Higher Spin Theory
and Holography-4
May-June, 2016

Ordinary-derivative approach to long conformal fields

R.R. Metsaev
Lebedev Institute

Plan

1) **Introduction - Review-Results**

2) **Modified Lorentz and de Donder gauges**

and (global) BRST Lagrangian for conformal fields

3) **Computation of partition functions**

of long and partial-short conformal fields

Totally symmetric conformal field in $R^{d-1,1}$

$$\phi^s = \phi^{a_1 \dots a_s}$$

$$\Delta = \frac{d}{2} - \kappa$$

$$\mathcal{L} = \phi^s \Box^\kappa \phi^s$$

Three arbitrary integer labels

$$\mathbf{s}, \qquad \kappa, \qquad \mathbf{d}$$

Short conformal field

$$\kappa = s + \frac{d-4}{2}, \qquad d - even$$

Partial-short conformal field

$$\kappa = s + \frac{d-4}{2} - t, \qquad d - even$$

$$t = 1, 2, \ldots, s-1$$

Long conformal field

$$\kappa > s + \frac{d-4}{2}, \qquad \kappa - integer$$

In **AdS**/**CFT**

Conformal field in $R^{d-1,1}$ with

$$\Delta = \frac{d}{2} - \kappa$$

is dual to -non-normalizable modes of field

in AdS_{d+1} with

$$E_0 = \frac{d}{2} + \kappa$$

Short conformal spin-2 field in $R^{3,1}$ (**Weyl Gravity**)

$$\mathbf{Z} = \frac{(D_1)^3}{(D_2)^2}$$

$$\mathbf{n^{DoF}} = 9 \times 2 - 4 \times 3 = \mathbf{6}$$

$$D_n \equiv \sqrt{\det(-\Box)}$$

for rank-n traceleess tensor field

Fradkin, Tseytlin 1981

also n^{DoF} by Dirac method

Nieuwenhuizen, Lee 1982

Short conformal arbitrary spin-s field in $R^{3,1}$ (Fradkin-Tseytlin fields)

$$\mathcal{L} = \phi^{\mathbf{s}} \Box^{\mathbf{s}} \phi^{\mathbf{s}}$$

$$\mathbf{Z} = \frac{(D_{s-1})^{s+1}}{(D_s)^s}$$

$$\begin{aligned} \mathbf{n}^{\mathbf{DoF}} &= s \times (s+1)^2 - (s+1) \times s^2 \\ &= \mathbf{s(s+1)} \end{aligned}$$

Fradkin, Tseytlin 1985

Short conformal arbitrary spin-s field in $R^{d-1,1}$

$$Z = \frac{(D_{s-1})^{\nu_s+1}}{(D_s)^{\nu_s}} \qquad \nu_s \equiv s + \frac{d-4}{2}$$

$$n^{DoF} = \nu_s n_s^{so(d)} - (\nu_s+1) n_{s-1}^{so(d)}$$

$$= \frac{1}{2}(d-3)(2s+d-2)(2s+d-4)\frac{(s+d-4)!}{(d-2)!s!}$$

Tseytlin 2013

also n^{DoF} by light-cone gauge method

RRM 2007

Partial-short maximal-dept conformal spin-s field in $R^{3,1}$

$$Z = \frac{(D_0)^{s+1}}{(D_s)}$$

$$n^{DoF} = s(s+1)$$

$$\kappa = s + \frac{d-4}{2} - t$$

$$\kappa = 1\,, \qquad d = 4, \qquad t = s-1$$

Beccaria, Tseytlin 2015

Long conformal arbitrary spin-s field in $R^{d-1,1}$

$$\mathbf{Z} = \frac{1}{(D_s)^\kappa}$$

$$n^{DoF} = \kappa\, \mathbf{n}_{\mathbf{s}}^{\mathbf{so(d)}}$$

$$\mathbf{n}_{\mathbf{s}}^{\mathbf{so(d)}} \equiv (\mathbf{2s} + \mathbf{d} - \mathbf{2})\frac{(s+d-3)!}{(d-2)!s!}$$

$\mathbf{n}^{\mathbf{DoF}} = \kappa \times$(**DoF massive field in (d+1) dimensions**)

Partial-short conformal spin-s field in $R^{d-1,1}$

$$Z = \frac{(D_{s-1-t})^{s+\frac{d-2}{2}}}{(D_s)^{\kappa}}$$

$$\kappa = s + \frac{d-4}{2} - t$$

$$n^{DoF} = \frac{(2s+d-2)(2s+d-4-2t)(s+d-4)!}{2(d-2)!s!}$$

$$\times \left(s + d - 3 - s\frac{(s-t)_t}{(s-t+d-3)_t} \right) ,$$

Arbitrary values of κ, s, d

$$\mathcal{L} = \phi^{\mathbf{s}} \Box^{\kappa} \phi^{\mathbf{s}}$$

ϕ^s traceless rank-s tensor field of $so(d-1,1)$

higher-derivative Lagrangian

Vasiliev 2009

spin-1

$$\mathcal{L} = -\frac{1}{4}F^{ab}F^{ab}$$

$$F^{ab} = \partial^a\phi^b - \partial^b\phi^a$$

$$\mathcal{L} = \frac{1}{2}\phi^a\Box\phi^a + \frac{1}{2}L^2$$

$$L = \partial^a\phi^a$$

$$\Box\phi^a = 0 \qquad L = 0$$

Apply Faddeev-Popov procedure

Step 1. Introduce

Faddeev-Popov fields

$$\bar{c} \qquad c$$

Nakanishi-Lautrup field

$$b$$

Step 2.

$$\mathcal{L}^{BRST} = \mathcal{L} + \mathcal{L}_{\mathbf{g.fix}}$$

$$\mathcal{L}_{\mathbf{g.fix}} = -\mathbf{bL} + \bar{\mathbf{c}}\,\Box\mathbf{c} + \frac{1}{2}\xi\mathbf{b}^{2}$$

$\xi = 0$ **Landau gauge**

$\xi = 1$ **Feynman gauge**

BRST

$$\mathsf{s}\phi^a = \partial^a c$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = b$$

$$\mathsf{s}b = 0$$

anti-BRST

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c}$$

$$\bar{\mathsf{s}}c = -b$$

$$\bar{\mathsf{s}}\bar{c} = 0$$

$$\bar{\mathsf{s}}b = 0$$

$$\mathsf{s}^2 = 0 \qquad \bar{\mathsf{s}}^2 = 0 \qquad \mathsf{s}\bar{\mathsf{s}} + \bar{\mathsf{s}}\mathsf{s} = 0$$

OFF-SHELL

Step 3.

Feynman gauge $\xi = 1$

Integrate out field $\mathbf{b}$

$$\mathcal{L} = \frac{1}{2}\phi^a \Box \phi^a + \bar{c}\,\Box c$$

$$\mathbf{Z} = \frac{D_0^2}{D_1}$$

BRST

$$\mathsf{s}\phi^a = \partial^a c$$

$$\mathsf{s}c = 0$$

$$\mathsf{s}\bar{c} = \mathbf{L}$$

anti-BRST

$$\bar{\mathsf{s}}\phi^a = \partial^a \bar{c}$$

$$\bar{\mathsf{s}}c = -\mathbf{L}$$

$$\bar{\mathsf{s}}\bar{c} = 0$$

$$\mathsf{s}^{\mathbf{2}} = 0 \qquad \bar{\mathsf{s}}^{\mathbf{2}} = 0$$

ON-SHELL for $\mathbf{c}$ and $\bar{\mathbf{c}}$

spin-2 in $R^{3,1}$ (Weyl gravity)

$$\phi^{ab}_{-1} \qquad \phi^{ab}_{1}$$

$$\phi^{a}_{0}$$

$$\Delta(\phi_{k'}) = \frac{d-2}{2} + k'$$

$$\mathcal{L} \;=\; \frac{1}{2}\phi^{ab}_{-1}\Box\phi^{ab}_{1} - \frac{1}{4}\phi^{aa}_{-1}\Box\phi^{bb}_{1} + \frac{1}{2}\phi^{a}_{0}\Box\phi^{a}_{0}$$

$$+\;\; \mathbf{L^{a}_{-1}L^{a}_{1}} + \frac{1}{2}\mathbf{L_0 L_0}$$

$$-\;\; \frac{1}{4}\phi^{\mathbf{ab}}_{\mathbf{1}}\phi^{\mathbf{ab}}_{\mathbf{1}} + \frac{1}{8}\phi^{\mathbf{aa}}_{\mathbf{1}}\phi^{\mathbf{bb}}_{\mathbf{1}}$$

$$L^{a}_{-1} = \partial^{b}\phi^{ab}_{-1} - \frac{1}{2}\partial^{a}\phi^{bb}_{-1} + \phi^{\mathbf{a}}_{\mathbf{0}}$$

$$L^{a}_{1} = \partial^{b}\phi^{ab}_{1} - \frac{1}{2}\partial^{a}\phi^{bb}_{1}$$

$$L_0 = \partial^{a}\phi^{a}_{0} + \frac{1}{2}\phi^{\mathbf{bb}}_{\mathbf{1}}$$

$$\delta\phi^{ab}_{-1} = \partial^{\mathbf{a}}\xi^{\mathbf{b}}_{\mathbf{-1}} + \partial^{\mathbf{b}}\xi^{\mathbf{a}}_{\mathbf{-1}} + \eta^{\mathbf{ab}}\xi_{\mathbf{0}}$$

$$\delta\phi^{ab}_{1} = \partial^{\mathbf{a}}\xi^{\mathbf{b}}_{\mathbf{1}} + \partial^{\mathbf{b}}\xi^{\mathbf{a}}_{\mathbf{1}}$$

$$\delta\phi^{a}_{0} = \partial^{\mathbf{a}}\xi_{\mathbf{0}} - \xi^{\mathbf{a}}_{\mathbf{1}}$$

Faddeev-Popov fields

$$\bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{-1}} \qquad \mathbf{c}^{\mathbf{a}}_{\mathbf{-1}}\,, \qquad \bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{1}} \qquad \mathbf{c}^{\mathbf{a}}_{\mathbf{1}} \qquad \bar{\mathbf{c}}_{\mathbf{0}} \qquad \mathbf{c}_{\mathbf{0}}$$

Nakanishi-Lautrup field

$$\mathbf{b}^{\mathbf{a}}_{\mathbf{-1}} \qquad \mathbf{b}^{\mathbf{a}}_{\mathbf{1}}\,, \qquad \mathbf{b}_{\mathbf{0}}$$

$$\mathcal{L}^{BRST} = \mathcal{L} + \mathcal{L}_{\mathbf{g.fix}}$$

$$\begin{aligned}
\mathcal{L}_{\mathbf{g.fix}} &= -\mathbf{b}^{\mathbf{a}}_{\mathbf{1}}\mathbf{L}^{\mathbf{a}}_{\mathbf{-1}} - \mathbf{b}^{\mathbf{a}}_{\mathbf{-1}}\mathbf{L}^{\mathbf{a}}_{\mathbf{1}} - \mathbf{b}_{\mathbf{0}}\mathbf{L}_{\mathbf{0}} \\
&+ \xi\mathbf{b}^{\mathbf{a}}_{\mathbf{1}}\mathbf{b}^{\mathbf{a}}_{\mathbf{-1}} + \frac{1}{2}\xi\mathbf{b}^{\mathbf{2}}_{\mathbf{0}} \\
&+ \bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{1}}\Box\mathbf{c}^{\mathbf{a}}_{\mathbf{-1}} + \bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{-1}}\Box\mathbf{c}^{\mathbf{a}}_{\mathbf{1}} + \bar{\mathbf{c}}_{\mathbf{0}}\Box\mathbf{c}_{\mathbf{0}} \\
&- \bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{1}}\mathbf{c}^{\mathbf{a}}_{\mathbf{1}}
\end{aligned}$$

Use Feynman gauge $\xi = 1$

Integrate out Nakanishi-Lautrup fields

$$
\begin{aligned}
\mathcal{L}^{BRST} &= \frac{1}{2}\phi^{ab}_{-1}\Box\phi^{ab}_{1} - \frac{1}{4}\phi^{aa}_{-1}\Box\phi^{bb}_{1} + \frac{1}{2}\phi^{a}_{0}\Box\phi^{a}_{0} \\
&- \frac{1}{4}\phi^{\mathbf{ab}}_{\mathbf{1}}\phi^{\mathbf{ab}}_{\mathbf{1}} + \frac{1}{8}\phi^{\mathbf{aa}}_{\mathbf{1}}\phi^{\mathbf{bb}}_{\mathbf{1}} \\
&+ \bar{c}^{a}_{1}\Box c^{a}_{-1} + \bar{c}^{a}_{-1}\Box c^{a}_{1} + \bar{c}_{0}\Box c_{0} \\
&- \bar{\mathbf{c}}^{\mathbf{a}}_{\mathbf{1}}\mathbf{c}^{\mathbf{a}}_{\mathbf{1}}
\end{aligned}
$$

Solution for **auxiliary fields**

$$\boldsymbol{\phi}_{\mathbf{1}}^{\mathbf{ab}} = \Box \phi_{-1}^{ab}$$

$$\mathbf{c}_{\mathbf{1}}^{\mathbf{a}} = \Box c_{-1}^{a}$$

$$\bar{\mathbf{c}}_{\mathbf{1}}{}^{a} = \Box \bar{c}_{-1}^{a}$$

$$\mathcal{L} = \frac{1}{2}\phi^{ab}_{-1}\Box^2\phi^{ab}_{-1} - \frac{1}{4}\phi^{aa}_{-1}\Box^2\phi^{bb}_{-1} + \frac{1}{2}\phi^{a}_{0}\Box\phi^{a}_{0}$$

$$+ \ \bar{c}^{a}_{-1}\Box^2 c^{a}_{-1} + \bar{c}_0\Box c_0$$

$$Z = \frac{D_1^4 D_0^2}{D_2^2 D_0^2 D_1}$$

$$= \frac{D_1^3}{D_2^2}$$

$$D_n = \sqrt{\det(-\Box)}$$

Arbitrary spin-s conformal field

double traceless fields

$$\phi_{\lambda,k'}^{a_1 \ldots a_{s'}}$$

$$s' = 0, 1, \ldots, s$$

$$\lambda \in [s - s']_2$$

$$k' \in [\kappa - 1 + \lambda]_2$$

$$p \in [q]_2 \qquad \Longleftrightarrow \qquad p = -q, -q + 2, \ldots, q - 2, q$$

Lagrangian

$$\mathcal{L} = \sum_{\lambda,k',s'} \mathcal{L}^{s'}_{\lambda,k'}$$

$$\mathcal{L}^{s'}_{\lambda,k'} = \phi^{s'}\Box\phi^{s'} + \phi^{\mathbf{s'}}\phi^{\mathbf{s'}}$$

$$+ \mathbf{L}^{\mathbf{s'}}\mathbf{L}^{\mathbf{s'}}$$

$$\mathbf{L}^{\mathbf{s'}} = \partial^{\mathbf{a}}\phi^{\mathbf{a}\mathbf{a}_1\ldots\mathbf{a}_{\mathbf{s'}}} - \frac{1}{2}\partial^{\mathbf{a}_1}\phi^{\mathbf{a}\mathbf{a}\mathbf{a}_2\ldots\mathbf{a}_{\mathbf{s'}}}$$

$$+ \phi^{\mathbf{a}_1\ldots\mathbf{a}_{\mathbf{s'}}}$$

$$+ \eta^{(\mathbf{a}_1\mathbf{a}_2}\phi^{\mathbf{a}_3\ldots\mathbf{a}_{\mathbf{s'}})}$$

$$
\begin{aligned}
\delta\phi^{a_1\ldots a_{s'}} &= \partial^{(\mathbf{a_1}}\xi^{\mathbf{a_2\ldots a_{s'}})} \\
&+ \xi^{\mathbf{a_1\ldots a_{s'}}} \\
&+ \eta^{(\mathbf{a_1 a_2}}\xi^{\mathbf{a_3\ldots a_{s'}})}
\end{aligned}
$$

Apply Faddeev-Popov procedure

Faddeev-Popov fields

$$\bar{c}^{a_1 \ldots a_{s'}} \qquad c^{a_1 \ldots a_{s'}}$$

Nakanishi-Lautrup fields

$$b^{a_1 \ldots a_{s'}}$$

$$\mathcal{L}^{BRST} = \mathcal{L} + \mathcal{L}_{\mathbf{g.fix}}$$

$$\mathcal{L}_{\mathbf{g.fix}} = \sum_{s'} \mathcal{L}_{\mathbf{g.fix}}^{\mathbf{s}'}$$

$$\begin{aligned} \mathcal{L}_{\mathbf{g.fix}}^{\mathbf{s}'} &= b^{s'}\mathbf{L}^{\mathbf{s}} + \frac{1}{2}\xi \mathbf{b}^{\mathbf{s}'}\mathbf{b}^{\mathbf{s}'} \\ &+ \bar{\mathbf{c}}^{\mathbf{s}'}\Box\mathbf{c}^{\mathbf{s}'} \\ &+ \bar{\mathbf{c}}^{\mathbf{s}'}\mathbf{c}^{\mathbf{s}'} \end{aligned}$$