Conserved charges in HS theory

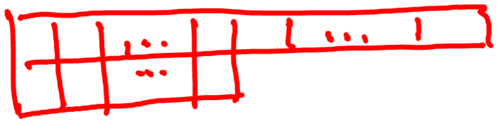
(with N. G. Misuna and M.A. Vasiliev)

- Introduction
- Structure of HS interactions
  Vasiliev equations
- Conserved and asymptotic charges
- Application
  HS black holes
- Conclusion

structure of HS interactions

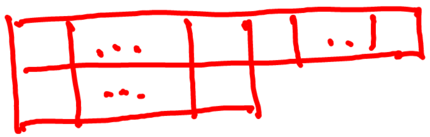
$$\underbrace{AdS \to \text{Fronsdal fields}}_{\text{HS algebra}} \to \text{Noether interaction}$$

Set of fields where HS algebra acts naturally (unfolding)



two-row Young diagrams (1- and 0- forms)

Major simplification in $d=4$



$\sim F(y,\bar{y})$; $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$

Vasiliev equations in $d=4$

- Free system $(d=4)$:

HS algebra Y_A $A=1,\dots,4$ $[Y_A, Y_B]_* = 2i\epsilon_{AB}$

Master fields $C(Y|x)$, $\omega(Y|x)$

AdS background $\Omega = \Omega_{AB} Y^A Y^B$ $\Omega_{AB} = (\omega_{\alpha\beta}, \bar{\omega}_{\dot\alpha\dot\beta}, e_{\alpha\dot\beta})$

equations:

$$\begin{cases} d\Omega + \Omega * \Omega = 0 \\ dC + \Omega * C - C * \pi(\Omega) = 0 \\ d\omega + \{\Omega, \omega\}_* = (e \wedge e)^{\alpha\beta} \dfrac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y,0) + c.c. \end{cases}$$

$$\pi(y, \bar{y}) = (-y, \bar{y})$$

- **Interactions**

$$\begin{cases} dC + \Omega * C - C * \pi(\Omega) = \mathcal{V}(\omega, C) + \mathcal{V}(\omega, C, C) + \ldots + \mathcal{V}(\omega, C, \ldots, C) + \ldots \\ d\omega + \{\Omega, \omega\}_* = \mathcal{V}(\omega, \omega) + \mathcal{V}(\omega, \omega, C) + \ldots + \mathcal{V}(\omega, \omega, C, \ldots, C) + \ldots \end{cases}$$

HS vertices:

$$\begin{cases} \mathcal{V}(\omega, C) = -\omega * C + C * \pi(\omega) \\ \mathcal{V}(\omega, \omega) = -\omega * \omega \end{cases}$$

– local

free level observation: $[z_A, z_B]_* = -[y_A, y_B]$; $[y_A, z_B]_* = 0$

$$\rightarrow \quad W(Y, Z) = \omega + z^\alpha \int_0^1 dt\,(1-t)\left[\Omega,\ z_\alpha C(-tz, \bar{y})\, e^{itzy}\right]_*$$

$$\rightarrow \quad dW + [\Omega, W]_* = 0 \qquad (\text{for any } z)$$

$$
\left.\begin{aligned}
C(Y|X) &\to B(z,Y|X) = C(Y|X) + \sum \gamma(C,\ldots,C)\, z,\ldots,z \\
\omega(Y|X) &\to W(z,Y|X) = \omega(Y|X) + \sum \gamma(\omega, C,\ldots,C)\, z,\ldots,z
\end{aligned}\right\}
\quad
\begin{cases}
dW + W*W = 0 \\
dB + W*B - B*\pi(W) = 0
\end{cases}
$$

Full nonlinear system:

$$
\begin{cases}
\hat{d}B + \hat{W}*B - B*\pi(\hat{W}) = 0 \\
\hat{d}W + \hat{W}*\hat{W} = dz_\alpha \wedge dz^\alpha\, B*\varkappa + d\bar{z}_{\dot{\alpha}} \wedge d\bar{z}^{\dot{\alpha}}\, B*\bar{\varkappa}
\end{cases}
$$

$$
\begin{cases}
d \to \hat{d} = d \oplus d_z \\
W \to \hat{W} = W \oplus A_B(Y,Z)\, dZ^B
\end{cases}
$$

$$
\begin{cases}
\varkappa * \varkappa = 1 \\
\varkappa * f(y,z) * \varkappa = f(-y,-z)
\end{cases}
\quad \text{Klein operator}
$$

$$
\varkappa = e^{i z_\alpha y^\alpha}
$$

Vasiliev's form of Vasiliev equations

$$A_A = S_A + \# z_A \quad ; \qquad \mathcal{W} = d + W + S \; ;$$

Extra klein operators : $\{k, dz_\alpha\} = \{k, y_\alpha\} = \{k, z_\alpha\} = 0 \quad ; \quad \bar{k} \; \ldots$

$k^2 = 1$

$$\begin{cases} \mathcal{W} * \mathcal{W} = -i\left(dz_A \wedge dz^A + \eta B * k\varkappa\, dz_\alpha \wedge dz^\alpha + \bar{\eta} B * \bar{k}\bar{\varkappa}\, d\bar{z}_{\dot\alpha} \wedge d\bar{z}^{\dot\alpha}\right) \\ [\mathcal{W}, B]_* = 0 \end{cases}$$

$$W = W(z, Y; k, \bar{k}) \qquad B = B(z, Y; k, \bar{k})$$

HS fields : $W(-k, -\bar{k}) = W(k, \bar{k}) \quad ; \quad B(-k, -\bar{k}) = -B(k, \bar{k})$

topolog. fields: $W(-k, -\bar{k}) = -W(k, \bar{k}) \quad ; \quad B(-k, -\bar{k}) = B(k, \bar{k})$

Solving HS equations

- AdS background

$$W_0(Z,Y) = \Omega(Y) = \Omega_{AB}Y^AY^B$$

$$B_0(Z,Y) = 0\,, \qquad S_0(Z,Y) = Z_A\, dZ^A$$

- free level and higher $\quad d_Z F = J(Z,Y)$

$$\begin{cases} d_Z B_n(Z,Y) = \mathcal{V}(C\dots C) \\ d_Z S_n(Z,Y) = \mathcal{V}(C\dots, C, dZ) \\ d_Z W_n(Z,Y) = \mathcal{V}(\omega, C\dots C) \end{cases}$$

$$F = (Z^A + \alpha Y^A)\int_0^1 dt\, J_A(tZ + \alpha(1-t)Y, Y)$$

$$\alpha = \begin{cases} 1 & \text{left} \\ 0 & \text{central} \\ -1 & \text{right} \end{cases} \quad \text{gauges}$$

Conserved HS charge in $d=4$

$$d\omega(Y) + \omega * \omega = \mathcal{V}(\omega,\omega,C) + \mathcal{V}(\omega,\omega,C,C) + \dots + \mathcal{V}(\omega,\omega,C,\dots C) + \dots = J$$

$$Q = \int (2\text{-form}) \qquad d(2\text{-form}) = 0 \qquad \delta_\varepsilon Q = 0$$

$$d\omega(0|x) = J(0|x) \rightarrow dJ(0|x) = 0$$

$$Q = \int J(0|x)\, d^2 S$$

$$Q = -\int W(z,y|x) * W(z,y|x)\Big|_{Y,Z,k,\bar{k}=0}$$

free level example

$$J(\xi|x) = i\mu\,(e\wedge e)^{\dot\alpha\dot\beta}\,\frac{\partial^2}{\partial\bar y^{\dot\alpha}\,\partial\bar y^{\dot\beta}}\Big(C^{top}(\xi) * C + C * \pi\big(C^{top}(\xi)\big)\Big)\Big|_{Y=0} + c.c.$$

$$D_0^{tw} C = 0 \;;\qquad D_0\, C^{top}(\xi) = 0 \;\rightarrow\; D_\Omega\, \underbrace{\xi_{A_1,\dots,A_n}}_{\text{Killing tensors}} = 0$$

- $\xi_{A_1\dots A_n} := \xi_{AB}$

$$J_{s=0} = 4i\mu(e\wedge e)^{\dot\alpha\dot\beta}\Big(\bar\xi_{\dot\alpha\dot\beta}\, C(x) - 2i\,\xi^{\beta}{}_{\dot\delta}\, C_{\beta\dot\beta}(x) - \frac{1}{2}\xi^{\beta\beta}\, C_{\beta\beta,\dot\alpha\dot\beta}\Big) + c.c.$$

$$J = J_{s=0} + J_{s=2}$$

$$J_{s=2} = 2i\mu(e\wedge e)^{\dot\alpha\dot\beta}\,\bar\xi^{\dot\gamma\dot\delta}\,\bar C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}(x) + c.c.$$

Application

- HS black holes

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a}{\Xi}\sin^2\theta\, d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\sin^2\theta\,\Delta_\theta}{\rho^2}\left(a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right)^2$$

Kerr solution in GR

$$g^{Kerr}_{\mu\nu} = g^{AdS}_{\mu\nu} + \frac{M}{U(r,\theta)} K_\mu K_\nu$$ — Kerr-Schild form

— solves free HS eqs, [D, Matveev, Vasiliev]

AdS global symmetry parameter (isometry)

$$D_\Omega K_{AB} = 0 \quad \longrightarrow \quad C^{Kerr}(Y|x) = \left[F(K_{AB}Y^A Y^B) * \delta^2(Y)\right]_{s=2} + c.c.$$

$$C_2 = \Lambda(1 - a^2\Lambda)\,; \quad C_4 - C_2^2 = -4a^2\Lambda^3\,; \quad C_2 = Tr\,K^2\,; \quad C_4 = Tr\,K^4$$

- Most symmetric cases

$$K_A{}^B = -\varepsilon\, \delta_A{}^B \;; \qquad \varepsilon = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

Schwarzschild
planar
hyperbolic

for generic K_{AB} at $O(M)$ - level the charge correctly reproduces ADM values, e.g.

$$Q_{s=1} = \frac{m_e}{1+\Lambda a^2}$$

(Kerr - Newman charge)

$K^2 = -1$ - is exactly solvable. Charge can be computed to all orders

$O(M^2)$ analysis

- Schwarzschild

$$B = \sum_s m_s B_s$$

if $\# m_s < \infty \rightarrow Q < \infty$

if $\# m_s = \infty \rightarrow \begin{cases} m_s \text{ factorialy decays with spin} \\ m_s = m \quad \text{(BPS)} \end{cases}$

- planar BH

if $\# m_s < \infty \rightarrow Q = \infty$

for $\# m_s = \infty$ is not clear so far.

Conclusion

- It was demonstrated that in $d=4$ HS theory one can define gauge invariant on-shell functionals
- 4-form functional is conjectured to be responsible for boundary correlation functions
- 2-form allows one calculating conserved charges on BH-like solutions
- $O(M^2)$ - analysis demonstrate that some known solutions deliver infinite charges.