# Cartan subalgebras of sp(4;C) and Exact Solutions to Vasiliev's 4D equations

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# Summary

- Basics of the 4D bosonic Vasiliev's equations
- Systematics of the construction of 6 families of exact solutions
  - Gauge function method
  - Expansion of the initial data on various $\mathfrak{so}(3,2)$-modules
  - Projectors and twisted projectors
- Choice of Cartan subalgebra of $\mathfrak{sp}(4;C)$ and physical significance of solutions
  - Compact/particle basis vs. conformal basis.
- Conclusions

# Kinematics

- Master-fields living on *correspondence space*, locally $\mathcal{X} \times \mathcal{Z} \times \mathcal{Y}$:

$$\widehat{W} = dx^\mu \widehat{W}_\mu(Y,Z|x) \longrightarrow$$ gauge fields of all spins + auxiliary

$$\widehat{B} = \widehat{B}(Y,Z|x) \longrightarrow$$ Weyl tensors and their derivatives → local dof

$$\widehat{S} = dz^\alpha \widehat{S}_\alpha(Y,Z|x) + d\bar{z}^{\dot{\alpha}} \widehat{\bar{S}}_{\dot{\alpha}}(Y,Z|x) \longrightarrow$$ Z-space connection, no extra local dof

- Commuting oscillators $Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot{\alpha}})$ , $Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}})$ → $\mathfrak{sp}$(4,R) quartets

$$[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i \begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix} , \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}} , \quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Star-product:

$$\widehat{F}(Y,Z) \star \widehat{G}(Y,Z) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4} e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}} \widehat{F}(Y+U, Z+U)\, \widehat{G}(Y+V, Z-V)$$

- Inner kleinian operator $\hat{\kappa}$:

$$\widehat{\kappa} = e^{iy^\alpha z_\alpha} , \qquad \widehat{\kappa} \star \widehat{f}(z,y) = \widehat{f}(-z,-y) \star \widehat{\kappa} , \qquad \widehat{\kappa} \star \widehat{\kappa} = 1$$

$$\widehat{\kappa} = \kappa_y \star \kappa_z , \quad \kappa_y \star \widehat{f}(z,y) = \widehat{f}(z,-y) \star \kappa_y , \quad \kappa_y \star \kappa_y = 1 ,$$

$$\kappa_y = 2\pi\delta^2(y) = 2\pi\delta(y_1)\delta(y_2)$$

# 4D bosonic Vasiliev equations

- Full equations:

$$\boxed{\begin{aligned} d\widehat{W}+\widehat{W}\star\widehat{W} &= 0\\ d\widehat{\Phi}+\widehat{W}\star\widehat{\Phi}-\widehat{\Phi}\star\pi(\widehat{W}) &= 0\\ d\widehat{S}_\alpha+[\widehat{W},\widehat{S}_\alpha]_\star &= 0\\ \widehat{S}_\alpha\star\widehat{\Phi}+\widehat{\Phi}\star\pi(\widehat{S}_\alpha) &= 0\\ [\widehat{S}_\alpha,\widehat{S}_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1-\mathcal{B}\star\widehat{\Phi}\star\widehat{\kappa})\\ [\widehat{S}_\alpha,\widehat{\bar{S}}_{\dot\beta}]_\star &= 0\ , \end{aligned}}$$

*(Vasiliev)*

$$\boxed{\hat{S}_\alpha \ = \ z_\alpha - 2i\hat{V}_\alpha}$$

- Manifestly consistent ←→ gauge invariant.

- *Z*-oscillators → auxiliary, non-commutative coordinates. Equations fix the evolution along Z in such a way that it gives rise to consistent interactions to all orders among physical fields.
  The latter are contained in the (Z-independent) initial conditions for the Z-evolution,

$$W \ = \ \widehat{W}_{|Z=0}\ , \qquad \Phi \ = \ \widehat{\Phi}_{|Z=0}\ .$$

# Exact solutions: gauge function method

- $\mathcal{X}$ x $\Upsilon$ x $\mathcal{Z}$-space eqns:
- $\Upsilon$ x $\mathcal{Z}$-space eqns:

$$\boxed{\begin{aligned}
\widehat{W} &= \widehat{L}^{-1}\star d\widehat{L} \\
\widehat{\Phi} &= \widehat{L}^{-1}\star\widehat{\Phi}'\star\pi(\widehat{L})\ , & d\widehat{\Phi}' = 0 \\
\widehat{S}_\alpha &= \widehat{L}^{-1}\star\widehat{S}'_\alpha\star\widehat{L}\ , & d\widehat{S}'_\alpha = 0 \\
\widehat{S}'_\alpha\star\widehat{\Phi}' + \widehat{\Phi}'\star\pi(\widehat{S}'_\alpha) &= 0 \\
[\widehat{S}'_\alpha,\widehat{S}'_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1-\mathcal{B}\star\widehat{\Phi}'\star\widehat{\kappa}) \\
[\widehat{S}'_\alpha,\widehat{\bar{S}}'_{\dot\beta}]_\star &= 0
\end{aligned}}$$

- Solve locally all equations with at least one spacetime component via some gauge function, here chosen as an *AdS*$_4$ coset element.

- *AdS*$_4$:

$$\widehat{\Phi} = 0\ ,\quad \widehat{S}_\alpha = \widehat{S}^{(0)}_\alpha = z_\alpha\ ,\quad \widehat{S}_{\dot\alpha} = \widehat{S}^{(0)}_{\dot\alpha} = \bar{z}_{\dot\alpha}\ ,\quad \widehat{W}_\mu = \Omega^{(0)}_\mu = L^{-1}\star\partial_\mu L$$

$$L(x;y,\bar{y}) = e_\star^{i\lambda\tilde{x}^\mu(x)\delta^a_\mu P_a}\ :\quad \mathcal{R}^{3,1}\longrightarrow\frac{SO(3,2)}{SO(3,1)}\quad\longrightarrow\quad \boxed{ds^2_{(0)} = \frac{4dx^2}{(1-x^2)^2}}$$

- Solve the twistor-space equations, then “dress” all fields with *x*-dependence by performing star-products with the gauge function.

# Solution in twistor-space: factorized Ansatz

- Ansatz:

$$\begin{aligned}
\widehat{\Phi}'(Y,Z) &= \Phi'(Y) = F'(Y)\star\kappa_y = \bar{F}'(Y)\star\bar{\kappa}_{\bar{y}}\ ,\\
\widehat{V}'_\alpha(Y,Z) &= \widehat{V}'_\alpha(Y,z) = \widehat{V}'_\alpha(F'(Y),z) = \sum_{k=1}^{\infty}(F'(Y))^{\star k}\star V^{(k)}_\alpha(z)\ ,\\
\widehat{\bar{V}'}_{\dot\alpha}(Y,Z) &= \widehat{\bar{V}'}_{\dot\alpha}(Y,\bar{z}) = \widehat{\bar{V}'}_{\dot\alpha}(\bar{F}(Y),\bar{z}) = \sum_{k=1}^{\infty}(\bar{F}'(Y))^{\star k}\star \bar{V}^{(k)}_{\dot\alpha}(\bar{z})
\end{aligned}$$

$$F' := \Phi'\star\kappa_y\ ,\quad \bar{F}' := \Phi'\star\bar{\kappa}_{\bar{y}}\ ,\qquad [F',\bar{F}']_\star = 0$$

- Holomorphicity in z + $[F',\bar{F}']=0$ → $[S',\bar{S}'] = 0$.
- $\pi(V') = - V'$ solves $\{S',\Phi'\}_\pi = 0$ .

- Remain:

$$\begin{aligned}
\partial_{[\alpha}\widehat{V}'_{\beta]} + \widehat{V}'_{[\alpha}\star\widehat{V}'_{\beta]} &= -\frac{i}{4}\,\epsilon_{\alpha\beta}\,bF'\star\kappa_z\ ,\\
\partial_{[\dot\alpha}\widehat{\bar{V}}'_{\dot\beta]} + \widehat{\bar{V}}'_{[\dot\alpha}\star\widehat{\bar{V}}'_{\dot\beta]} &= -\frac{i}{4}\,\epsilon_{\dot\alpha\dot\beta}\,\bar{b}\bar{F}'\star\bar{\kappa}_{\bar{z}}
\end{aligned}$$

But F' is Z-constant → eqs. analogue to deformed oscillator problem with a δ-function deformation (just like for HS bh). With basis spinors $u^\pm_\alpha$ , $(u^{+\alpha} u^-_\alpha = 1)$

$$z^\pm := u^{\pm\alpha}z_\alpha\ ,\quad w_z := z^+z^-\ ,\quad [z^-,z^+]_\star = -2i \quad\rightarrow\quad \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}e^{-i\frac{1}{\varepsilon}z^+z^-} = \kappa_z$$

$$\widehat{V}'_\alpha = 2i\sum_{k=1}^{\infty}\binom{1/2}{k}\left(-\frac{b}{2}\right)^k\int_{-1}^{1}\frac{dt}{(t+1)^2}\,\frac{(\log(1/t^2))^{k-1}}{(k-1)!}\,z_\alpha e^{i\frac{t-1}{t+1}w_z}\star(F')^{\star k}$$

# Expansion in so(3,2)-representations

- Idea: decompose Φ' into (enveloping-algebra realization of) $\mathfrak{so}$(3,2)-modules .
  Consider functions of the Y-oscillators with definite eigenvalues of under the two commuting generators ($K_+$ , $K_-$) spanning the Cartan subalgebra of $\mathfrak{sp}$(4;C),

$$\widehat{\Phi}'(Y,Z) \;=\; \Phi'(Y) \;\in\; \mathcal{M} \;=\; \bigoplus_{n_1,n_2,n'_1,n'_2} \mathbf{C}\otimes P_{n_1,n_2|n'_1,n'_2}$$

  The generalized projectors $P_{\mathbf{n}|\mathbf{m}} = P(K_+, K_-)$ are non-polynomial functions of Y such that

$$P_{\mathbf{n}|\mathbf{n}'}\star P_{\mathbf{m}|\mathbf{m}'} \;=\; \delta_{\mathbf{n}',\mathbf{m}}P_{\mathbf{n}|\mathbf{m}'}\ , \qquad P_{\mathbf{n}|\mathbf{m}'} \;\sim\; |\mathbf{n}><\mathbf{m}|$$

  and obeying proper reality conditions.

- On expanding the (internal) master fields as

$$\widehat{\mathcal{O}}(Y,Z) \;=\; \sum_{\mathbf{n},\mathbf{n}'}\sum_{k=0,1} P_{\mathbf{n}|\mathbf{n}'}(Y)\star\kappa_y^{\star k}\star\check{\mathcal{O}}_{k;\mathbf{n}|\mathbf{n}'}(Z)$$

  Vasiliev's equations are turned into matrix equations, and we can construct a solution by known methods.

# Systematics of projectors

- Focus on certain kinds of generalized projectors:

  - First, restrict our attention to diagonal $P_{\mathbf{n}|\mathbf{n}} = |n_1,n_2\rangle\langle n_1,n_2|$ and "skew-diagonal" $P_{\mathbf{n}|-\mathbf{n}} = |n_1,n_2\rangle\langle -n_1,-n_2|$

  - Moreover, we shall only consider "symmetry-enhanced" projectors $\mathcal{P}_n$ , depending only on one of the two Cartan generators

$$
\begin{aligned}
\mathcal{P}_n(K_{(q)}) \;&=\; \sum_{\substack{n_2 + q n_1 = n \\ \epsilon_1 \epsilon_2 = q}} P_{n_1,n_2} \;=\; 4(-)^{n-\frac{1+\varepsilon}{2}}\, e^{-4K_{(q)}} L^{(1)}_{n-1}(8K_{(q)}) \\
&=\; 2(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta K_{(q)}} \;, \qquad n \in \mathbb{Z}
\end{aligned}
$$

# I. Weyl 0-form moduli and sectors of HSGRA

- Different families of solutions are selected by different choices of the Cartan subalgebra: there are 3 inequivalent embeddings of $\mathfrak{so}(2)_{(+)} \oplus \mathfrak{so}(2)_{(-)}$ in $\mathfrak{sp}(4,C)$: $(K_{(+)}, K_{(-)})$ = (E , J) ; (J , iB) ; (iB , iP) . [E := $M_{0'0} = P_0$ ; J: = $M_{23}$ ; B := $M_{01}$ ; P := $P_3$]

- Consider the $\mathcal{P}_n$(K): kor the generators that anticommute with $\kappa_y$, K = E, P, one can obtain skew-diagonal, symmetry-enhanced projectors by ★-multiplication with $\kappa_y$, → "twisted projectors" $\widetilde{\mathcal{P}}_n(K) = \mathcal{P}_n(K) \star \kappa_y$, satisfying the generalized projector algebra

$$\begin{aligned} \mathcal{P}_n \star \mathcal{P}_m &= \delta_{nm} \mathcal{P}_n \ , & \widetilde{\mathcal{P}}_n \star \widetilde{\mathcal{P}}_m &= \delta_{n,-m} \mathcal{P}_n \\ \mathcal{P}_n \star \widetilde{\mathcal{P}}_m &= \delta_{nm} \widetilde{\mathcal{P}}_n \ , & \widetilde{\mathcal{P}}_n \star \mathcal{P}_m &= \delta_{n,-m} \widetilde{\mathcal{P}}_n \ , \end{aligned}$$

$$\Longrightarrow$$

$$\boxed{\widetilde{\mathcal{P}}_n(E) \simeq |n/2;0\rangle\langle -n/2;0| \in \mathcal{D}_0 \otimes \widetilde{\mathcal{D}}_0^*}$$

- Projectors and twisted projectors form an ideal of the star-product algebra → possible to expand the master-fields on this enlarged basis. Physical significance changes with the chosen Cartan generator.

# Projectors and twisted projectors

- Universal form of the projectors:

$$
\begin{aligned}
\mathcal{P}_n(K) &= \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta K} \\
&= \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-\frac{1}{2}y^\alpha \varkappa_{\alpha\beta} y^\beta - \frac{1}{2}\bar{y}^{\dot{\alpha}} \bar{\varkappa}_{\dot{\alpha}\dot{\beta}} y^{\dot{\beta}} - y^\alpha v_{\alpha\dot{\beta}} \bar{y}^{\dot{\beta}}}
\end{aligned}
$$

where $K$ is one of the possible Cartan generators,

$$
K = \frac{1}{8} Y^{\underline{\alpha}} K_{\underline{\alpha\beta}} Y^{\underline{\beta}} , \qquad K_{\underline{\alpha\beta}} = \begin{pmatrix} \varkappa_{\alpha\beta} & v_{\alpha\dot{\beta}} \\ \bar{v}_{\dot{\alpha}\beta} & \bar{\varkappa}_{\dot{\alpha}\dot{\beta}} \end{pmatrix} .
$$

- Universal form of the twisted projectors:

$$
\boxed{\widetilde{\mathcal{P}}_n = \frac{\mathcal{N}_n}{\sqrt{\det \varkappa}} \oint_{C(\epsilon)} \frac{d\eta}{2\pi i \eta} \left(\frac{\eta+1}{\eta-1}\right)^n \exp\left\{ \frac{1}{\eta} \left[ \tfrac{1}{2} y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta + \tfrac{1}{2} \bar{y}^{\dot{\alpha}} \bar{\varkappa}^{-1}_{\dot{\alpha}\dot{\beta}} \bar{y}^{\dot{\beta}} + i y^\alpha \bar{y}^{\dot{\beta}} \varkappa^{-1}_{\alpha\beta} v^{\beta}{}_{\dot{\beta}} \right] \right\}}
$$

- $K = E$, $iP$ are purely off-diagonal → $\widetilde{\mathcal{P}}_n$ are singular! Projectors depending on K= J, iB are instead "eigenvectors" of $\kappa_{y.}$

# Projectors and twisted projectors

- The expansion of the initial datum in projectors and twisted projectors,

$$\boxed{\Phi'(Y) \;=\; \sum_n (\nu_n \widetilde{\mathcal{P}}_n + \widetilde{\nu}_n\, \mathcal{P}_n)}$$

yields the x-dependent master-field

$$\Phi(x|Y) \;=\; L^{-1} \star \Phi' \star \kappa_y \star L \star \kappa_y \;=\; \sum_n (\nu_n\, L^{-1} \star \mathcal{P}_n \star L + \widetilde{\nu}_n\, L^{-1} \star \widetilde{\mathcal{P}}_n \star L) \star \kappa_y$$

- The L-rotation introduces x-dependence: for singular $\widetilde{\mathcal{P}}_{\mathrm{n}}$, coordinates smoothen the singularities by entering (in combinations dictated by the residual symmetries) as parameters in the limit representation of the δ-function.
- Difference in residual symmetries: solutions based on twisted projectors inherit the symmetries of the projectors

$$\delta\Phi(x|Y) \;=\; -[\epsilon(x|Y), \Phi(x|K)]_{\star,\pi} \;=\; 0 \;\Leftrightarrow\; [\epsilon'(Y), \mathcal{P}_n(K)]_\star \;=\; 0 \;\Rightarrow\; \epsilon'_{\mathrm{r.s.}} \in \mathfrak{c}_{\mathfrak{sp}(4,\mathbb{R})}(K)$$

- The analysis of residual symmetries of solutions based on projectors is slightly more complicated.

# Particle basis

- The couple (E,J) with |E|> J corresponds to choosing the "particle (compact) basis" of AdS UIRs. Its LWS corresponds to the projector $\mathcal{P}_1 := 4\, e^{-4E}$.

$$\begin{aligned} \mathcal{P}_1 \star \mathcal{P}_1 &= \mathcal{P}_1 \\ E \star \mathcal{P}_1 &= \mathcal{P}_1 \star E = \frac{1}{2}\mathcal{P}_1 , \\ L_r^- \star \mathcal{P}_1 &= 0 = \mathcal{P}_1 \star L_r^+ , \\ M_{rs} \star \mathcal{P}_1 &= 0 \end{aligned} \quad \Longrightarrow \quad \boxed{\mathcal{P}_1(E) \simeq |1/2;0\rangle\langle 1/2;0| \in \mathcal{D}_0 \otimes \mathcal{D}_0^*}$$

and from the point-of-view of the two-sided, twisted-adjoint action $K\star \mathcal{P}_1 - \mathcal{P}_1 \star \pi(K)$,

$$\boxed{\mathcal{P}_1(E) \simeq |1;0\rangle \in \mathcal{D}(1,0)}$$

*(C.I., P. Sundell '08)*

- The $\mathfrak{so}(3,2)$ twisted adjoint action on $\mathcal{P}_1$ reconstructs the projectors on all other $\mathcal{D}(1,0)$ modes → the $\mathfrak{hs}(3,2)$ action gives the modes $P_{n1n2} = P_{n1n2}(E,J)$ of all spin-s AdS massless particles.
- The $\mathcal{P}_n(E)$ correspond to $\mathfrak{so}(3)$-invariant massless scalar modes.
- The star product with $\kappa_y$ flips the sign of energy and thus generates states with zero energy, static → *soliton*-like solutions.
- In fact, such states provide the local data for HS generalizations of Schwarzschild black holes !

# Weyl 0-form for HS black holes

- $\Phi'(Y)$ expanded in twisted projectors: $$\Phi'(Y) \;=\; \sum_n \nu_n\, \mathcal{P}_n(E) \star \kappa_y \;=\; \sum_n \nu_n\, \widetilde{\mathcal{P}}_n(Y)$$

- This expansion enforces the Kerr-Schild property in gauge fields: the latter are reconstructed from curvatures in powers of $(\Phi' \star \kappa_y)^{\star n} = \mathcal{P}^{\star n} = \mathcal{P}$.

- Projectors are $f(E)$ → symmetries of an AdS Schwarzschild bh → $\mathfrak{so}(2)_E \oplus \mathfrak{so}(3)_{M_{rs}}$ :

$$\mathcal{P}_n(E) \;=\; 4(-1)^{n-\frac{1+\varepsilon}{2}}\, e^{-4E} L^{(1)}_{n-1}(8E) \;=\; 2(-1)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta E}$$

- Indeed, reinstating the x-dependence:

$$\Phi(x|Y) \;=\; L^{-1}(x) \star \Phi' \star \pi(L)(x) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta E} \star L(x) \star \kappa_y}$$

a tower of type-D Weyl tensors of all spins: $$\Phi^{(n)}_{\alpha(2s)} \;\sim\; \frac{i^{n-1}\mu_n}{r^{s+1}}\, (u^+ u^-)^s_{\alpha(2s)}$$

*(Didenko-Vasiliev, '09, C.I.-Sundell, '11)*

# Weyl 0-form for massless scalar

- $\Phi'(Y)$ expanded on projectors:

$$\Phi'(Y) \;=\; \sum_n \widetilde{\nu}_n \, \widetilde{\mathcal{P}}_n(Y) \star \kappa_y \;=\; \sum_n \widetilde{\nu}_n \, \mathcal{P}_n(E) \;, \qquad (\widetilde{\nu}_n)^* \;=\; \widetilde{\nu}_{-n}$$

$$\widetilde{\mathcal{P}}_n(E) \;:=\; \mathcal{P}_n(E) \star \kappa_y \;=\; 4\pi(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y - i\eta\sigma_0\bar{y})$$

- Weyl zero-form only contains a scalar (modes of an AdS massless scalar):

$$\Phi(x|Y) \;=\; L^{-1}(x) \star \Phi' \star \pi(L)(x) \;=\; \sum_n \tilde{\nu}_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star \delta^2(y - i\eta\sigma_0\bar{y}) \star L(x) \star \kappa_y}$$

$$\Phi(x|Y) \;=\; (1-x^2) \sum_n \mathcal{N}_n \tilde{\nu}_n \oint_{C(\epsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \frac{e^{iy^\alpha M_\alpha{}^{\dot\beta}(x,\eta)\bar{y}_{\dot\beta}}}{1 - 2i\eta x_0 + \eta^2 x^2}$$

$$\boxed{4\widetilde{\nu}_1 \frac{1-x^2}{1-2ix_0+x^2} \;\sim\; \widetilde{\nu}_1 \frac{e^{-it}}{(1+r^2)^{1/2}}}$$

*(C.I., P. Sundell, to appear)*

- Differently from bhs, one does not expect a free scalar to solve the full equations. However, the completion to full solutions is precisely given by the bh sector!

# Cylindrically-symmetric solutions

- For K = J , the element

$$\mathcal{P}_1(J) \;:=\; 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'_{\underline{\alpha\beta}}Y^{\underline{\beta}}} \;=\; 4e^{-4J}$$

  again behaves as a ground state of a 2D Fock-space (a non-compact ultra-short irrep, singleton-like but with roles of E and J exchanged, |E| < |J| instead of |E| > |J|).

- The symmetry this time is $\mathfrak{so}(2)_{\mathrm{J}} \oplus \mathfrak{so}(2,1)_{\mathfrak{h}(\mathrm{J})}$ and no singularity in the Weyl tensors Is expected.

- Indeed, the same steps yield

$$\Phi(x|Y) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta J} \star L(x) \star \kappa_y}$$

$$\boxed{\Phi^{(n)}_{\alpha(2s)} \;\sim\; \frac{i^{n+s+1}\mu_n}{(1+r^2\sin^2\theta)^{(s+1)/2}}\,(\tilde{u}^+\tilde{u}^-)^s_{\alpha(2s)}}$$

# Conformal basis

- The couple (*iB*, *iP*), with |P| > |B| corresponds to the so-called conformal basis for AdS (U)IRs, where one singles out the Lorentz isometry algebra of the Minkowski boundary. The highest-weight state is

$$\mathcal{P}_1(iP) \;:=\; 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'_{\underline{\alpha\beta}}Y^{\underline{\beta}}} \;=\; 4e^{4iP}$$

- The expansion of $\Phi'(Y)$ in projectors with *K* = *iP* encodes the scalar bulk-to-boundary propagator

$$\Phi(x|Y) \;=\; \sum_n \tilde{\nu}_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n L^{-1}(x) \star \delta^2(y-\eta\sigma_3\bar{y}) \star L(x) \star \kappa_y$$

$$\varphi(x) \;=\; 4\widetilde{\nu}_1 \frac{1-x^2}{1-2x_3+x^2} \;\sim\; \widetilde{\nu}_1 \frac{z}{z^2+y^\nu y_\nu}$$

- The relation between the particle basis and the conformal basis is a (non-unitary) coherent-state transformation

$$\mathcal{P}_1(iP) \;\sim\; e^{-L_3^+} \star \mathcal{P}_1(E) \star e^{L_3^-}$$

# Conclusions and outlook

- Six infinite families of solutions, with various symmetries and physical significance have been found via a specific factorized Ansatz and by expanding the local data in projectors with so(3,2) eigenvalues.

- The different types of solutions are singled out by the choice of Cartan subalgebra and of generalized projectors.

- How to physically characterize the solutions? Important to better understand and evaluate various HS invariants. Behaviour of certain observables on the solutions may probe properties of the theory (e.g., existence of multi-body solutions?)

- Important related issues concerning functional classes of twistor-space elements, and related questions of the admissibility of gauge parameters, regularity under star-product, regularity of observables,…