Grassmannian integral for form factors in N=4 SYM.

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N=4 SYM theory.

- **N=4 SYM - one may hope that this theory is <u>exactly solvable.</u>**
- **The correlation functions in this theory can be studied in the weak and strong regimes (via AdS/CFT).**
- **The computation of anomalous dimensions of local operators in N=4 SYM in planar limit can be reduced to the problem of solving some integrable system.**
- **There are numerous results for perturbative expansions of amplitudes (S-matrix) with some results valid in all orders of PT (BDS ansatz for 4,5 points, collinear OPE).**
- **Some perturbative results for generalisations of amplitudes (form factors) with arbitrary number of on-shell states.**
- **N=4 SYM is perfect theoretical laboratory development and tests of new ideas, methods and representations for D=4 gauge theories**

Integral over Grassmannian for amplitudes.

One of such remarkable ideas is representations of amplitudes and leading singularities in N=4 SYM in terms of Grassmannian integral and development of on-shell diagram formalism and geometrical interpretation (“amplituhidron”):

On shell information about external particles

$$A_n^{(k)}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \int_\Gamma \frac{d^{n\times k}C_{al}}{Vol[GL(k)]} \frac{1}{M_1 \dots M_n} \prod_{a=1}^{k} \delta^2\left(\sum_{l=1}^{n} C_{al}\tilde{\lambda}_l\right) \delta^4\left(\sum_{l=1}^{n} C_{al}\eta_l\right) \times$$

$$\times \prod_{b=k+1}^{n} \delta^2\left(\sum_{l=1}^{n} \tilde{C}_{al}\lambda_l\right).$$

Hodges ~08

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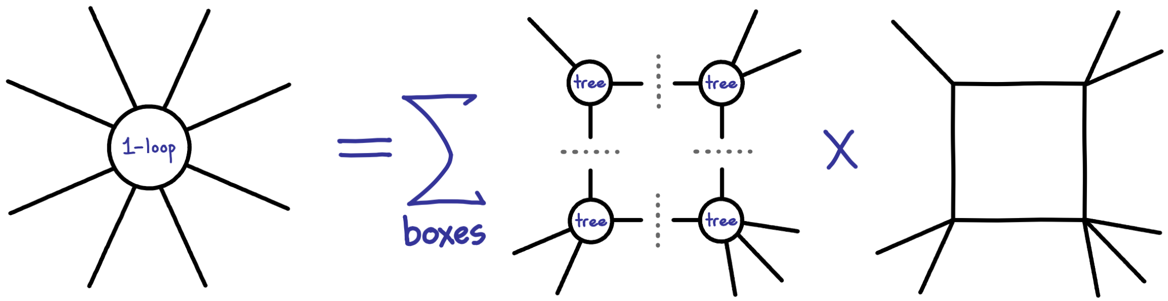
C - is nXk matrix - a point in Grassmannian

$$C\tilde{C}^T = \sum_{i=1}^{n} C_{ai}\tilde{C}_{bi} = 0.$$

This is multidimensional integral over multiple complex variables, which can be computed by residues. Different choices of integration contour gives different BCFW representations for <u>tree amplitudes</u> and <u>leading singularities</u> to all loop order (!). Also this is the most general form of rational Yangian invariant. And do not forget about <u>twistor strings</u>!

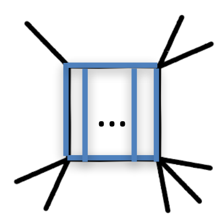
About leading singularities …

1-loop example. Roughly speaking leading singularities are coefficients before master integrals in loop corrections to the amplitudes.



l-loop example:

$$\sum_{\#} \left(\int_{\Gamma_{\#}} \frac{d^{(k-2)(n-k-2)}\tau}{(12\ldots k)\ldots(n1\ldots k-1)} \delta^4(C\cdot\tilde{\eta}) \right) \times$$



pictures
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Grassmannian integral, on shell diagrams and (decorated) permutations.

$$\int_{\Gamma} \frac{d^{n\times k}C_{al}}{Vol[GL(k)]}\frac{1}{M_1 \ldots M_n}\prod_{a=1}^{k}\delta^2\left(\sum_{l=1}^{n}C_{al}\tilde{\lambda}_l\right)\delta^4\left(\sum_{l=1}^{n}C_{al}\eta_l\right)\times$$
$$\times\prod_{b=k+1}^{n}\delta^2\left(\sum_{l=1}^{n}\tilde{C}_{al}\lambda_l\right).$$

There is one to one correspondence between the following objects:

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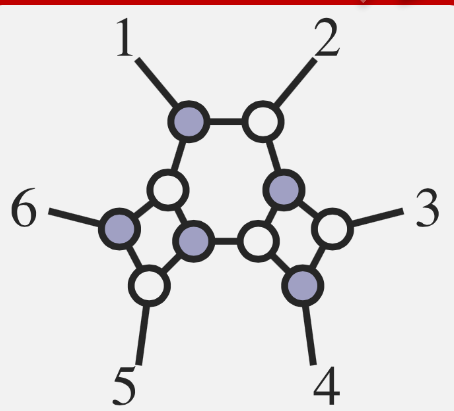

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & 3 & 2 & 6 & 1 & 4 \end{pmatrix}$$

Grassmannian integral and Form Factors ?

- **Considerable progress with understanding the structure of form factors in N=4 SYM (at lest from PT point of view)** **Brandhuber, Travaglini *et-al* 10, Bork, Kazakov 10-14, Zhiboedov 10, Nandan, Wen 12, Wen *et-al* 14, Wilhelm & Co 15, Bork, Onishchenko 15 and more!**
- **Most of “on shell methods” for the amplitudes, such as BCFW at tree level, generalized unitarity, can be applied for form factors as well.**
- **What about Grassmannian integral representation ?**
- **Motivation: arguments (proofs) in favour of equivalence between different BCFW representations, cancellation of spurious poles; leading singularities and also (possibly) BCFW recursion for integrands.**
- **Here (for simplicity) we will consider q^2=0, stress tensor supermultiplet form factors only and form factors of operators of Willson line insertion (gauge invariant off shell gluon). (See also Wilhelm & Co 15 for the form factors with off shell momentum q.)**

½ BPS and Wilson line operator form factors in N=4 SYM.

$$\mathcal{F} = \langle \Omega_1, \ldots, \Omega_N | \mathcal{O}(p) | 0 \rangle$$

$$\mathcal{T}_p = Tr([W^{++}(x,\theta^+,u)]^p)$$

We are most interested in p=2!

$$\mathcal{O}(p) = \int d^4x e^{ix\cdot k} \mathrm{Tr}\left\{ \frac{1}{\pi g} t^c \, \mathcal{P} \exp\left[\frac{ig}{\sqrt{2}} \int_{-\infty}^{\infty} ds \; p\cdot A_b(x+sp) t^b \right] \right\}$$

Ogivetsky, Ivanov, Sokatchev ~*80*

$$|\Omega_{1\ldots n}\rangle = \prod_{i=1}^{n} \Omega_i |0\rangle$$

$$|\Omega_i\rangle = \left(g_i^+ + \eta_A \Gamma_i^A + \frac{1}{2!}\eta_A\eta_B\phi_i^{AB} + \frac{1}{3!}\eta_A\eta_B\eta_C\varepsilon^{ABCD}\bar{\Gamma}_{i,D} + \frac{1}{4!}\eta_A\eta_B\eta_C\eta_D\varepsilon^{ABCD} g_i^- \right) |0\rangle$$

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$$W^{++}(x,\theta^+,u) = \phi^{++} + i\sqrt{2}\theta_\alpha^{+a}\epsilon_{ab}\epsilon^{\alpha\beta}\psi_\beta^{+b} + -i\frac{\sqrt{2}}{2}\theta_\alpha^{+a}\epsilon_{ab}\theta_\beta^{+b}F^{\alpha\beta} + \ldots \qquad \frac{SU(4)}{SU(2)\times SU(2)'\times U(1)}$$

$$\phi^{++}(x,u) = -1/2 u_A^{+a}\epsilon_{ab}u_B^{+b}\phi^{AB}$$

Most natural and simple way to treat form factors in N=4 SYM is to use superspace and on-shell techniques

½ BPS form factors in N=4 SYM in on shell momentum superspace - general structure.

In more details, using supersymmetry arguments one can write form factor as (*Brandhuber et al 10-14, Bork et-al 10-14*):

$$Z_{\mathbf{p},n}(\{\lambda,\tilde{\lambda},\eta\},\{q,\gamma_+\}) \;=\; \delta^4(\sum_{i=1}^{n}\lambda_{\alpha,i}\tilde{\lambda}_{\dot{\alpha},i}-q_{\alpha\dot{\alpha}})\delta^{-4}(q_{+a\alpha}+\gamma_{+a\alpha})\delta^{+4}(q_{-a'\alpha})\mathcal{X}_{p,n}\left(\{\lambda,\tilde{\lambda},\eta\}\right)$$

$$\mathcal{X}_{p,n} \;=\; \mathcal{Y}_{p,n}^{(2p-4)}+\mathcal{Y}_{p,n}^{(2p)}+\ldots+\mathcal{Y}_{p,n}^{(4n+2p-12)} \qquad q_{+a\alpha}=\sum_{i=1}^{n}\lambda_{\alpha,i}\eta_{+a,i},\; q_{-a'\alpha}=\sum_{i=1}^{n}\lambda_{\alpha,i}\eta_{-a',i}$$

“MHV” form factors
Total helisity n-p

“NMHV” form factors
Total helisity n-(p+2)

etc

We are most interested in p=2!

$$\mathcal{Y}_{2,n}^{(0)}=\mathcal{X}_n^{(0)},\;\mathcal{X}_n^{(0)}=\frac{1}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Tree level, stress tensor (p=2) supermultiplet Form factors.

$$\mathcal{Y}_{3,n}^{(2)}=\mathcal{X}_n^{(0)}\sum_{i<j=1}^{n}\langle ij\rangle\frac{1}{2}\eta_{+a,i}\epsilon^{ab}\eta_{+b,j}$$

Tree level, ½-BPS (p=3) supermultiplet Form factors.

Soft theorems for form factors.

$$Z_{2,n+1}\left(\{\epsilon\lambda_s, \tilde{\lambda}_s, \eta_s\}, \{\lambda_1, \tilde{\lambda}_1, \eta_1\}, \dots, \{\lambda_n, \tilde{\lambda}_n, \eta_n\}; q, \gamma\right) =$$
$$\left(\frac{\hat{S}_1}{\epsilon^2} + \frac{\hat{S}_2}{\epsilon}\right) Z_{2,n}\left(\{\lambda_1, \tilde{\lambda}_1, \eta_1\}, \dots, \{\lambda_n, \tilde{\lambda}_n, \eta_n\}; q, \gamma\right) + reg., \ \ \epsilon \to 0$$

$$\hat{S}_1 = \frac{\langle 1n \rangle}{\langle ns \rangle \langle s1 \rangle}, \ \ \hat{S}_2 = \frac{\tilde{\lambda}_s^{\dot{\alpha}}}{\langle s1 \rangle} \frac{\partial}{\partial \tilde{\lambda}_1^{\dot{\alpha}}} + \frac{\tilde{\lambda}_s^{\dot{\alpha}}}{\langle sn \rangle} \frac{\partial}{\partial \tilde{\lambda}_n^{\dot{\alpha}}} + \frac{\eta_s^{\Lambda}}{\langle s1 \rangle} \frac{\partial}{\partial \eta_1^{\Lambda}} + \frac{\eta_s^{\Lambda}}{\langle sn \rangle} \frac{\partial}{\partial \eta_n^{\Lambda}}.$$

Full analogy with amplitudes ! The same for other (1/2-BPS and Konishi form factors). In contrast when q->0:

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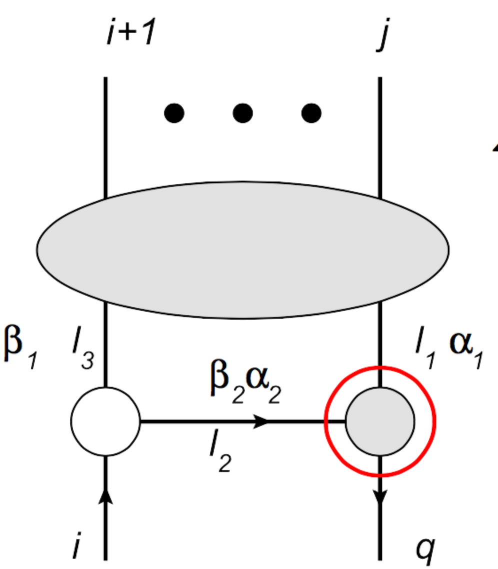
$$Z_{2,n}(\{\lambda, \tilde{\lambda}, \eta\}; 0, 0) = g \frac{\partial A_n(\{\lambda, \tilde{\lambda}, \eta\})}{\partial g}$$

Regular soft limit with respect to q momentum

On shell diagrams and form factors with q^2=0.



One can write the first none vanishing form factor as:

$$Z_2^{(2)}(1,2) = \int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\, Reg.(1,2|q)\, \delta^2\left(\tilde{\lambda}_1+\alpha_1\tilde{\lambda}_3\right)\delta^2\left(\tilde{\lambda}_2+\alpha_2\tilde{\lambda}_3\right)$$
$$\times\ \delta^2\left(\lambda_3+\alpha_1\lambda_1+\alpha_2\lambda_2\right)\hat{\delta}^4\left(\eta_1+\alpha_1\eta_3\right)\hat{\delta}^4\left(\eta_2+\alpha_2\eta_3\right).$$

$$Reg(i,i+1|q) \equiv S^{-1}(i,q,i+1) = \frac{\langle iq\rangle\langle qi+1\rangle}{\langle ii+1\rangle}$$

In some very specific case for carefully chosen coordinates ("canonical") on Grassmannian:

$$Reg(l_1[\alpha,\beta],l_2[\alpha,\beta]|q) = \frac{\langle ql_1[\alpha,\beta]\rangle\langle l_2[\alpha,\beta]q\rangle}{\langle l_1[\alpha,\beta]l_2[\alpha,\beta]\rangle} = \langle iq\rangle\alpha_2\beta_2.$$

However one can conjecture that in general case (M - are minors of C matrix):

$$Reg = \sum_i \langle iq\rangle \frac{M_a^{(i)}}{M_b^i}.$$

Is Reg. really the regulator ?

So for every on-shell diagram with form factor vertex one can obtain the following Grassmannian integral:

$$\Omega_n = \int \prod_{i=1}^{n_w} \frac{d\alpha_{1i}}{\alpha_{1i}} \frac{d\alpha_{2i}}{\alpha_{2i}} \prod_{j=1}^{n_b} \frac{d\beta_{1i}}{\beta_{1i}} \frac{d\beta_{2i}}{\beta_{2i}} \prod_{m=1}^{n_I} \frac{1}{U(1)_m} Reg(l_1[\alpha,\beta], l_2[\alpha,\beta]|q) \times$$
$$\times \quad \delta^{4|4}(1,\ldots,i,q,i+1,\ldots,n),$$

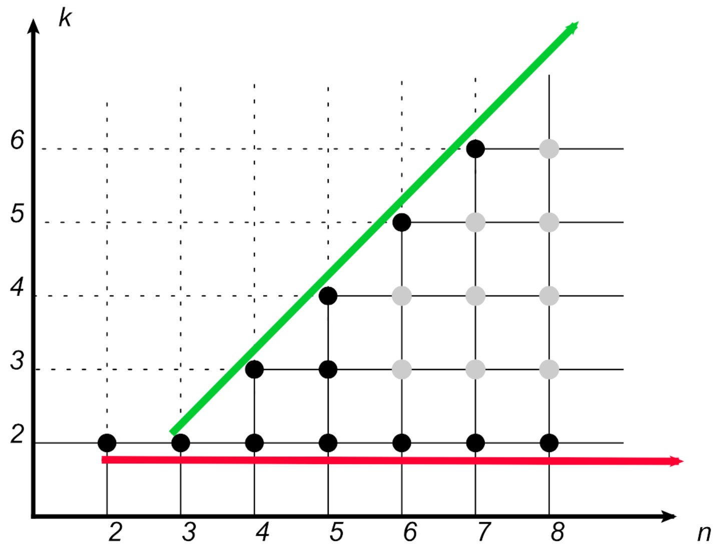
where

$$\delta^{4|4}(1,\ldots,i,q,i+1,\ldots,n) =$$
$$= \prod_{a=1}^{k} \delta^2\left(\sum_{l=1}^{n+1} C_{al}[\alpha,\beta]\tilde{\lambda}_l\right) \delta^4\left(\sum_{l=1}^{n+1} C_{al}[\alpha,\beta]\eta_l\right) \prod_{b=k+1}^{n} \delta^2\left(\sum_{l=1}^{n+1} \tilde{C}_{al}[\alpha,\beta]\lambda_l\right)$$

Where for some specific diagrams one can get:

$$Reg(l_1[\alpha,\beta], l_2[\alpha,\beta]|q) = \frac{\langle q l_1[\alpha,\beta]\rangle \langle l_2[\alpha,\beta] q\rangle}{\langle l_1[\alpha,\beta] l_2[\alpha,\beta]\rangle} = \langle iq\rangle \alpha_2 \beta_2.$$

Grassmannian integral representation for form factors with q^2=0. The conjecture.



One can solve BCFW ([1,2> shift is implemented) for MHV (red) and n=k+1 N^kMHV sectors (green). The result for N^kMHV can be written as:

$$Z_n^{(0)MHV} = \delta^8(q+\gamma)\mathcal{X}_n^{(0)} \quad \text{k=2}$$

$$\mathcal{X}_n^{(0)} = \frac{1}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

and:

$$Z_{k+1}^{(k)} = \sum_{j=4}^{k+1} \frac{\langle q|p_1+p_j+\ldots+p_{k+1}|2]}{[q2]} A_{k+2}^{(k)}(1,\ldots,j-1,q,j,\ldots,k+1) +$$

$$+ \frac{\langle q|p_1|2]}{[q2]} A_{k+2}^{(k)}(1,\ldots,k+1,q), \;\; k\geq 3.$$

Henn ~09

Grassmannian integral representation for form factors with q^2=0. The conjecture.

Our conjecture is that the following grassmannian integral for appropriate choice of the contour can reproduce tree level form factors:

$$\Omega_n^{(k)}[\Gamma] = \sum_{j=4}^{k+1} \int_\Gamma \frac{d^{n\times k}C_{al}}{Vol[GL(k)]} \frac{Reg_j^{R,(k)}}{M_1 \ldots M_n} \delta^{4|4}(1,\ldots,j-1,q,j,\ldots,n) + \int_\Gamma \frac{d^{n\times k}C_{al}}{Vol[GL(k)]} \frac{Reg_n^{L,(k)}}{M_1 \ldots M_n} \delta^{4|4}(1,\ldots,n,q),$$

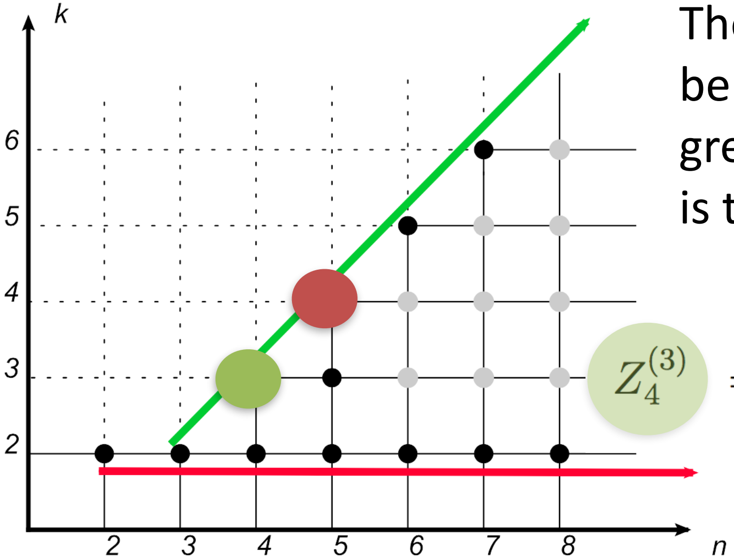
Where we choose Reg. functions as follows. (i…j) are minors of C constructed from i …j columns of C matrix:

$$Reg_j^{R,(k)} = \langle q1\rangle \frac{(kk+2\ 3\ldots k)}{(13\ldots j-1\ j+1\ldots n+1)} + \sum_{i=j}^{k+1}\langle qi\rangle \frac{(13\ 4\ldots i\ i+2\ldots k+2)}{(13\ldots j-1\ j+1\ldots n+1)}$$

$$Reg_n^{L,(k)} = \langle q1\rangle \frac{(nn+1\ 3\ldots k)}{(1n\ 3\ldots k)},$$

For k=2 the first line is zero.

Buy the way …



The explicit form of n=5,k=4 form factor for example can be seen below. Our Grassamnnian integral reproduces all green and red form factors. But the Grassmannian integral is trivial in such cases.

$$Z_4^{(3)} = \delta^8(q_{1\dots4}+\gamma)\frac{\hat{\delta}^4(123)}{\langle 4q\rangle^4}\left(\frac{\langle 1q\rangle[12][q3][14]+\langle 3q\rangle[23][34][q1]}{[1q][2q][3q][4q]\mathcal{P}^*(1234)}\right)$$

Here:

$$\hat{\delta}^4(ijk) = \hat{\delta}^4(\eta_i[jk]+\eta_j[ki]+\eta_k[ij]).$$

$$Z_5^{(4)} = \delta^8(q_{1\dots5}+\gamma)\Big(\frac{\hat{\delta}^4(12q)\hat{\delta}^4(345)}{(p^2_{345})^4}\frac{\langle q|p_1|2]}{[q2]\mathcal{P}^*(12345q)}+\frac{\hat{\delta}^4(125)\hat{\delta}^4(34q)}{(p^2_{125})^4}\frac{\langle q|p_3|5]}{[q5]\mathcal{P}^*(12q345)}$$
$$+\frac{\hat{\delta}^4(125)\hat{\delta}^4(34q)}{(p^2_{125})^4}\frac{\langle q|p_4|2]}{[q2]\mathcal{P}^*(1234q5)}\Big),$$

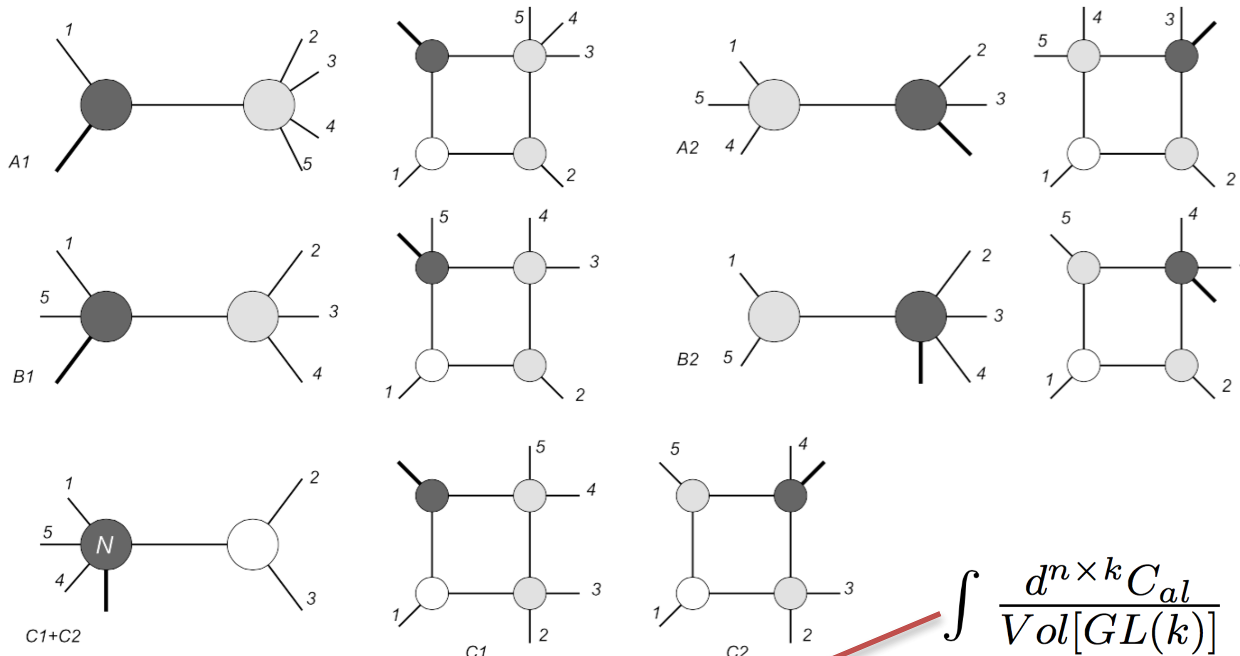
Where: $\mathcal{P}^*(1\dots n) = [12][23]\dots[n1]$

Not so trivial example. NMHV 5 point form factor.

$$Z_5^{(3)} = Z_5^{(2)} \left(R_{132}^{(1)} + R_{142}^{(1)} + R_{153}^{(1)} + R_{152}^{(2)} + R_{142}^{(2)} + R_{153}^{(2)} \right)$$

[1,2> Shift

The 5 point NMHV form factor are give by the following set of BCFW terms ([1,2> shift).



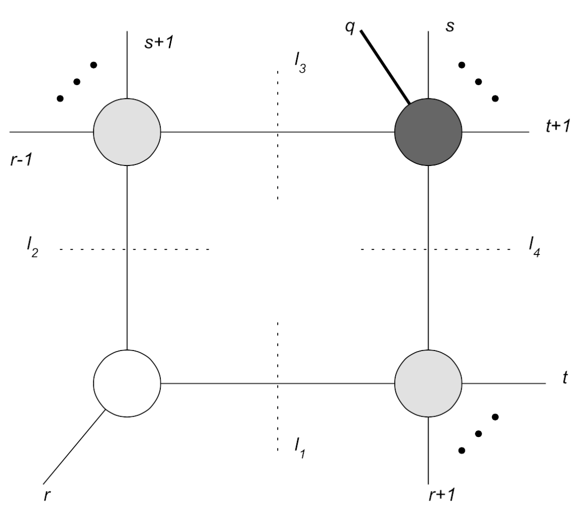
Can our Grassamnnian integral (which is no longer trivial) reproduce this result ?

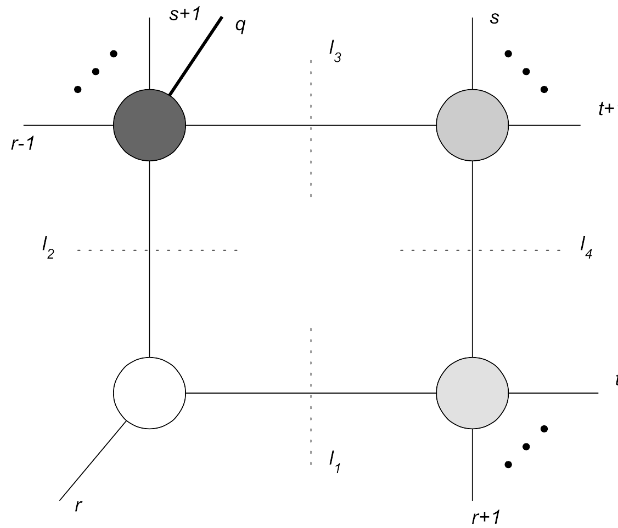
$$\int \frac{d^{n\times k}C_{al}}{Vol[GL(k)]} \equiv \int$$

$$\Omega_5^{(3)} = \int \left(\langle 1q\rangle \frac{(345)}{(135)} + \langle 4q\rangle \frac{(134)}{(135)} \right) \frac{\delta^{4|4}(1,2,3,q,4,5)}{M_1 \dots M_6} + \int \left(\langle 1q\rangle \frac{(356)}{(135)} \right) \frac{\delta^{4|4}(1,2,3,4,5,q)}{M_1 \dots M_6}.$$

Not so trivial example. NMHV 5 point form factor. R-functions definitions.

Some more details about NMHV 5 point form factor. The R functions are five by:





$$R^{(1)}_{rst} = \frac{\langle s+1 s\rangle\langle t+1 t\rangle \hat{\delta}^4\left(\sum_{i=r+1}^{t}\eta_i\langle i|p_{s+1\ldots t}p_{s+1\ldots r+1}|r\rangle - \sum_{i=r}^{s+1}\eta_i\langle i|p_{s+1\ldots t}p_{t\ldots r+1}|r\rangle\right)}{p^2_{s+1\ldots t}\langle r|p_{r\ldots s+1}p_{t\ldots s+1}|t+1\rangle\langle r|p_{r\ldots s+1}p_{t\ldots s+1}|t\rangle\langle r|p_{t\ldots r}p_{t\ldots s+1}|s+1\rangle\langle r|p_{t\ldots r}p_{t\ldots s+1}|s\rangle};$$

$$R^{(2)}_{rst} = \frac{\langle s+1 s\rangle\langle t+1 t\rangle \hat{\delta}^4\left(\sum_{i=t}^{r+1}\eta_i\langle i|p_{s\ldots t+1}p_{r+1\ldots s}|r\rangle + \sum_{i=r}^{s+1}\eta_i\langle i|p_{s\ldots t+1}p_{t\ldots r+1}|r\rangle\right)}{p^2_{s\ldots t+1}\langle r|p_{r\ldots s}p_{s\ldots t+1}|t+1\rangle\langle r|p_{r\ldots s}p_{s\ldots t-1}|t\rangle\langle r|p_{t\ldots r+1}p_{s\ldots t+1}|s+1\rangle\langle r|p_{t\ldots r+1}p_{s\ldots t+1}|s\rangle}.$$

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Not so trivial example. NMHV 5 point form factor. The choice of the contour.

$$\Omega_5^{(3)} = \int \left(\langle 1q \rangle \frac{(345)}{(135)} + \langle 4q \rangle \frac{(134)}{(135)} \right) \frac{\delta^{4|4}(1,2,3,q,4,5)}{M_1 \ldots M_6} + \int \left(\langle 1q \rangle \frac{(356)}{(135)} \right) \frac{\delta^{4|4}(1,2,3,4,5,q)}{M_1 \ldots M_6}.$$

In the case under consideration Grassamannian integral can be reduced to the one parameter integral over complex variable τ

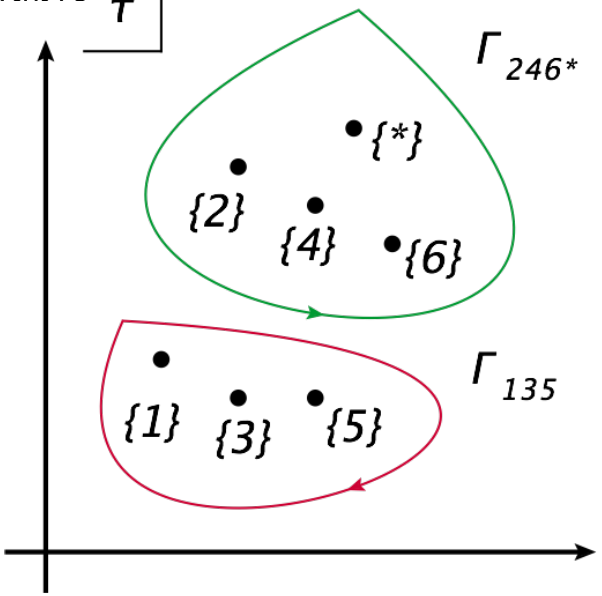
$$\int \frac{d^{n\times k} C_{al}}{Vol[GL(k)]} \equiv \int$$

By appropriate choice of integration contour one can obtain the following identity:

$$\Omega_5^{(3)}[\Gamma_{135}] = Z_5^{(3)\ [1,2\rangle}.$$

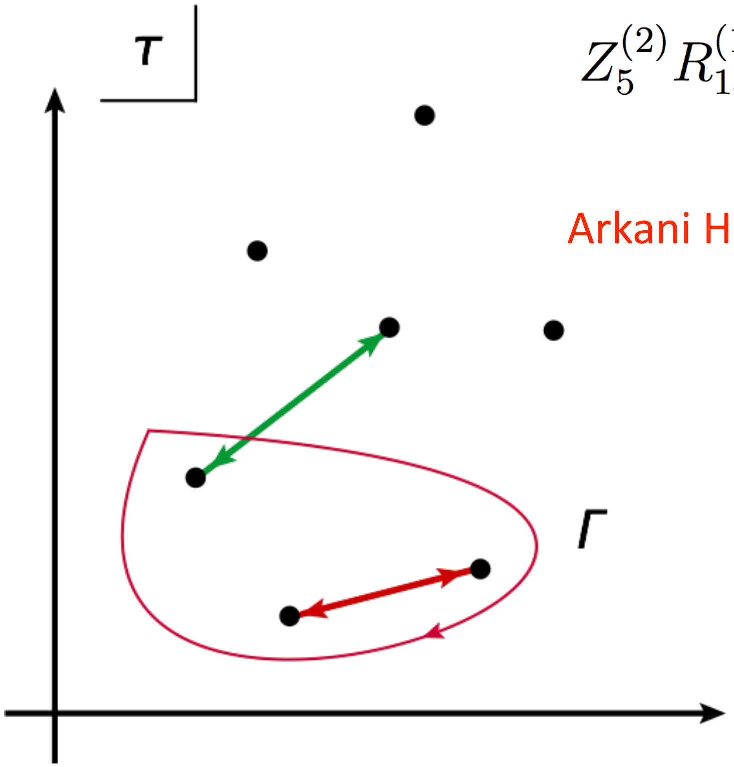
And (with some subtleties) with different contour:

$$\Omega_5^{(3)}[\Gamma_{246*}] = Z_5^{(3)\ [2,3\rangle}$$



Full analogy with NMHV 6 point amplitude case!

NMHV 5 point form factor example. Spurious poles cancellation.



Typical form of individual BCFW term:

$$Z_5^{(2)} R_{132}^{(1)} = \frac{\langle 3q\rangle \hat{\delta}^4(234)}{\langle 45\rangle\langle 15\rangle[3q][2q]\langle 1|5+4|q]\langle 4|5+1|2]p_{154}^2}$$

Spurious poles. Phys. pole.

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Poles like $\langle 3|5+4|q]$ or $\langle 3|1+2|5]$ should cancel in the full result (sum of all BCFW terms).

Poles like p_{123}^2 should remain in the full answer.

In Grassmannian picture spurious poles can be avoided by the choice of integration contour, while physical poles cannot! (see NMHV 5 example).

What about Wilson line operator form factors (gauge invariant off-shell amplitudes) ?

Using similar ideas as in q^2=0 case one can obtain:

$$A^{*(k)}_{n+1} = \hat{\Pi}_{n+1,n+2} \int \frac{d^{k\times(n+2)}C'}{\mathrm{Vol}[GL(k)]} \frac{Reg}{M_1\cdots M_{n+1}M_{n+1}} \times \delta^{4|4}(1,\ldots,n,p,\xi)$$

$$Reg \sim \langle \xi p\rangle \frac{M_{n+2}}{(n+1\;1\cdots k-1)}$$

From this, for example, using NMHV6 tree contour one can obtain :

$$A^{*(3)}_{4+1}(1^+2^+3^-4^-|q) = \frac{1}{\kappa^*}\frac{\langle p|3+4|2]^3}{\langle p1\rangle[23][34]p^2_{2,4}\langle 1|2+3|4]} + \frac{1}{\kappa}\frac{\langle 3|1+2|p]^3}{\langle 12\rangle\langle 23\rangle[4p]p^2_{1,3}\langle 1|2+3|4]}$$

$$q = p + q^{\perp},\; p^2 = q^{\perp 2} = 0,\; \kappa^*\kappa = q^2$$

Result coincides with BCFW. Also other consistency tests were performed.

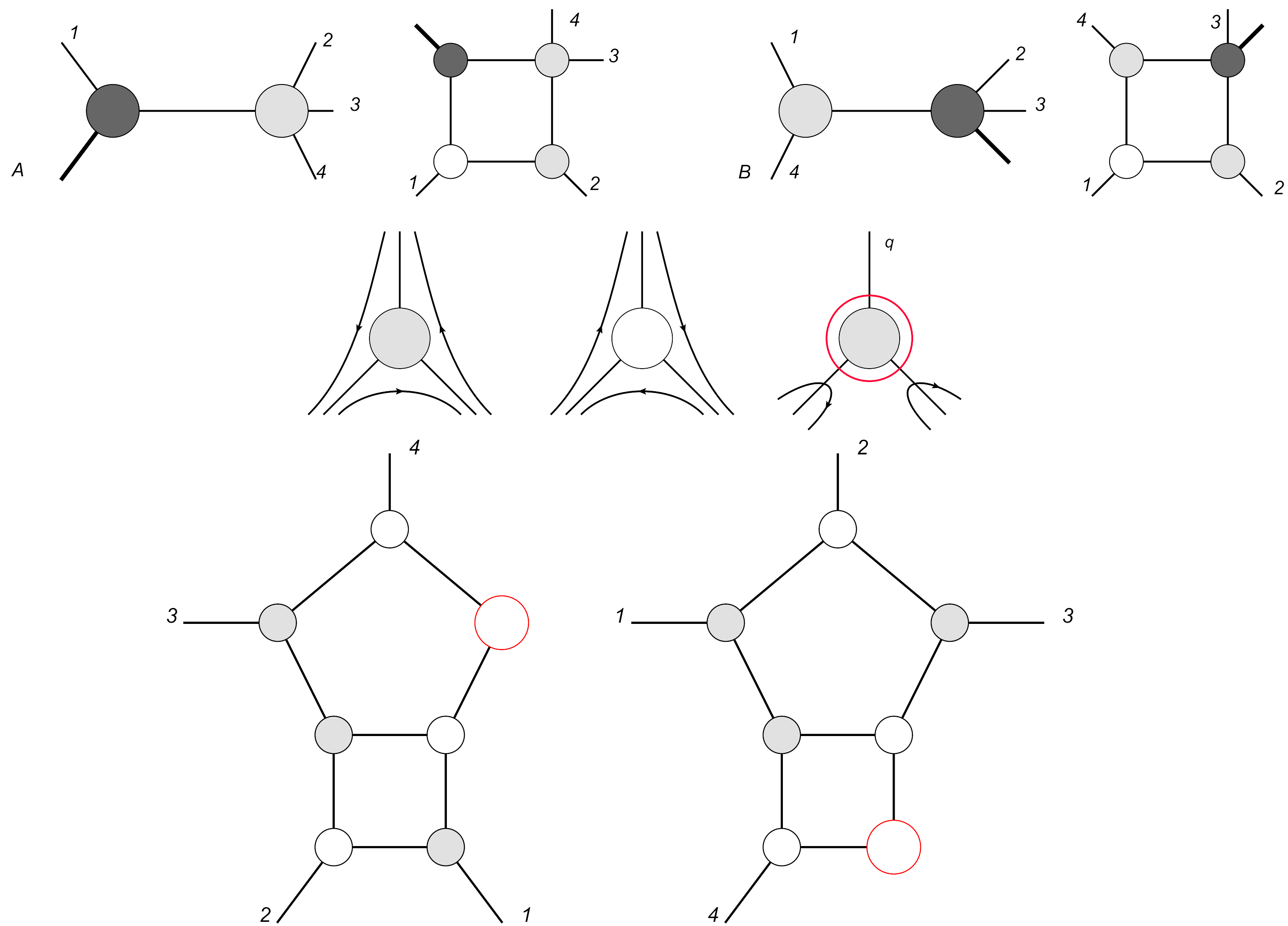
Conclusions.

- The powerful on-shell methods in N=4 SYM can be applied to partially off-shell objects and objects with different colour structures as well.
- The Grassmannian representation can be formulated for form factors in

N=4 SYM more or less in the same way as for the amplitudes.

- Duality between operators and states in N=4 SYM ?
- Leading singularities structure of form factors from Grassmannian integral ?
- Twistor string description of form factors as well as amplitudes ?
- Additional simplifications in suggested Grassmannian representation (true top-cell object, not linear combination of amplitude like top-cells) ?
- There is hope that N=4 SYM is integral theory and its S-matrix can be completely fixed by the symmetry arguments. If this is the case, then one may expect even more rich structure for form factors (like in 2D sin-gordon models).

$$Z_4^{(3)} = \delta^8(q_{1\ldots4}+\gamma)\frac{\hat{\delta}^4(123)}{\langle 4q\rangle^4}\left(\frac{\langle 1q\rangle[12][q3][14]+\langle 3q\rangle[23][34][q1]}{[1q][2q][3q][4q]\mathcal{P}^*(1234)}\right)$$



Inverse soft limit recursion and integrability for form factors.

New recursion relations from soft limit (Dhritiman Nandan, Congkao Wen):

$$Z_n(1,...n|q,\gamma) = \sum_{i;R,L} [(\prod_R \mathcal{S}_R) Z_{i+1}(\hat{1},...,\hat{i+1}|q,\gamma) + (\prod_L \mathcal{S}_L) Z_{n-i}(\hat{i},...,\hat{n}|q,\gamma)]$$

Where (+ substitutions of spinors):

$$\mathcal{S}_+(n,i-1,i) = \frac{\langle ni \rangle}{\langle ni-1 \rangle \langle i-1i \rangle},$$

$$\mathcal{S}_-(n,i-1,i) = \frac{1}{[ni-1][i-1i][in]^3} \hat{\delta}^4(\eta_n[ii-1] + \eta_i[ni-1] + \eta_{i-1}[in])$$

With interpretations as R-matrixes of **gl**(4|4) spin chain!

Inverse soft limit recursion and integrability for form factors.

$$\mathcal{S}_+(n,i-1,i) = \frac{\langle ni\rangle}{\langle ni-1\rangle\langle i-1i\rangle},$$
$$\mathcal{S}_-(n,i-1,i) = \frac{1}{[ni-1][i-1i][in]^3}\hat{\delta}^4(\eta_n[ii-1]+\eta_i[ni-1]+\eta_{i-1}[in])$$

$$(\mathcal{S}_+(n1n-1))^*X|_{sub.} = R(0)_{n1}R(0)_{nn-1}X\delta^2(\lambda_n),$$

$$\mathcal{S}_-(n1n-1)X|_{sub.} = R(0)_{n1}R(0)_{12}X\delta^2(\tilde{\lambda}_n)\hat{\delta}^4(\eta_n).$$

The subtleties with (pseudo)vacuum state for form factors. Form factors (in general) are eigenvectors of Tr(M) while amplitudes are eigenvectors of M.

Where R's are R-matrixes of **gl**(4|4) spin chain (Chicerin,Derkachov,Krichner,Staudacher et.all 13-14):

$$R_{12}(u) = \int \frac{dz}{z^{1-u}} exp[-z(\mathbf{p}_1\mathbf{x}_2)],$$

Where $\mathbf{x}_i = (\lambda_i, \partial/\partial\tilde{\lambda}_i, \partial/\partial\eta_i)$ and $\mathbf{p}_i = (\partial/\partial\lambda_i, -\tilde{\lambda}_i, -\eta_i)$. $L(u)$ matrix is given by

$$L(u) = u\mathbb{I} + \mathbf{x}\otimes\mathbf{p},$$