# Global Holographic Effects of Defects

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# Our goal:

- **Holographic diagnostic of the black hole creation**

- **Explicit model for thermalization**
  (non-equilibrium correlators that in some limit approach to thermal correlators)

*This is related with the problem of quark-gluon plasma formation*.

**I.A., ``Holographic approach to quark-gluon heavy plasma in ion collisions,''Phys.Usp.57 (2014) 527**

# Two main points

- **Thermal Green** functions in the holographic approach correspond to a bulk with a **black hole** (or a black brane)

- **Thermalization** holographically means

  a **black hol**e **creation**

# Our modest goal:

**How this works in D=3**

# Why D=3 ?

**Simplest matter - point particles**

**In D=3 point particles admit a simple description**

# Point particle in $AdS_3$

- In 3 dim Einstein gravity:
  point particle produces a conical singularity, holonomy **u** of which is defined **by the mass of the particle and its kinetics**

**S. Deser, R. Jackiw, and G. 't Hooft, 1984**

# Point particle in AdS$_3$

$$G^{\mu\nu} - \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}$$

$$ds^2_{DJ} = -N^2(R)dt^2 + \Phi(R)(dR^2 + R^2 d\tilde{\phi}^2), \qquad T^{00} = \frac{m}{\sqrt{-g}} N(R)\delta(R)$$

**Solution**:

$$\Phi(R) = \frac{4A^2}{\Lambda R^2((R/R_0)^A + (R/R_0)^{-A})^2}, \qquad A = 1 - 4Gm$$

$$N(R) = \frac{(R/R_0)^A - (R/R_0)^{-A}}{(R/R_0)^A + (R/R_0)^{-A}} \qquad A \to -A \implies N \to -N$$

$$\cancel{T^{00} < 0}$$

**Change of the variable** $\sinh\chi = \cosh\left(\frac{R}{R_0}\right)^A, \quad \phi = A\tilde{\phi}$

$$ds^2_{DJ} = -\cosh^2\chi dt^2 + d\chi^2 + \sinh^2\chi d\phi^2$$

**Metric as AdS$_3$ in the global coordinates, but** $\phi \in (0, 2\pi A)$

# AdS$_3$

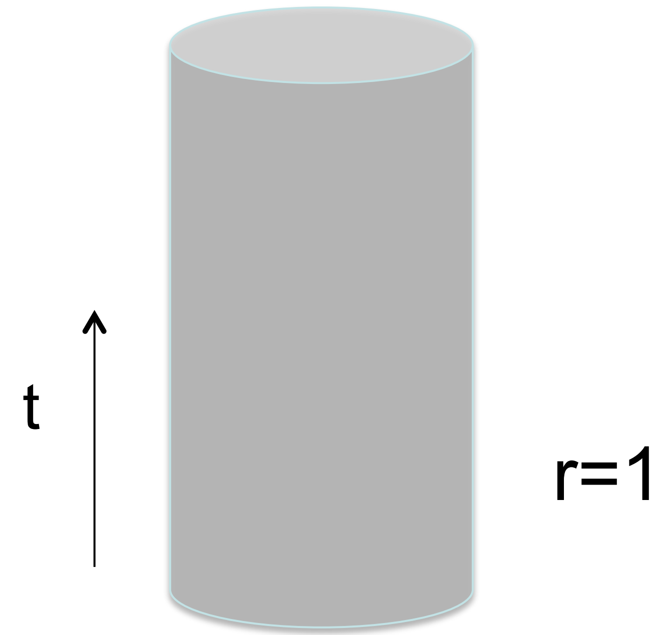
$$x_3^2 + x_0^2 - x_1^2 - x_2^2 = 1.$$

**Parametrization**

$$x_3 = \cosh\chi\ \cos t$$
$$x_0 = \cosh\chi\ \sin t$$
$$x_1 = \sinh\chi\ \cos\phi$$
$$x_2 = \sinh\chi \sin\phi$$

$$\phi \in (0, 2\pi)$$

**Time evolution of the Poincare-disk**




$$ds^2 = -\cosh^2\chi dt^2 + d\chi^2 + \sinh^2\chi d\phi^2$$

**Relations with the Poincare-disk** $r = \tanh\chi/2$

$$ds^2 = -\left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 + \left(\frac{2}{1-r^2}\right)^2 (dr^2 + r^2 d\phi^2)$$

# In 3-dim

## What we do in D=3:

- We take the cylinder, put there particles, accelerate them and
  examine when we can get the black hole

# In 3-dim

- An "explicit" model of BH creation in $AdS_3$

- H.-J.Matschull, ``Black hole creation in (2+1)-dimensions,''  Class. Quant.Grav,16, 1069 (1999)

# Refs:

- This is a long term project:
- I.A., A. Bagrov, "Holographic dual of a conical defect", TMP, 182 (2015), 1–22
- I.A., A. Bagrov, P. Saterskog, K. Schalm, Holographic dual of a time machine, arXiv:1508.04440
- I.A., D.Ageev, M.Tihanovskaya, M.Khramtsov, 1512.03362
  1512.03363; 1604.08905
- I.A.,M.Khramtsov, JHEP 1604, 121 (2016)
- Work in progress,

# AdS$_3$ as the SL(2,R) group manifold

SL(2,R)  2x2 matrices with unit determinant

$$\mathbf{x} = x_3 \mathbf{1} + \sum_{\mu=0,1,2} \gamma_\mu x^\mu \qquad \gamma_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

**The condition for the determinant to be one is** $x_3^2 + x_0^2 - x_1^2 - x_2^2 = 1.$

**The relation with the global coordinates is**

$$\mathbf{x} = \cosh\chi\, \mathbf{\Omega}(\tau) + \sinh\chi\, \mathbf{\Gamma}(\phi)$$

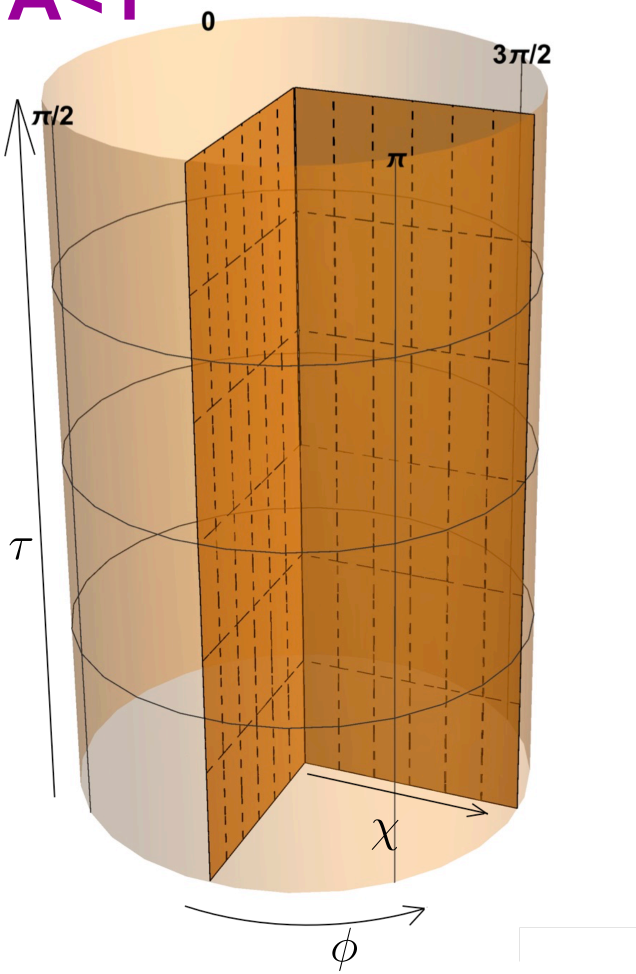
$$\mathbf{\Omega}(\tau) = \cos\tau \mathbf{1} + \sin\tau\, \gamma_0 \qquad \mathbf{\Gamma}(\phi) = \cos\phi\gamma_1 + \sin\phi\gamma_2$$

$$ds^2_{AdS_{2+1}} = \frac{1}{2}\mathrm{Tr}(\mathrm{x}^{-1}\, d\,\mathrm{x}\,\mathrm{x}^{-1}\, d\,\mathrm{x})$$

# Stationary point particle in $AdS_3$ at the origin

Prescription: one removes a wedge bounded by two faces separated by an angle $\bar{\alpha} \sim m, \quad \bar{\alpha} = 2\pi - \alpha$

**A<1**



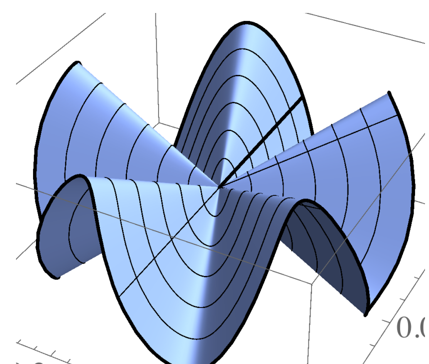

$$\mathbf{x}_{1-st\,face} = \cosh\chi\,\mathbf{\Omega}(t) + \sinh\chi\,\mathbf{\Gamma}(-\alpha/2)$$

$$\mathbf{x}_{2-nd\,face} = \mathbf{\Omega}(\alpha/2)\cdot\mathbf{x}_{1-st\,face}\cdot\mathbf{\Omega}(-\alpha/2)$$

**Fixed points represent a point like source**

$$\mathbf{\Omega}(\alpha/2)\mathbf{\Omega}(t)\mathbf{\Omega}(-\alpha/2) = \mathbf{\Omega}(t)$$

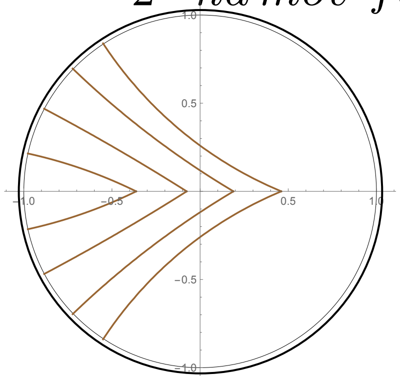
**A>1**



Picture from:
1603.08925

# Moving massive particle

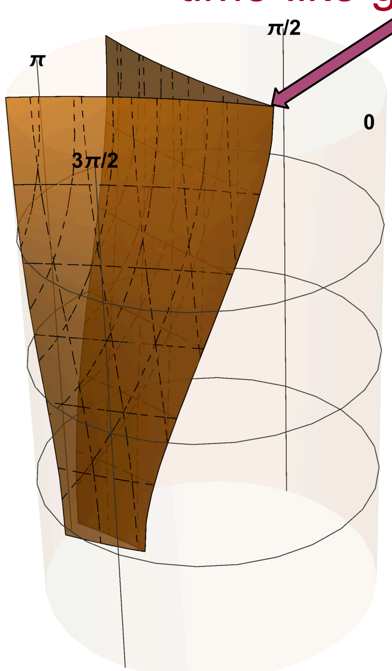
$$\mathbf{x}_{2-nd\,mov\;face} = \mathbf{\Omega_u}(\alpha/2,\xi/2)\cdot\mathbf{x}_{1-st\,mov\;face}\cdot\mathbf{\Omega_u}(-\alpha/2,\xi/2)$$



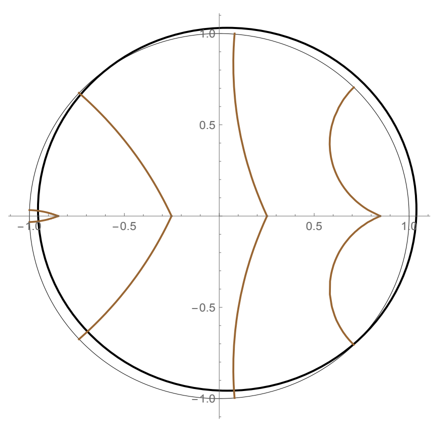

$$\mathbf{\Omega_u}(-\alpha/2,\xi/2) = \mathbf{u}^{-1}(\xi/2)\mathbf{\Omega}\,(-\alpha/2)\,\mathbf{u}(\xi/2)$$

$$\mathbf{u}(\xi/2) = \cosh(\xi/2)\,\mathbf{1} - \sinh(\xi/2)\,\gamma_2$$

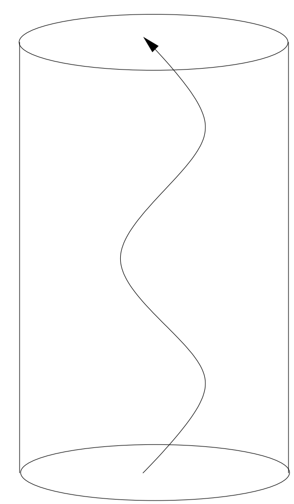
time-like geodesics




**Increasing rapidity**




**Massive particle cannot reach the boundary**



# Ultrarelativistic particle

A massless particle with a light-like momentum vector pointing into the x-direction. Its holonomy is

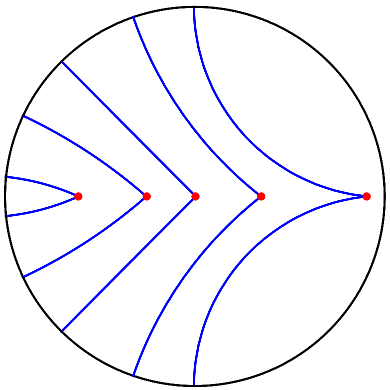
$$u = 1 + \tan\epsilon \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad u^{-1} = 1 - \tan\epsilon \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

The isometry transformation

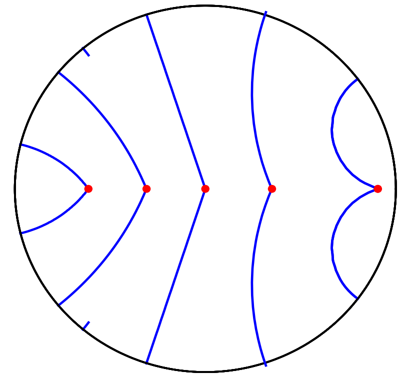
$$\mathbf{x} \to \mathbf{x}^* = u^{-1} \mathbf{x}\, u$$

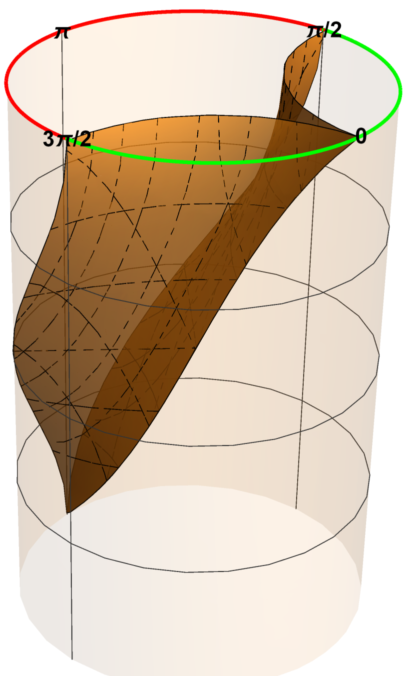
**The fixed points lie on the world l**ine $\qquad r = \tan(t/2), \quad \phi = 0.$

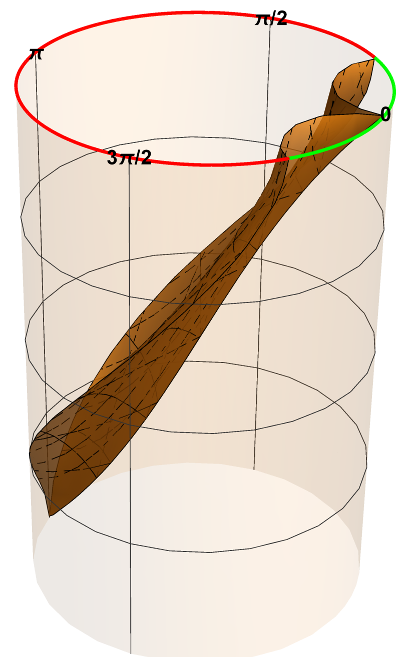
# Ultrarelativistic particle



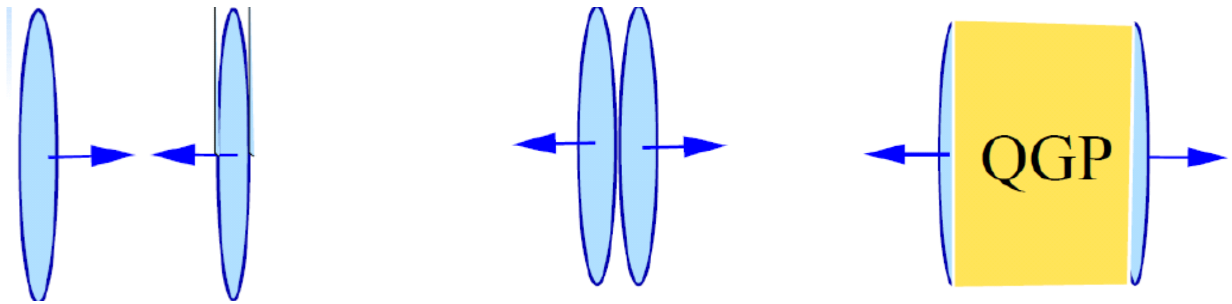
**Increasing energy**





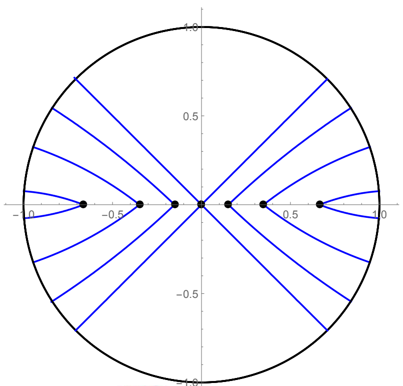
# Analogy with HIC

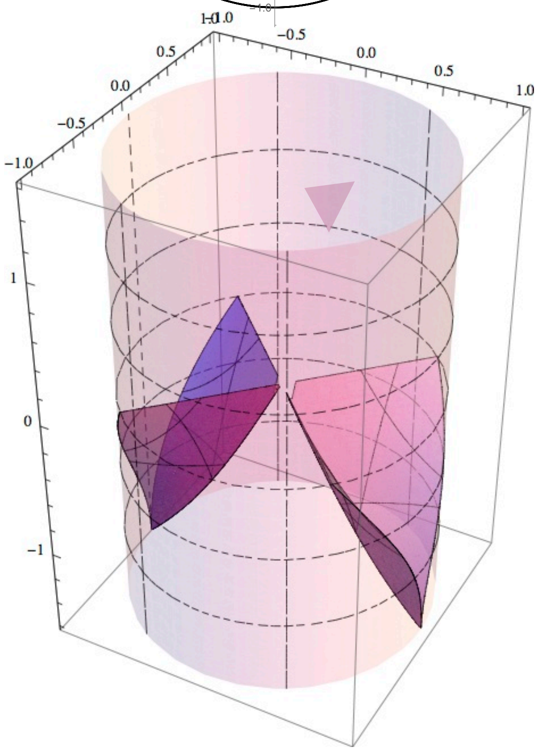


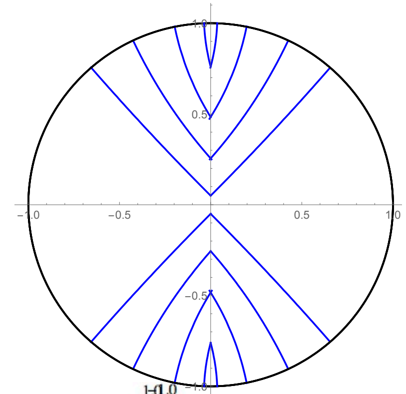

Thermalization,
Isotropization,
Hydrodynamization

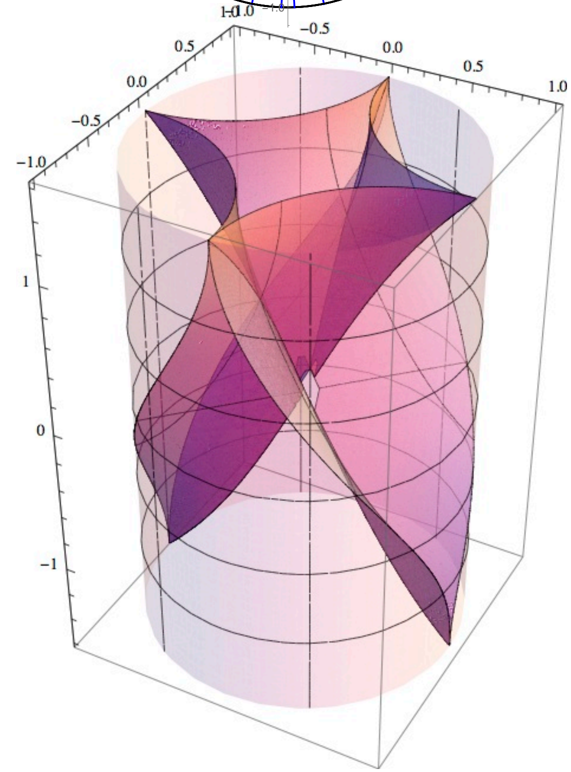
Container with flexible wall

# Two wedges, $\epsilon = \pi/4$



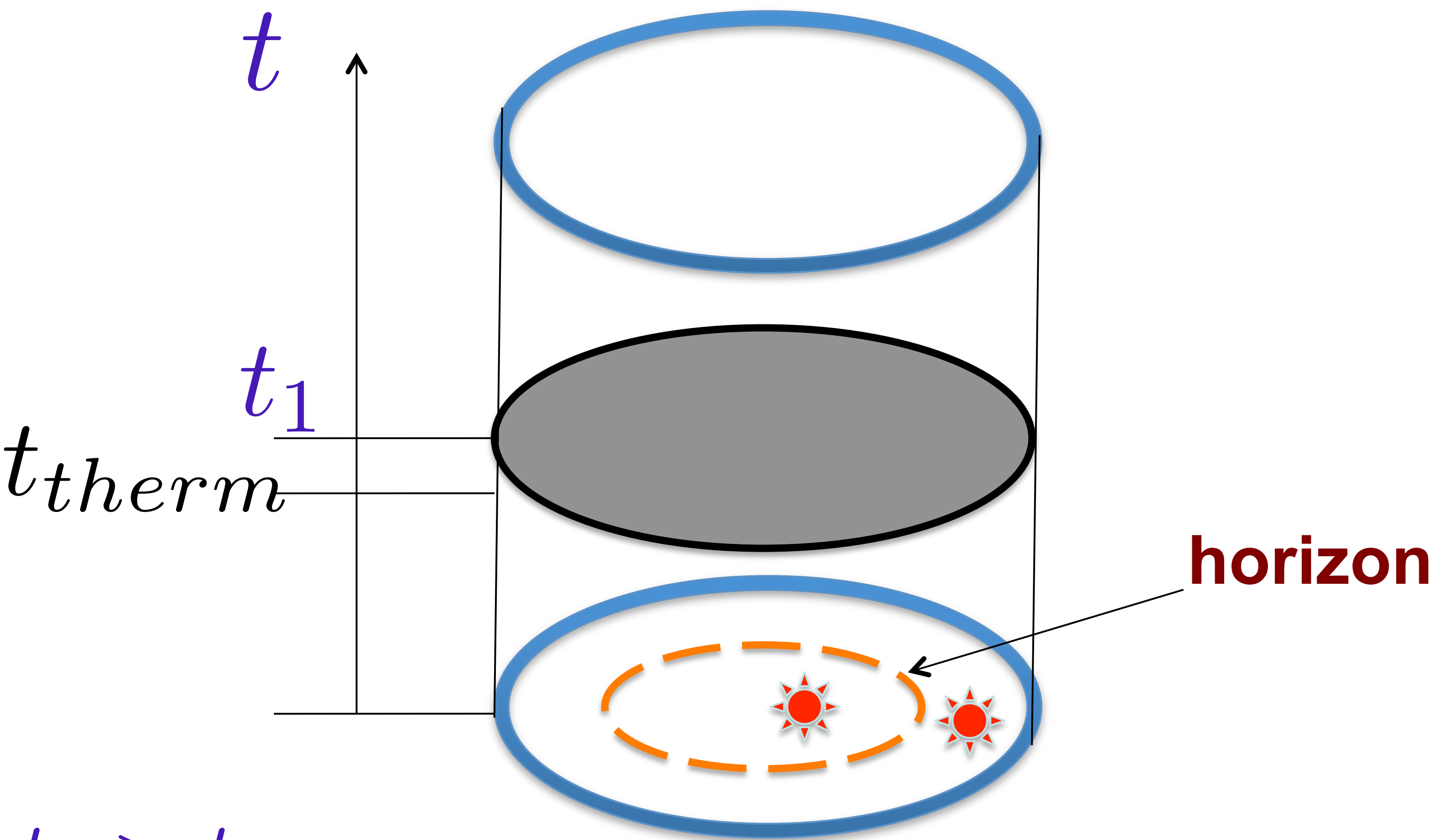






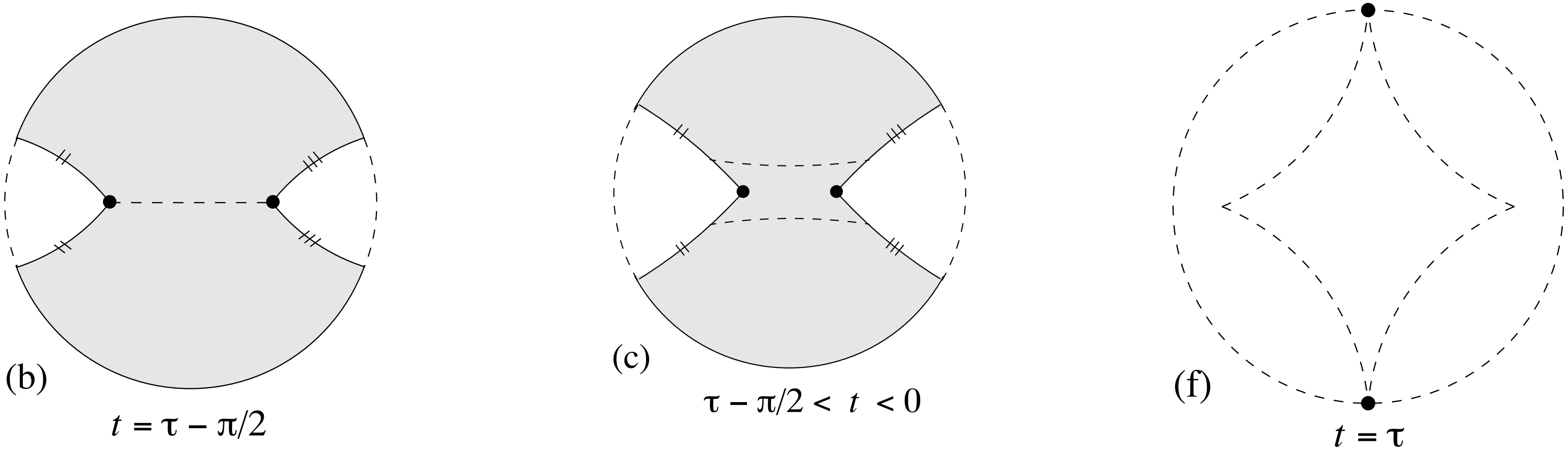
**Inevitable consequences**

# Appearance of the horizon




$t_1 > t_{therm}$

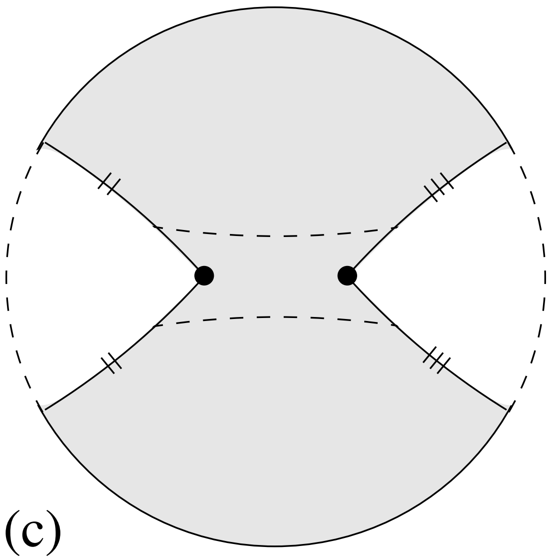
# Appearance of the horizon




Pictures from **Matschull,1991**

# Holographic diagnostic of the BH creation

- **Correlators on the boundary until it exists ☹**



(c)

$\tau - \pi/2 < t < 0$

# Boundary-bulk correspondence, $AdS_3$

Banks, Douglas, Horowitz, Martinec, hep-th/9808016

Hamilton, Kabat, Lifschytz, A. Lowe, hep-th/0606141

Harlow, Stanford, 1104.2621

**Cl.** $\Phi_a(t,\chi,\phi) \quad \Longleftrightarrow \quad \mathcal{O}_{\Delta_a}(t,\phi)$

**Q.** $\hat{\Phi}_a(t,\chi,\phi) \quad \Longleftrightarrow \quad \hat{\mathcal{O}}_{\Delta_a}(t,\phi)$

$$\hat{\mathcal{O}}_{\Delta_a}(t,\phi) = \hat{\mathcal{O}}_{L,a}(t+\phi)\cdot\hat{\mathcal{O}}_{R,a}(t-\phi)$$

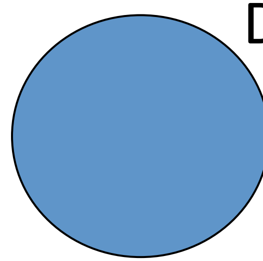
$$\hat{\mathcal{O}}_{L,\Delta}(t,\phi) = \sum_{n\in Z} \mathcal{O}_{n-\frac{\Delta}{2}} e^{i(n-\frac{\Delta}{2})(t+\phi)}$$

$$\hat{\mathcal{O}}_{R,\Delta}(t,\phi) = \sum_{n\in Z} \tilde{\mathcal{O}}_{n-\frac{\Delta}{2}} e^{i(n-\frac{\Delta}{2})(t-\phi)}$$

# Boundary-bulk correspondence

Bulk local operators

$$(\Box - m_a^2)\Phi_a = 0$$



Dual local CFT operators

$$\Delta_a = 1 + \sqrt{1 + m_a^2}$$

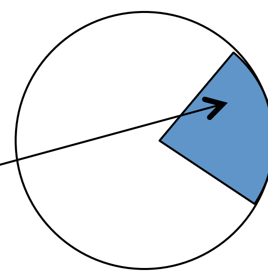
$$\Phi_a(t,\chi,\phi) = \int_{\partial AdS_3} K(t,\chi,\phi;t',\phi')\mathcal{O}_a(t',\phi')\,dt'd\phi'$$

$$\lim_{\chi\to\infty} e^{-\Delta_a\chi}\Phi_a(t,\chi,\phi) = \mathcal{O}_a(t,\phi)$$

**Orbifold**

Fundamental domain

$AdS_3/\Gamma$



Leaving space:

$\partial(AdS_3/\Gamma)$

$$\Phi_a(t,\chi,\phi) = \sum_{n=1}^{m} \int_{\partial AdS_3/Z_m} K(t,\chi,\phi;t^{'(n)},\phi^{'(n)})\mathcal{O}_a(t^{'},\phi^{'})\,dt'd\phi'$$

$$\phi^{'(n)} = \phi^{'} + 2\pi n/m, \quad t^{'(n)} = t^{'}$$

# Boundary-bulk correspondence

Boundary state

$$< \Psi|\mathcal{O}_a(x_b)\mathcal{O}_a(x_b')|\Psi >= \lim_{x\to x_b,\ x\to x_b'} \mathcal{Z} < \Phi_a(x)\Phi_a(x') >_{AdS_3}$$

$$|\Psi >= |0 >$$

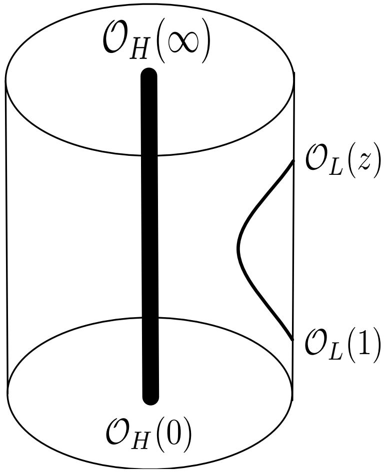
$$\lim_{\chi\to\infty} e^{-\Delta_a \chi}\Phi_a(t,\chi,\phi) = \mathcal{O}_a(t,\phi)$$

**Orbifold**

$$< \Psi_\Gamma|\mathcal{O}_a(x_b)\mathcal{O}_a(x_b')|\Psi_\Gamma >= \lim_{x\to x_b,\ x\to x_b'} \mathcal{Z} < \Phi_a(x)\Phi_a(x') >_{AdS_3/\Gamma}$$

$$\Gamma = Z_n$$

$$|\Psi_\Gamma >= \hat{\Phi}_{\Delta_n}(0,0,\phi)|0 >$$




Picture from: 1603.08925

# Backup page

Goto, Miyaji,Takayanagy, 1605.02835

$$K(0,0,\phi_0|t,\phi) = \frac{e^{i\Delta t}}{2\pi^2} \cdot {}_2F_1(1,1,\Delta,-e^{2it}) = \frac{e^{i\Delta t}}{2\pi^2}\Gamma(\Delta)\sum_{n=0}^{\infty}\frac{\Gamma(n+1)}{\Gamma(n+\Delta)}(-1)^n e^{2int}$$

$$\hat{\Phi}_\Delta(0,0,\phi)|0>=\sum_{n=0}^{\infty}\frac{\Gamma(n+1)\Gamma(\Delta)}{\Gamma(n+\Delta)}(-1)^n \cdot \mathcal{O}_{\Delta,-n-\frac{\Delta}{2}}\tilde{\mathcal{O}}_{\Delta,-n-\frac{\Delta}{2}}|0> .$$

$$(L_0-\tilde{L}_0)|\Psi_\Delta>=(L_{\pm1}+\tilde{L}_{\mp1})|\Psi_\Delta>=0$$

Ishibashi's state

$$L_0 = i\partial_+, \quad \tilde{L}_0 = i\partial_-,$$

$$L_{\pm1} = ie^{\pm ix^+}\left[\frac{\cosh 2\chi}{\sinh 2\chi}\partial_+ - \frac{1}{\sinh 2\chi}\partial_- \mp \frac{i}{2}\partial_\rho\right],$$

$$\tilde{L}_{\pm1} = ie^{\pm ix^-}\left[\frac{\cosh 2\chi}{\sinh 2\chi}\partial_- - \frac{1}{\sinh 2\chi}\partial_+ \mp \frac{i}{2}\partial_\rho\right]$$

Algebraic proof based on

$$\mathcal{O}_{-k-\frac{\Delta}{2}}|0>_L = \frac{1}{\Gamma(k+1)}(L_{-1})^k|0>_L$$

## Moving massive particle

$$\Gamma = \mathbf{u}(\xi/2)\mathbf{\Omega}(2\pi/n)\mathbf{u}(-\xi/2)$$

$$\mathbf{u}\Phi_a(t,\chi,\phi)\mathbf{u}^{-1} = \int_{\mathbf{u}\partial AdS_3/Z_n} K(t,\chi,\phi;t',\phi')\mathcal{O}_a(t'_{\mathbf{u}},\phi'_{\mathbf{u}})\,dt'_{\mathbf{u}}d\phi'_{\mathbf{u}}$$

$$< \Psi_{\mathbf{u}}|\mathcal{O}_a(x_b)\mathcal{O}_a(x'_b)|\Psi_{\mathbf{u}} >=< \Psi|\mathcal{O}_{a,\mathbf{u}}(x_b)\mathcal{O}_{a,\mathbf{u}}(x'_b)|\Psi >$$

## Two moving massive particle

$$< \Psi|\mathcal{O}_{a,\mathbf{u_2}}(x_b)\mathcal{O}_{a,\mathbf{u_1}}(x'_b)|\Psi >$$

# Geodesics approximation

## Bulk local operators

$$< \Phi_a(x)\Phi_a(x') >_{AdS_3} = \int e^{i\Delta_a \int_0^1 \sqrt{g_{MN}\dot{x}^N \dot{x}^M} ds} \prod_{x(0)=x; x(1)=x'} dx, \;\; x = \{t, \chi, \phi\}$$

$$\sim e^{-\Delta_a \mathcal{L}(x,x')}$$

Boundary state

$$< \Psi|\mathcal{O}_a(x_b)\mathcal{O}_a(x_b')|\Psi > = \lim_{x\to x_b,\;\; x\to x_b'} \mathcal{Z} < \Phi_a(x)\Phi_a(x') >_{AdS_3}$$

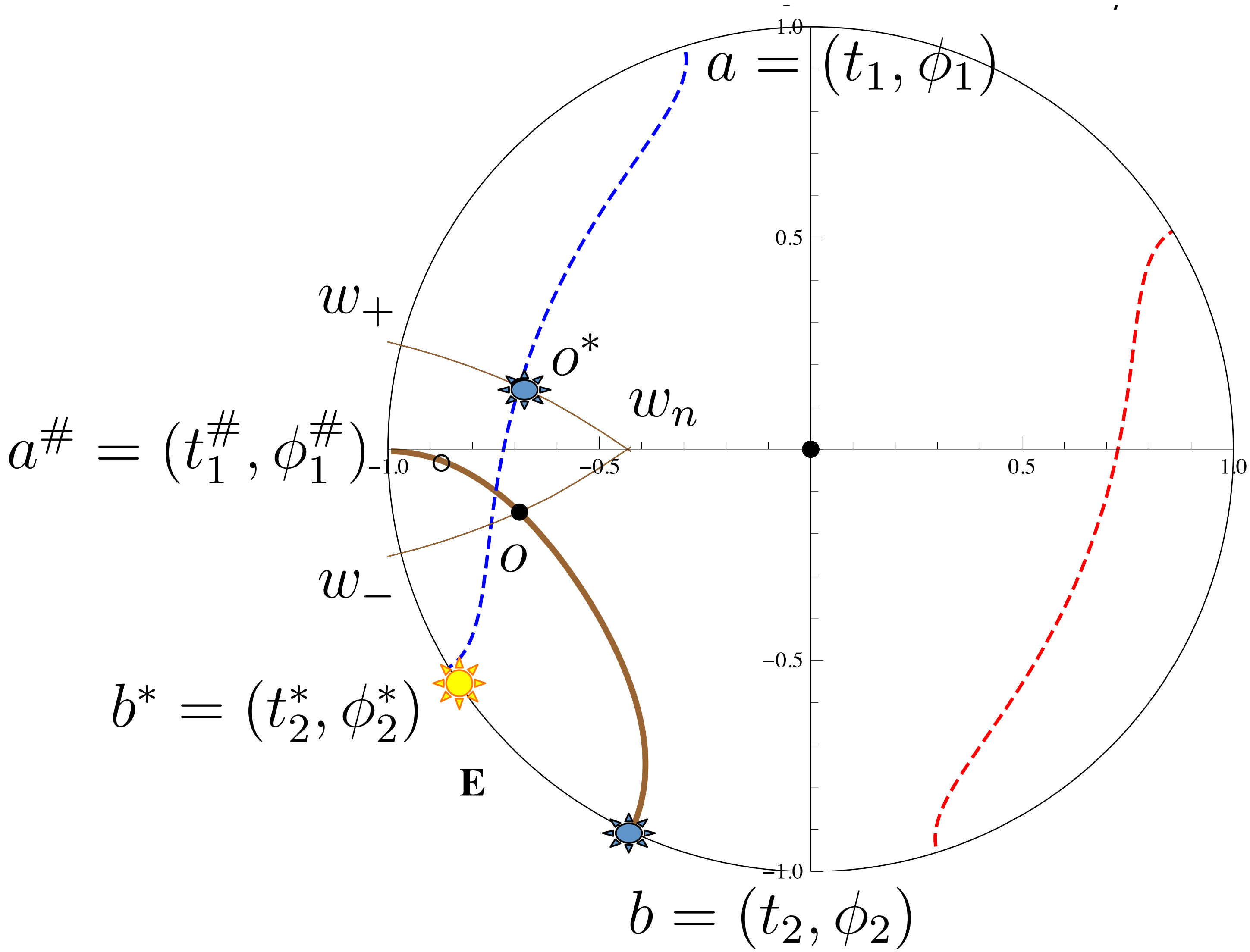
## Orbifold

$$< \Phi_a(x)\Phi_a(x') >_{AdS_3/\Gamma} = \sum_n \int e^{i\Delta_a \int_0^1 \sqrt{g_{MN}\dot{x}^N \dot{x}^M} ds} \prod_{x(0)=x; x(1)=x_n'} dx, \;\; x = \{t, \chi, \phi\}$$

$$\phi^{'(n)} = \phi^{'} + 2\pi n/m, \quad t^{'(n)} = t^{'}$$

$$\sim \sum_n e^{-\Delta_a \mathcal{L}(x,x_n')}$$

# Contribution coming from the geodesic crossing the wedge



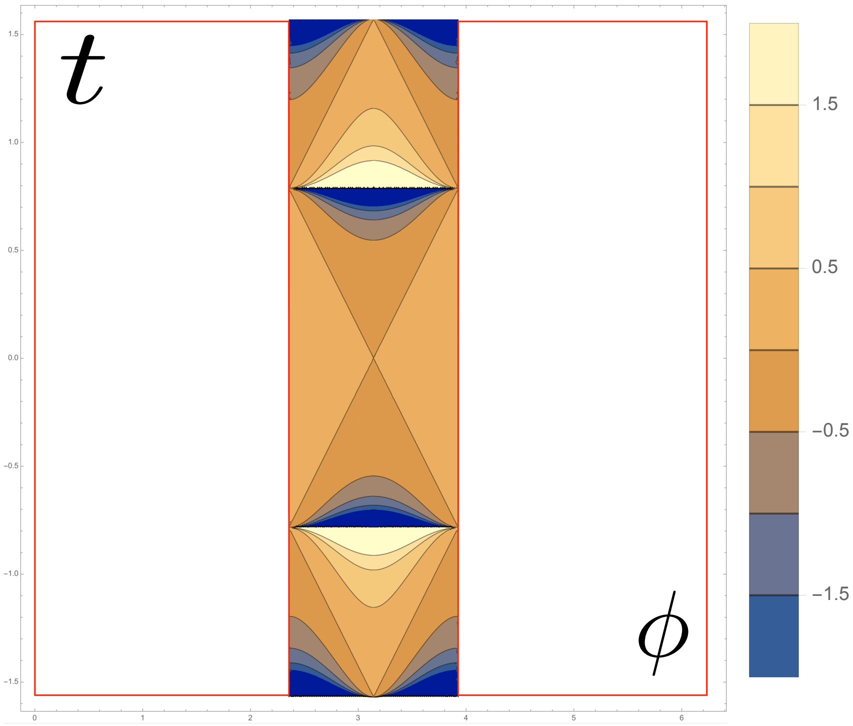

Geodesics length $x_\mu^{(1)} x^{(2)\mu} \sim \frac{1}{4} e^{\chi_1+\chi_2} \left(\cos(t_1 - t_2) - \cos(\phi_1 - \phi_2)\right)$

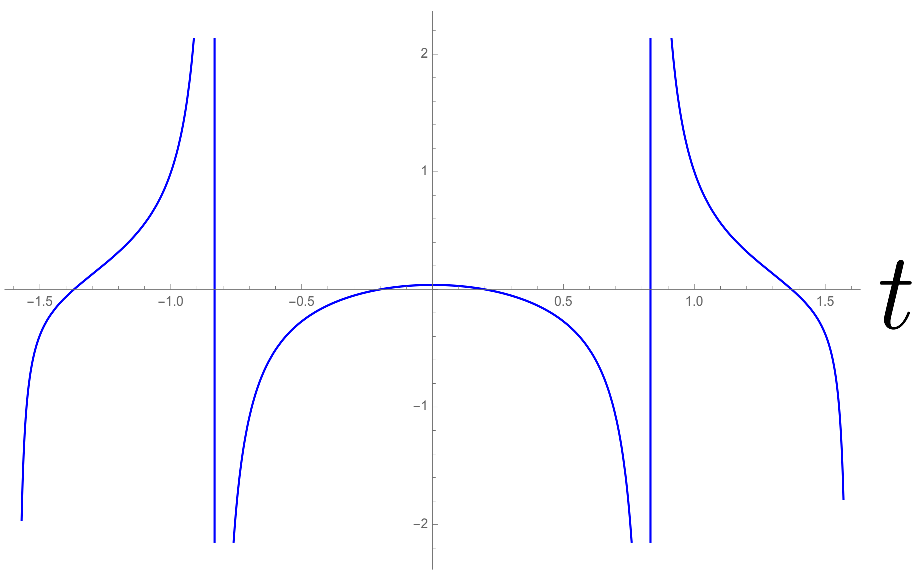
# Correlator for heavy static particle

Orbifold $\alpha = 2\pi/n$ $n = 1, 2, 3, \ldots$

,

$G^{-1}$




$\alpha = \pi/2$

# Correlator geodesic in the orbifold case

Orbifold $\alpha = 2\pi/n$ , $n = 1, 2, 3, \ldots$

$$l_n = iA\, \mathrm{e}^{in\frac{w}{A}} \partial_w \equiv A\, L_{\frac{n}{A}}$$

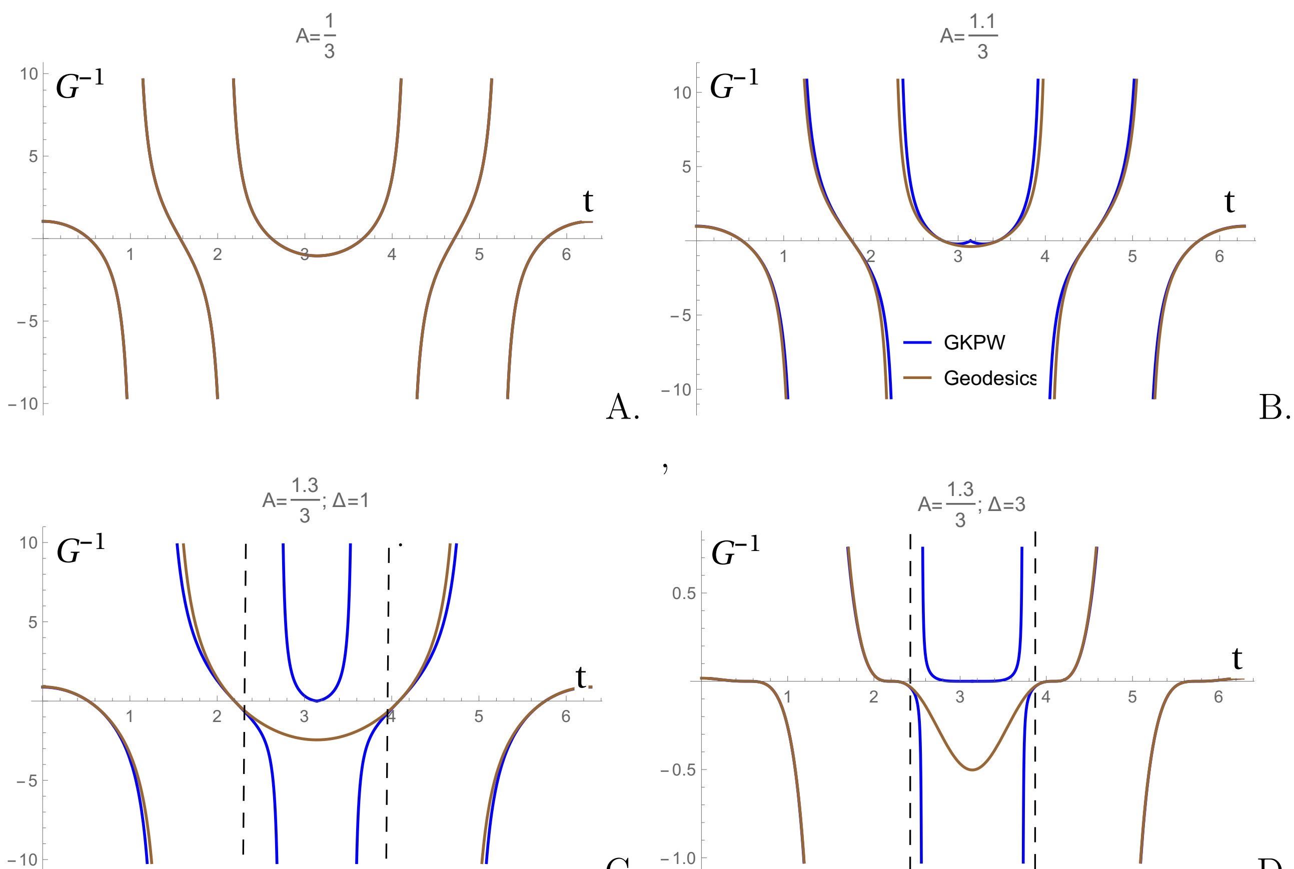
$$\langle \mathcal{O}_\Delta(t,\phi)\mathcal{O}_\Delta(0,0)\rangle \sim \sum_{k=0}^{n-1} \left( \frac{1}{2\left(\cos(t+i\epsilon) - \cos\left(\phi + 2\pi\frac{k}{n}\right)\right)} \right)^\Delta$$

**In the agreement with GKPW-prescription**

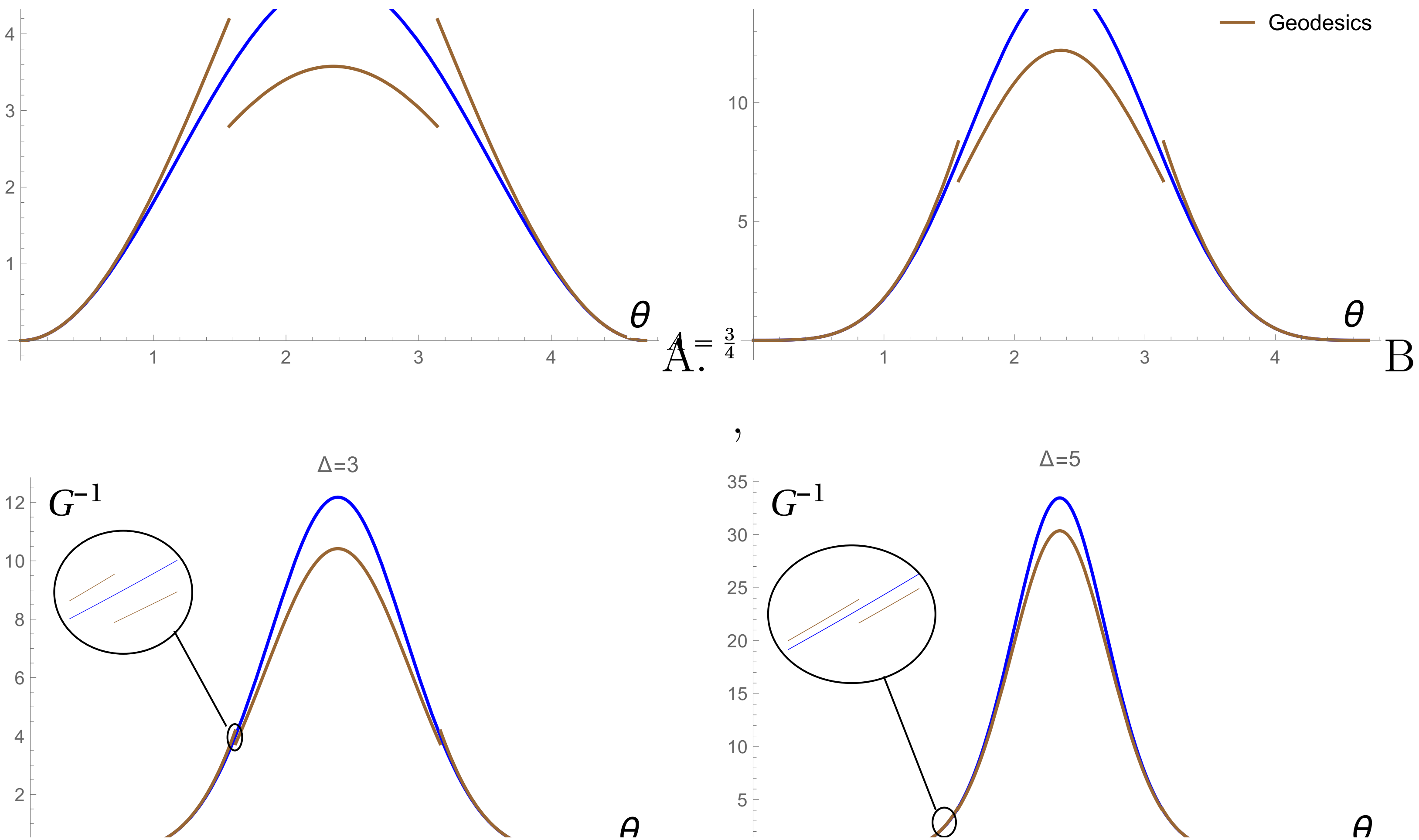
# Correlator geodesic in "near" orbifold case

Orbifold $\alpha = 2\pi/n + \delta$ $n = 1, 2, 3, \ldots$

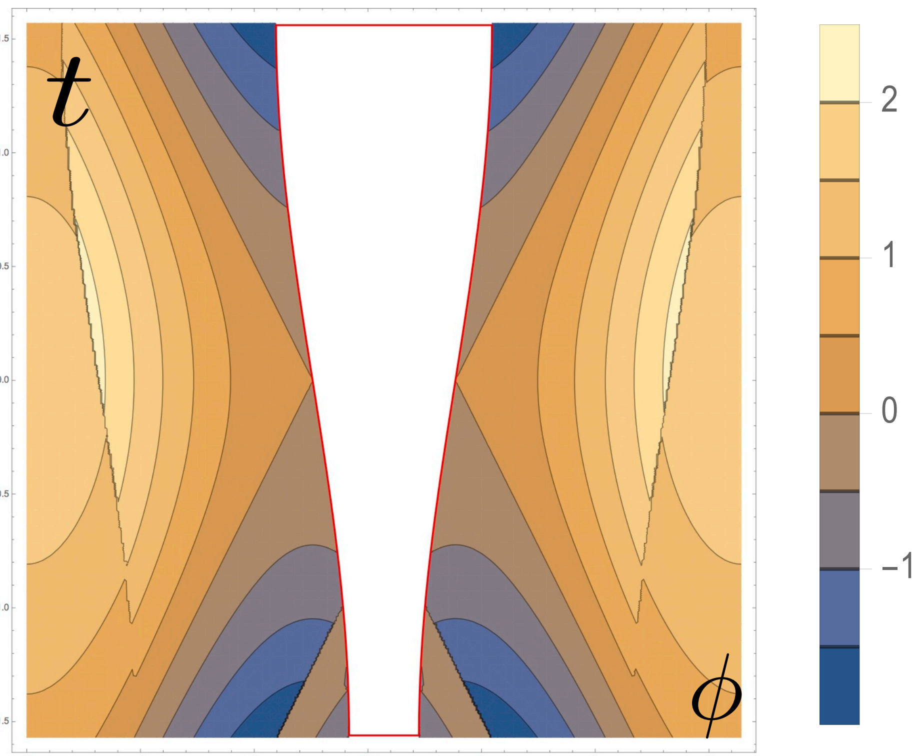
**Comparison with GKPW-prescription**

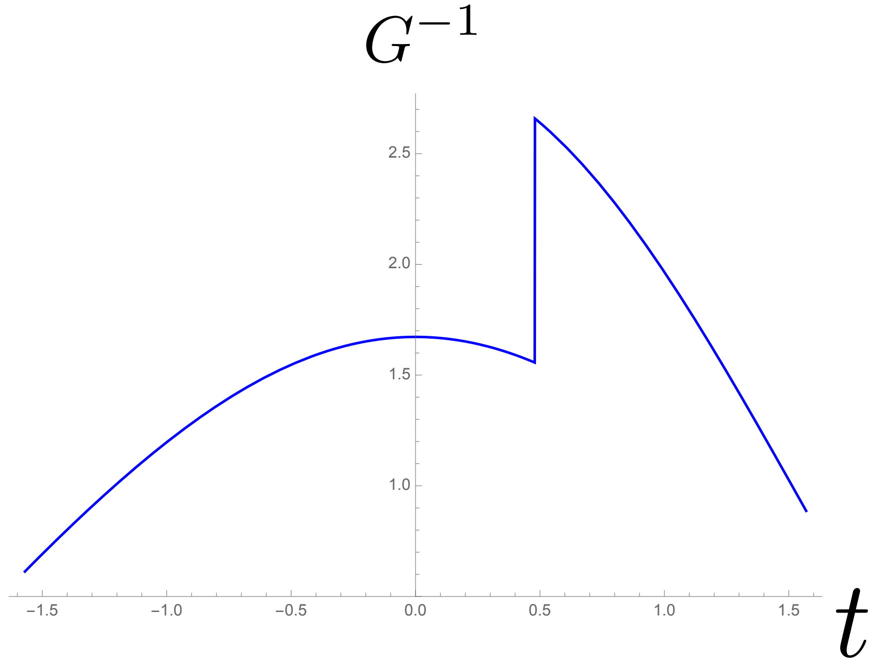


# Light static particle, $A = 3/4$



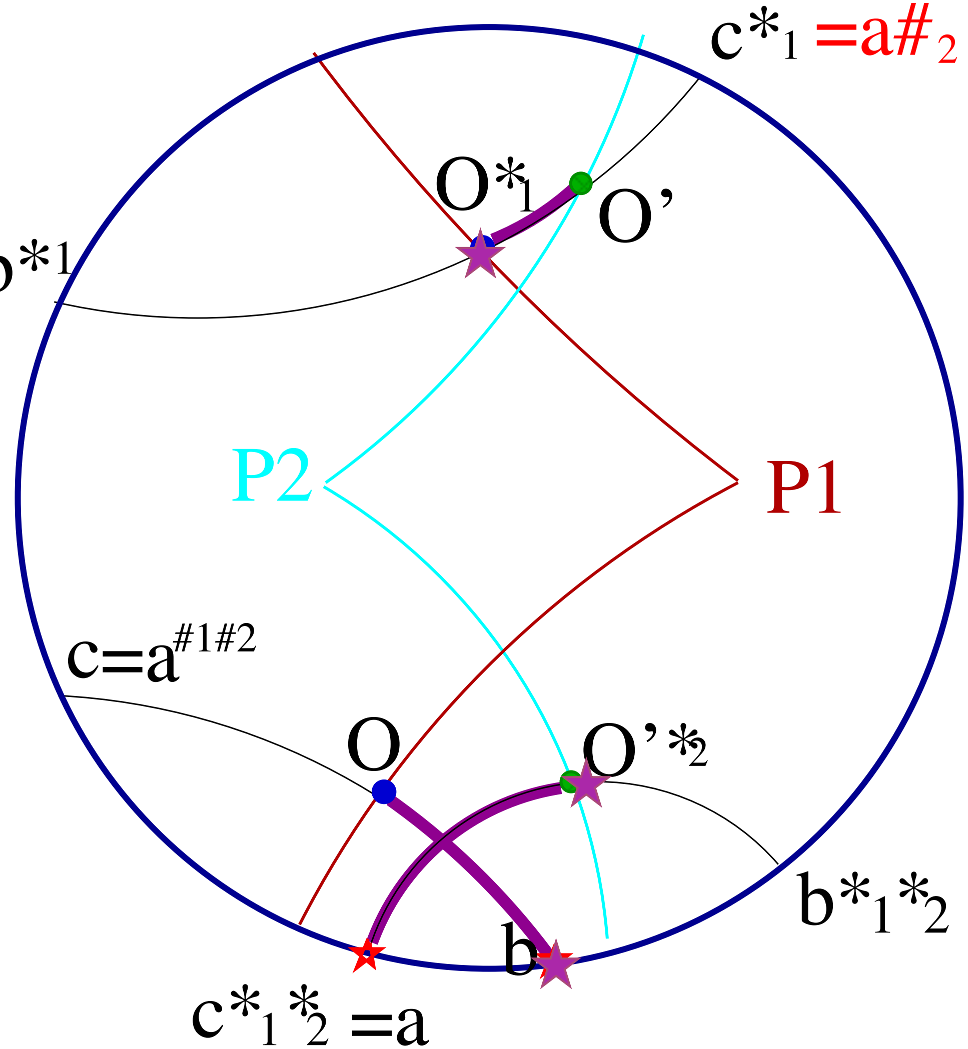
# Zone structure of correlation function on the boundary of the spacetime with light moving particle




$$\xi = 0.6, \qquad \alpha = 1.1 \neq 2\pi/n$$

# 2 wedges and 2 geodesics contribution for 2 URPs



# Infinite # of images and compactification/thermalization

$$\sum_{n=-\infty}^{\infty} \frac{1}{(x+n)^2} = \frac{\pi^2}{1-\cos^2 \pi x}$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{(x+i\pi n)^2} = \frac{1}{\cosh^2 x - 1}$$

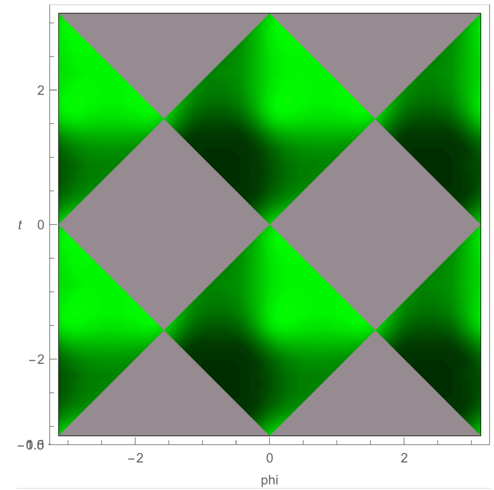
$$ds^2_{BTZ} = -(\hat{r}^2-1)d\hat{t}^2 + \frac{1}{\hat{r}^2-1}\hat{d}r^2 + \hat{r}^2 d\hat{\phi}^2$$

$$< \Psi|\mathcal{O}(\hat{t}_1,\hat{\phi}_1)\mathcal{O}(\hat{t}_2,\hat{\phi}_2)|\Psi > = \sum_n \frac{1}{(\cosh(\hat{\phi}_1-\hat{\phi}_2-2\pi n)-\cosh(\hat{t}_1-\hat{t}_2+i\epsilon))^{\Delta}}$$

# BTZ

$$x_0 = -\hat{r}\cosh\hat{\phi} = \cosh\chi\sin t$$
$$x_2 = \hat{r}\sinh\hat{\phi} = \sin\phi\sinh\chi$$
$$x_3 = \sqrt{\hat{r}^2-1}\sinh\hat{t} = \cos t\cosh\chi,$$
$$x_1 = \sqrt{\hat{r}^2-1}\cosh\hat{t} = \cos\phi\sinh\chi$$

$$\sin^2 t \geq \sin^2\phi$$



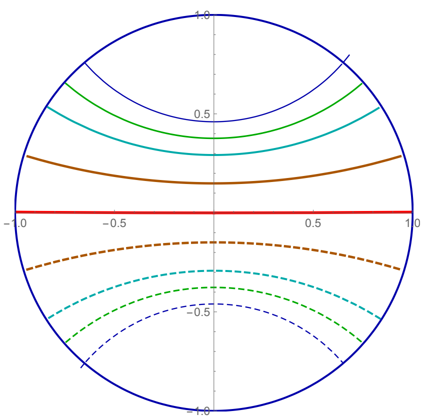

$$x^{(1)}_\mu x^{(2)\mu} \quad \sim \quad \hat{r}_1\hat{r}_2\left(\cosh(\hat{\phi}_1-\hat{\phi}_2)-\cosh(\hat{t}_1-\hat{t}_2)\right)$$

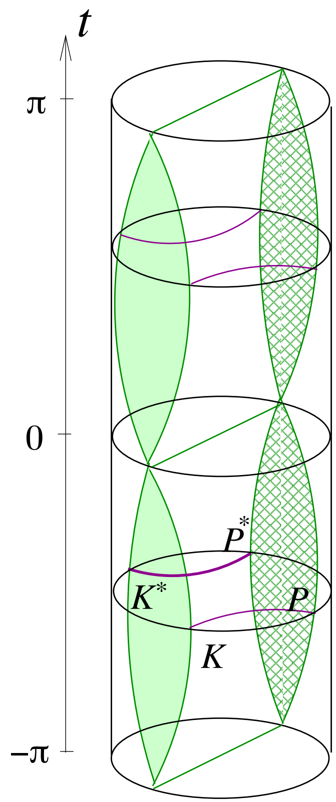
$$\sim \quad \frac{1}{4}e^{\chi_1+\chi_2}\left(\cos(t_1-t_2)-\cos(\phi_1-\phi_2)\right)$$

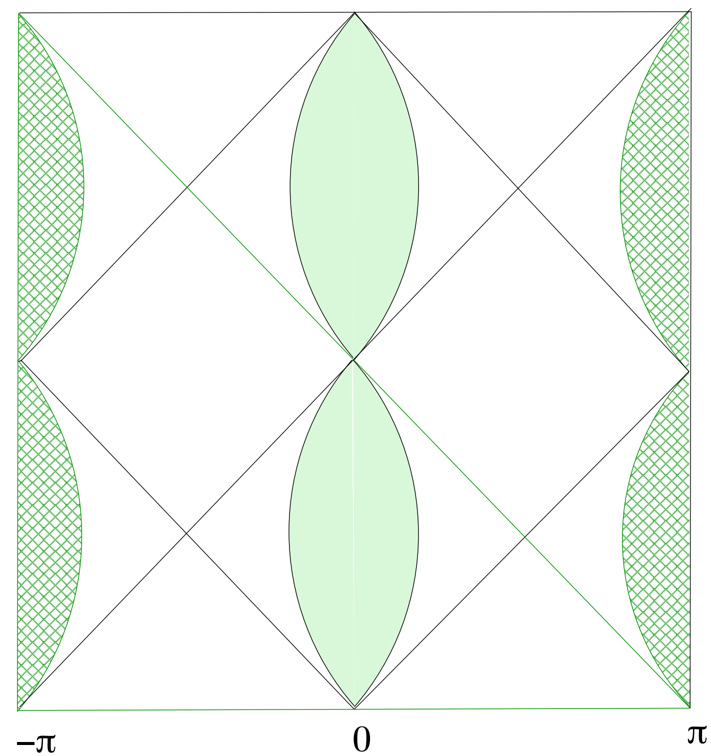
BTZ: $\hat{\phi}\sim\hat{\phi}+2\pi n$

# BTZ

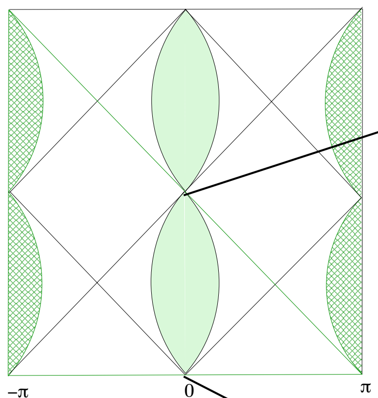
$$\mathbf{u} = -e^{-\mu\,\gamma_1} \qquad \frac{1}{2}\mathrm{Tr}\,\mathbf{u} = -\cosh\mu$$

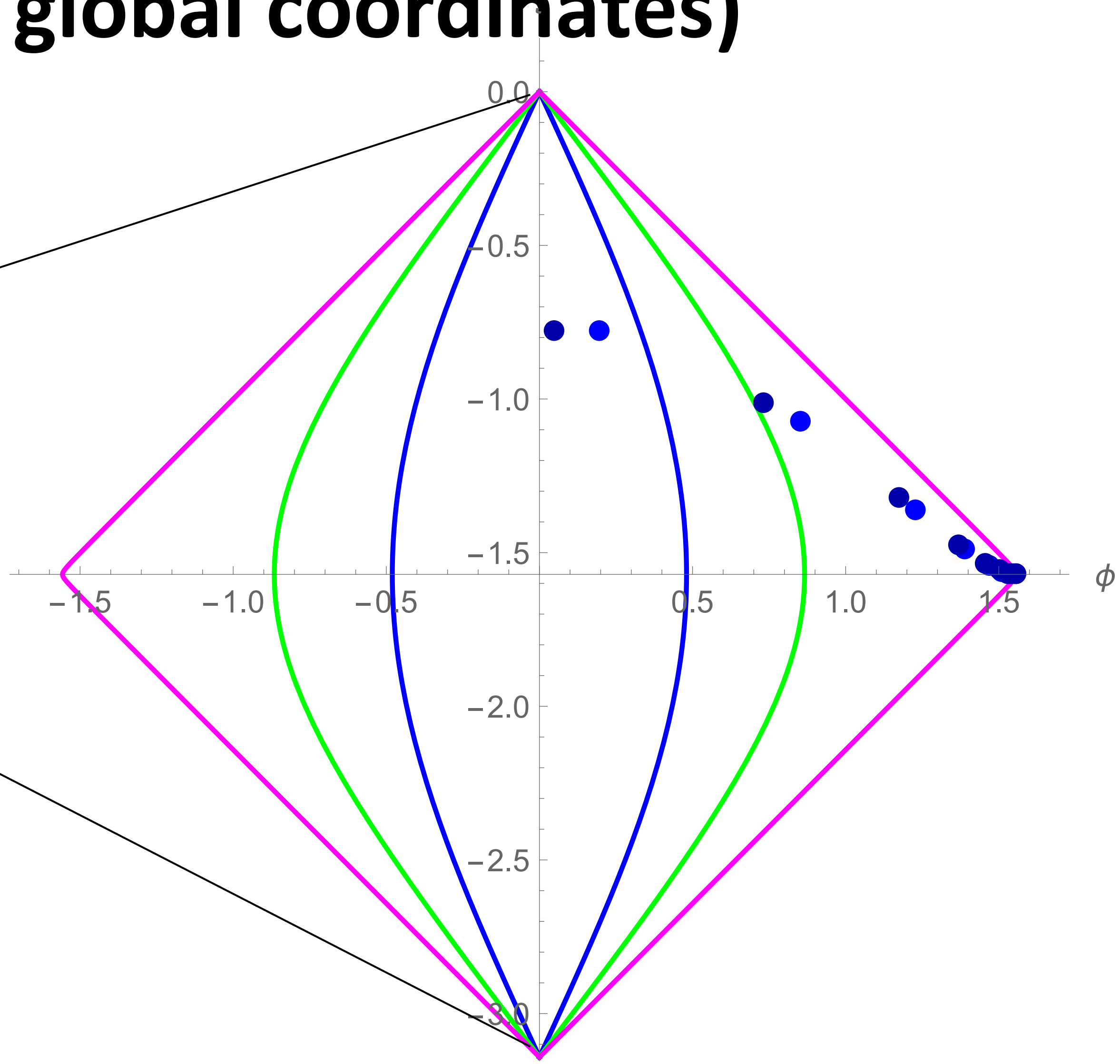


# BTZ (images in global coordinates)

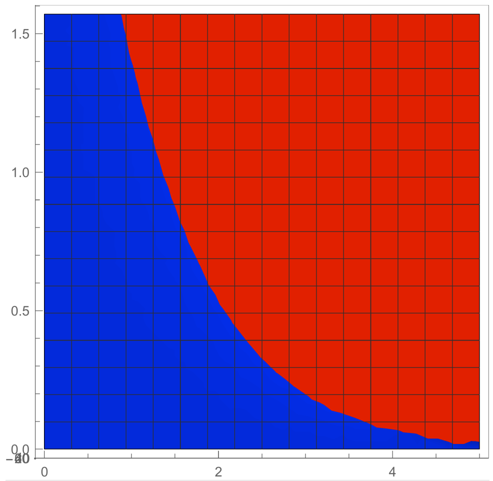



Few steps $\mathbf{u}^n$ to get the fixed point $(-\pi/2, \pi/2)$

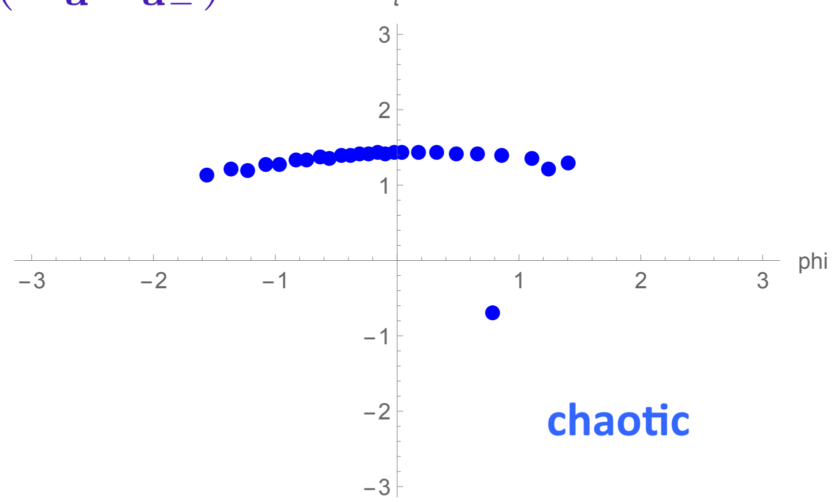
# Collision of two massive particles

$$\mathrm{Tr}\boldsymbol{\Omega}_{\mathbf{u}}\boldsymbol{\Omega}_{\mathbf{u}_-} = 2\cos(\alpha)\cosh^2(\xi) - 2\sinh^2(\xi) = -\cosh\mu$$

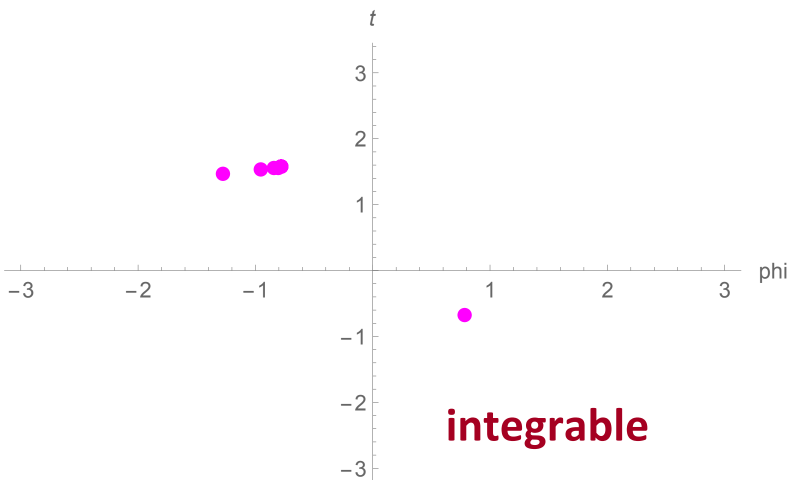



**threshold**

$$(\boldsymbol{\Omega}_{\mathbf{u}}\boldsymbol{\Omega}_{\mathbf{u}_-})^n : Bnd \to Bnd$$

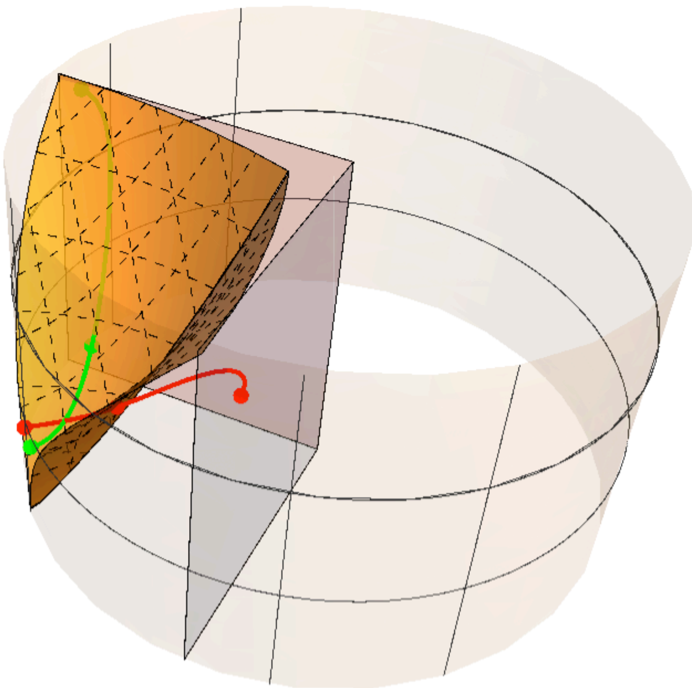


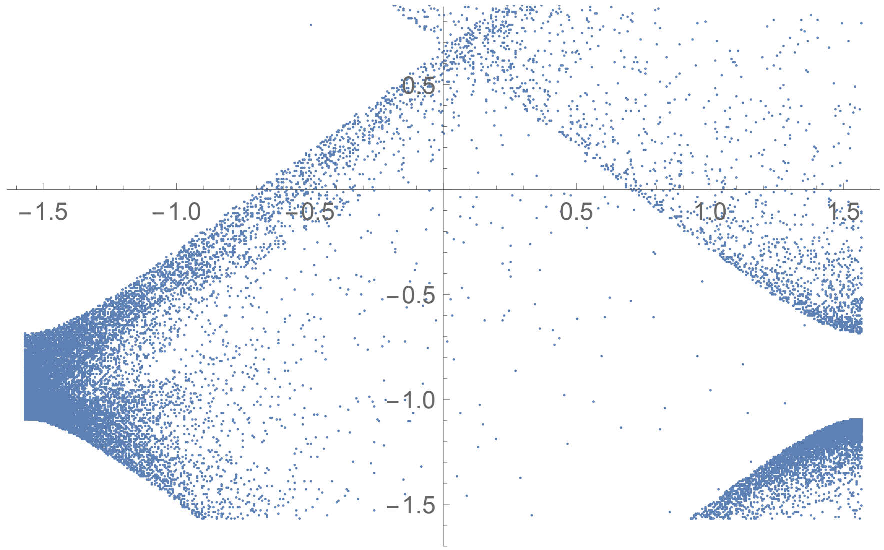

chaotic




integrable

# Collision of  massive and static particles






chaotic

# Conclusion

**Different behaviour of winding geodesics:**

**below threshold chaotic**

**above threshold like to an integrable model**

**We have to check what is going in the CFT language**