[D, f_{\infty,j}] \approx 0$$

Locality: a pseudo-local proposal I

Use the inverse limit to propose a pseudo-local functional class for which the cohomology is non-trivial (!)

$$\begin{tikzcd}
 & & U \arrow[d, "g"] \arrow[ddl, "g_{j+1}"'] \arrow[ddr, "g_j"] & & \\
 & & V_\infty \arrow[dl, "f_{\infty,j+1}"] \arrow[dr, "f_{\infty,j}"'] & & \\
{} \arrow[r] & V_{j+1} \arrow[rr, "f_{j+1,j}"'] & & V_j \arrow[r] & {}
\end{tikzcd}$$

- Decomposition in local cohomology well defined for each $j < \infty$
- For each $j < \infty$ one can rewrite:

Conserved

$$f_{\infty,j}(J_s) = b^{(s)}_{j,J} \tilde{J}_s + \mathrm{Impr.}$$

$$\tilde{J}_s \in \mathbb{H}^{(2)}_s(D, CC)$$ *Primary-canonical*

- One can then take the limit:

$$\boxed{J_s = \lim_{j\to\infty} f_{\infty,j}(J_s) = \left(\lim_{j\to\infty} b^{(s)}_{j,J}\right) \tilde{J}_s + \mathrm{Impr.}}$$

- Read off the coefficient of the cohomology representative: $\lim_{j\to\infty} b^{(s)}_{j,J}$
- If the limit exists and is finite the coupling admits a Metsaev frame

Locality: a pseudo-local proposal II

- *Trivial* pseudo-local currents are: $J_s \in V^\infty$ $\quad \Rightarrow \quad$ $\boxed{\lim_{j\to\infty} b^{(s)}_{j,J} = 0}$

$$f_{\infty,j}(J_s) = b^{(s)}_{j,J}\tilde{J}_s + \text{Impr.} \quad \& \quad \text{Impr.} \in V_j$$

- Given a basis one can compute the coefficients b_j

primary canonical

$$j_{s,l} = [C_{\alpha(s)\dot{\alpha}(s)\beta(l)\dot{\beta}(l)}C^{\beta(l)\dot{\beta}(l)} + \ldots]y^{\alpha(s)}\bar{y}^{\dot{\alpha}(s)} = c^{(s)}_l\tilde{J}_s + \text{Impr.}$$

$$c^{(s)}_l \sim l^{2s}(l!)^2$$

$$\boxed{J_s = \sum_{l=0}^{\infty} g_l\, j_{s,l} = \lim_{j\to\infty}\left(\sum_{l=0}^{j} g_l\, c^{(s)}_l\right)\tilde{J}_s + \text{Impr.}}$$

$$b^{(s)}_{j,J}$$

- Converges if: $\quad g_l \prec \dfrac{1}{l^{2s+1}}\left(\dfrac{1}{l!}\right)^2$

This defines a pseudo-local functional class preserving the cohomology

Locality: String Field Theory

0-0-s vertex in open string theory is also pseudo-local!

$$\mathcal{V}_3 \sim |y_{12}y_{13}y_{23}| \exp\left[\alpha' \sum_{i\neq j} p_i \cdot p_j \ln|y_{ij}| - \frac{\sqrt{2\alpha'}}{y_{31}} p_3 \cdot \alpha_{+1}\right] \Phi^\star(p_2)\Phi(p_3)\Phi_{\underline{m}(s)}(p_1)\alpha_{-1}^{\underline{m}(s)}$$

One can split the latter in improvements and canonical currents (local cohomology):

s=1:

$$J^{\underline{m}} \sim \exp\left(-\alpha'\Box \ln\left|\tfrac{y_{23}}{y_{31}y_{12}}\right|\right) \Phi^\star \overset{\leftrightarrow}{\partial}{}^{\underline{m}}\Phi = \Phi^\star \overset{\leftrightarrow}{\partial}{}^{\underline{m}}\Phi - \alpha' \left(\ln\left|\tfrac{y_{23}}{y_{31}y_{12}}\right|\right)\Box\, \underbrace{(\Phi^\star \overset{\leftrightarrow}{\partial}{}^{\underline{m}}\Phi)} + \dots$$

- Canonical current comes with a finite coefficient compatible with Virasoro
- All infinite tail contributes! (<u>no</u> redefs involved)
- The locality proposal works for ST

$$J^{\underline{m}(s)} \sim \frac{1}{s!} \Phi^\star \overset{\leftrightarrow}{\partial}{}^{\underline{m}(s)} \Phi + \text{Impr.}$$

M.T. 2010, Sagnotti & M.T. 2010

Need to write the currents as primary plus Impr.

Locality and Witten Diagrams (3d)

We have computed the Witten diagram associated to each basis element: ➡ $\Box^l J^{\mathrm{can.}}$

$$J_s = \sum_l g_l \, \Box^l J^{\mathrm{can}}$$

$$\boxed{\langle J_s \mathcal{O}^* \mathcal{O} \rangle \sim \left(\sum_l g_l \, c_l^{(s)} \right) \frac{1}{|x_{12}|} \left(\frac{x_{12}^+}{x_{31}^+ x_{23}^+} \right)^s}$$

$$\int_{AdS} \sum_l \overset{?}{\neq} \sum_l \int_{AdS}$$

This locality proposal ensures that infinite sum and AdS integral commute – otherwise some analyticity issue may occur (??)

Summary

We propose a functional class for pseudo-local theories

Properties:

- This functional class space works in String Field Theory
- It ensures that the integration over space time and infinite sum over derivatives commute (preserves holographic S-matrix)
- It is ***tuned*** to make primary canonical currents non-trivial cohomologies – otherwise one can consider:

$$\tilde{J}_s^{\mathrm{can}} = \lim_{l\to\infty}\left(\tilde{J}_s^{\mathrm{can}} - \frac{1}{c_l^{(s)}}\Box^l\,\tilde{J}_s^{\mathrm{can}}\right) = \lim_{l\to\infty}\left(D\xi_l\right) \overset{?}{=} D\xi$$

- *Puzzle*: for divergent coefficient of the cohomology the Metsaev frame *cannot be reached or is singular* – Intrinsically non-local cubic?? Clash with Metsaev's classification??

Resumming Vasiliev's Backreaction

Puzzle: Metsaev's field frame seems singular

$$\frac{\cos(2\theta)}{12}\left(\sum_{n=1}^{\infty} l\right) J^{\text{can.}}_{s=2}$$

$$\frac{\cos(2\theta)}{3\cdot 7!}\left(\sum_{l=1}^{\infty} \frac{l(l+1)(l+2)^2(3l+11)(5l(l+4)+3)}{(l+3)(l+4)}\right) J^{\text{can.}}_{s=4}$$

The series diverge as l^{2s-3}

Skvortsov & M.T. 2015

Possible ways out:

- Vasiliev's theory may not admit a Metsaev frame (Vasiliev vs. Metsaev)
- The prescription of the equations should be improved
- The functional class needs to be enlarged (hard since we only not allow the redefinitions that would remove the cohomology)