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based on *arXiv:1511.03024* with I. Bandos, L. Martucci and M. Tonin

VOLKOV-AKULOV GOLDSTINI, BRANES AND SPONTANEOUSLY BROKEN SUPERGRAVITY

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OVERVIEW

- Recent studies of inflation and susy breaking (ala Volkov-Akulov) within N=1, 4D sugra:

  Ketov & Starobinski '12; Ellis, Nanopoulos & Olive '13, Kehagias, Farakos & Riotto '13, …
  Ferrara, Kallosh, Linde, Porrati, Van Proeyen, Frè, Sorin '13, Antoniadis, Dudas, Ferrara, Sagnotti '14,….
  …Roest & Scalisi, Dall'Agata & Zwirner; Carrasco, Kallosh, Linde'15, …

  E.g. models with one or two chiral supermultiplets: inflaton T and goldstino S

- Another (related) aim of recent activity: to provide a 4D effective field theory ground for phenomenological models with spontaneously broken susy constructed in the framework of 10D string theory (e.g. KKLT-like models involving anti-D3-branes to generate de Sitter vacua)

- Most of the constructions of D=4 sugra with spontaneously broken susy use constrained (nilpotent) superfield description of the Volkov-Akulov goldstini (scalar partners are composed of goldstino bilinears) – this avoids the presence of extra scalar moduli and their stabilization

  Antoniadis, Dudas, Ferrara, Kehagias, Farakos, Kallosh, Linde, Porrati, Sagnotti, Scalisi, dall'Agata, Zwirner; Bergshoeff, Freedman, Kallosh, Van Proeyen; Hasegawa,Yamada; Kuzenko; Antoniadis, Markou; …,
  Aparicio, Quevedo, Valandro, …

- Original Volkov-Akulov construction '72 is directly related to worldvolume actions for superbranes *(Hughes & Polchinski '86, Kallosh '98, … Kallosh, Quevedo, Uranga '15…)*

  Its full coupling to N=1, D=4 sugra was given only recently: *Bandos, Martucci, D.S, Tonin '15*

- Earlier studies of local susy breaking & super-Brout-Englert-Higgs effect in sugra:

  Volkov & Soroka '73, Deser & Zumino '77,…, Lindstrom & Rocek '79, Samuel & Wess '83, Ivanov & Kapustnikov '84 -'90,…

ORIGINAL VOLKOV-AKULOV MODEL '72 AS A 3-BRANE

"Can neutrino be a Goldstone particle?"

- Goldstino as the manifestation of susy breaking

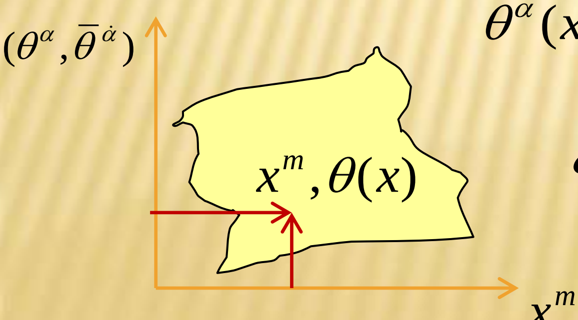
$$\chi^\alpha(x) \to \chi'^\alpha(x) = \chi^\alpha(x) + \varepsilon^\alpha + \text{nonlinear terms} \quad \Rightarrow \quad \{\mathrm{Q}, \overline{\mathrm{Q}}\}\chi = \sigma^m \partial_m \chi$$

- Volkov-Akulov superspace $\quad z^M = (x^m, \theta^\alpha, \bar\theta^{\dot\alpha}) \quad m = 0,1,2,3 \quad \alpha, \dot\alpha = 1,2$

SUSY transform. $\quad \delta\theta^\alpha = \varepsilon^\alpha, \quad \delta\bar\theta^{\dot\alpha} = \bar\varepsilon^{\dot\alpha}, \quad \delta x^m = i\theta\sigma^m\bar\varepsilon - i\varepsilon\sigma^m\bar\theta, \quad \sigma^m - \text{Pauli matrices}$

SUSY invariant VA (Cartan) 1-form: $\quad E_0^m = dx^m + i\theta\sigma^m d\bar\theta - id\theta\sigma^m\bar\theta$

Consider a 4d worldvolume (of a 3-brane) placed in superspace $M_{(4,4)}$



$$\theta^\alpha(x) \equiv f^{-1}\chi^\alpha(x), \quad \bar\theta^{\dot\alpha}(x) \equiv f^{-1}\,\bar\chi^{\dot\alpha}(x)$$

spinor field appears on the brane worldvolume

$$\delta\chi^\alpha(x) = f\varepsilon^\alpha - \delta x^m \partial_m \chi^\alpha(x) = f\varepsilon^\alpha + \underline{if^{-1}(\varepsilon\sigma^m\bar\chi - \chi\sigma^m\bar\varepsilon)\partial_m\chi^\alpha(x)}$$

non-linear susy transform.

susy breaking parameter
3-brane tesnion

VOLKOV-AKULOV ACTION

- volume of 4d surface in superspace (*space-filling non-BPS 3-brane*)

$$g_{ij}(x) = E^m_{0i} E^m_{0j} \eta_{mn}$$ - induced metric in the brane 4d worldvolume

$$E^m_0 = dx^i E^m_{0i}(\chi(x)) = dx^m + if^{-2}(\chi\sigma^m d\bar{\chi} - d\chi\sigma^m\bar{\chi}) = dx^i[\delta^m_i + if^{-2}(\chi\sigma^m\partial_i\bar{\chi} - \partial_i\chi\sigma^m\bar{\chi})]$$

$$S_{VA} = -f^2 \int_{M_4} d^4x \sqrt{\text{-}\det g_{ij}(x)} = -f^2 \int_{M_4} d^4x \det E^m_{0i}(\chi(x))$$

$$= -\int_{M_4} d^4x \left(f^2 + \underbrace{i(\chi\sigma^m\partial_m\bar{\chi} - c.c.)}_{\text{Dirac Lagrangian}} + f^{-2}(\chi\partial\chi)^2 + f^{-4}(\chi\partial\chi)^3 + \cancel{f^{-6}(\chi\partial\chi)^4} \right)$$

positive cosm. constant

- no bosonic superpartners
- no (independent) Wess-Zumino term $\int C_4$
- no kappa-symmetry

$\rightarrow$ N=1, D=4 susy is completely broken in the vacuum

Unique (modulo field redefinitions) **susy invariant** effective **low-energy** goldstino **action**

CONSTRAINED SUPERFIELD REALIZATION OF THE VA GOLDSTINO

(IVANOV, KAPUSTNIKOV '77; ROCEK 78', ...,SAMUEL, WESS '83; CASALBUONI ET AL. 89', ..., KOMARGODSKI, SEIBERG 09',..)

Simplest example:

$S(x_L,\theta)=s(x_L)+\theta\chi(x_L)+\theta^2 F(x_L)$ chiral superfield constrained by: $S^2=0 \Rightarrow s=\dfrac{\chi\chi}{4F}$

$$x_L^m=x^m+i\theta\sigma^m\bar\theta, \qquad z_L^M=(x_L^m,\theta^\alpha); \qquad z^M=(x^m,\theta^\alpha,\bar\theta^{\dot\alpha})$$

$$S_{NS}=\int d^8z\, S\bar S+f\left(\int d^4 z_L S+c.c.\right)=\int d^4x\left(-i\chi\sigma^m\partial_m\bar\chi-\partial\frac{\chi\chi}{4F}\partial\frac{\bar\chi\bar\chi}{4\bar F}+f(F+\bar F)+F\bar F\right)$$

$+\int dz_L^6\Lambda S^2+c.c.$ - Lagrange multiplier term

Upon integrating F: $S_{NS}=-\int d^4x\left(f^2+i\chi\sigma^m\partial_m\bar\chi+f^{-2}\chi\chi\partial^2(\bar\chi\bar\chi)+f^{-6}\chi^2\bar\chi^2\partial^2\chi^2\partial^2\bar\chi^2\right)$

$$S_{VA}=-\int_{M_4}d^4x\left(f^2+i\chi\sigma^m\partial_m\bar\chi+f^{-2}(\chi\partial\chi)^2+f^{-4}(\chi\partial\chi)^3+\cancel{f^{-6}(\chi\partial\chi)^4}\right)$$

From general theory of non-linear realizations it follows that the VA model is universal:
all models of spontaneous susy breaking involving goldstino should be related to the VA model by a non-linear transformation of $\chi^\alpha(x)$
whose general form was obtained only quite ricently *Kuzenko & Tyler* 10'-11'

$$\chi^\alpha_{VA}=\chi^\alpha(1+f^{-4}(\chi\partial\chi)^2+f^{-6}(\chi\partial\chi)^3)+f^{-2}(\sigma^m\bar\chi)^\alpha\partial_m\chi^2$$

COUPLING VA GOLDSTINO TO N=1, D=4 SUGRA, SUPER-BEHIGGS EFFECT AND DE SITTER VACUA

- *Volkov -Soroka model '73*
- *Deser, Zumino '77,…, Lindstrom, Rocek '79; Ferrara et al; Samuel, Wess '83; Ivanov, Kapustnikov '84 -'90,…*

2013-16 Antoniadis, Dudas, Ferrara, Kehagias, Farakos, Kallosh, Linde, Porrati, Sagnotti, dall'Agata, Zwirner, Farakos; Bergshoeff, Freedman, Kallosh, Van Proeyen; Hasegawa,Yamada; Kuzenko; Antoniadis, Markou;…, Schillo, van der Woerd, Wrase, …

Recent constructions are mainly based on the superconformal (tensor calculus) approach to supergravity *(see Freedman and Van Proeyen Book for a review)*

Alternatively, one can use superspace and superfield forlmalism
(e.g. Lindstrom & Rocek 79', *Samuel, Wess'83*)

Sugra coupled to scalar Φ(z), vector V(z), W_α(z), and goldstino S(z) superfields

$$S_{NS} = \frac{3}{4\kappa^2}\int d^8z\, \mathrm{Ber}\, E_M^A\, e^{-\frac{\kappa^2}{3}K(\bar\Phi, e^V\Phi, S, \bar S)} + \frac{m}{2\kappa^2}\int d^6 z_L \mathcal{E}\left(W(\Phi,S) + \mathbf{tr}\, g(\Phi,S)\mathbf{W}_\alpha \mathbf{W}^\alpha + \Lambda S^2\right)$$

Kahler potential   chiral measure   superpotential   SYM kin.term   Lagrange multiplier

$$E^A = dz^M E_M{}^A(z), \qquad T^A = dE^A - E^B\Omega_B{}^A \qquad z^M = (x^m, \theta^\alpha, \bar\theta^{\dot\alpha})$$

$$T^a = -iE^\alpha\sigma^a_{\alpha\dot\beta}\bar E^{\dot\beta}, \quad T^\alpha = iE^a(\sigma_{ab})_\beta{}^\alpha E^\beta G^b(z) - iE^a\bar E^{\dot\beta}\sigma_{a\dot\beta}{}^\alpha R(z) + \tfrac{1}{2}E^a E^b T^\alpha_{ab}$$ - sugra constraints

(Old minimal sugra, Wess & Zumino, *Stelle & West, Ferrara & van Nieuwenhuizen '78)*

COUPLING VA GOLDSTINO TO N=1, D=4 SUGRA, SUPER-BEHIGGS EFFECT AND DE SITTER VACUA

- upon integrating out the auxiliary fields component pure supergravity + VA goldstino *(Bergshoeff at. all; Hasegawa,Yamada '15)*

$$S=\frac{1}{2\kappa^2}\int d^4x\, e\Big(\overbrace{R(e,\omega)-\psi_l\sigma^{lmn}\nabla_m\bar\psi_n-m(\psi_a\sigma^{ab}\psi_b+\bar\psi_a\sigma^{ab}\bar\psi_b)+6m^2}^{\text{AdS sugra action (Townsend '77)}}-2\kappa^2 f^2\Big)$$
$$+\int d^4x\, e\Big(if\chi\sigma^a\bar\psi_a+\tfrac{i}{2}\chi\sigma^a\nabla_a\bar\chi-2m\chi\chi+c.c\Big)$$
$$+\int d^4x\, e\Big(\bar\chi\bar\chi\psi\psi-\chi\sigma_a\bar\chi\psi_b\sigma^{abc}\bar\psi_c+\tfrac{3\kappa^2}{2f^2}\chi\chi\bar\chi\bar\chi+c.c\Big)+O(\chi^4,\ldots,\chi^8)$$

In the unitary gauge $\chi=0\,(\delta\chi=\varepsilon(x)+\ldots)$ the remaining first line in the action describes a massive gravitino field coupled to gravity with the cosmological constant $\lambda=f^2-\frac{3m^2}{\kappa^2}$ which can be positive (dS-type)

$$S=\frac{1}{2\kappa^2}\int d^4x\, e\Big(R(e,\omega)-\psi_l\sigma^{lmn}\nabla_m\bar\psi_n-m(\psi_a\sigma^{ab}\psi_b+\bar\psi_a\sigma^{ab}\bar\psi_b)+2\kappa^2\lambda\Big)$$

1973 Volkov – Soroka action for spontaneously broken supergravity

VOLKOV-AKULOV BRANE COUPLED TO SUPERGRAVITY

- For finding a more direct relation of 4D effective theories with spontaneously broken local susy to 10D stringy constructions which use anti-D3-branes to induce a de Sitter vacuum, it may be useful to couple to supergravity the original Volkov-Akulov 3-brane model, without resorting to the constrained superfields
- The action has a suggestive geometric form of the sum of three different volumes in N=1, D=4 curved superspace

$$S_{SG+VA} = \tfrac{3}{4\kappa^2}\int d^8 z \,\mathrm{Ber}\, E_M^A + \tfrac{m}{2\kappa^2}\int d^6 z_L \mathcal{E} + f^2 \int d^4\xi \,\det \mathrm{E}_i^a(z^M(\xi))$$

$$\mathrm{E}_i^a(z^M(\xi)) = \partial_i x^m(\xi) E_m^a(x,\theta,\bar\theta) + \partial_i \theta^\mu(\xi) E_\mu^a(x,\theta,\bar\theta) + \partial_i \bar\theta^{\dot\mu}(\xi) E_{\dot\mu}^a(x,\theta,\bar\theta)$$

pullback on 3-brane worldvolume

in the static gauge for worldvolume diffeomorphisms: $x^m(\xi) = \xi^i \delta_i^m.\quad \theta(x) = f^{-1}\chi(x)$

All the couplings of VA goldstino to sugra fields is encoded in $\mathrm{E}_i^a(z^M(x))$. In the WZ gauge we obtain:

$$E^a(z^M) = dx^m e_m^a(x) + (2i\theta\sigma^a\bar\psi(x) + i\theta\sigma^a\nabla\bar\theta + c.c.) + 2\theta\sigma^c\bar\theta(\delta_c^a e^b G_b - e^{[a}G^{c]}(x)) + e^a(\theta\theta\,\bar R + \bar\theta\bar\theta R(x))$$
$$+O(\theta^3) + O(\theta^4) - \tfrac{i}{6}\theta\theta\bar\theta\bar\theta\left(d\theta\sigma_b(\bar T^{ab} - \tfrac{i}{2}\varepsilon^{abcd}\bar T_{cd}) - c.c.\right)$$

$$T_{ab}^\alpha = 2e_a{}^m e_b{}^n \nabla_{[m}\psi_{n]} - i(\psi_{[a}\sigma_{b]}\tilde\sigma^c)^\alpha G_c - i\psi_{[a}^\alpha G_{b]} - 2i(\bar\psi_{[a}\tilde\sigma_{b]})^\alpha R$$

VOLKOV-AKULOV BRANE COUPLED TO SUPERGRAVITY

- Integrating out the auxiliary fields $R(x)$ and $G_a(x)$ from the action we get

$$
\begin{aligned}
S_{SG+3br} = \tfrac{1}{2\kappa^2}\int d^4x\, e\Big(R(e,\omega) - \psi_l\sigma^{lmn}\nabla_m\overline{\psi}_n - m\psi_a\sigma^{ab}\psi_b + 6m^2 - 2\kappa^2 f^2\Big) \\
+ \int d^4x\, e\Big(if\chi\sigma^a\overline{\psi}_a + \tfrac{i}{2}\chi\sigma^a\nabla_a\overline{\chi} + 2m\chi\chi + c.c\Big) \\
+ \int d^4x\, e\Big(\overline{\chi}\overline{\chi}\psi\psi - \chi\sigma_a\overline{\chi}\psi_b\sigma^{abc}\overline{\psi}_c + \tfrac{3\kappa^2}{2f^2}\chi\chi\overline{\chi}\overline{\chi} + c.c\Big) + \underline{O(\chi^4,\ldots,\chi^8)}
\end{aligned}
$$

$$
\begin{aligned}
\delta e_m^a &= 2\mathrm{i}(\epsilon\sigma^a\bar{\psi}_m - \psi_m\sigma^a\bar{\epsilon}) , \\
\delta\psi_m &= \hat{\nabla}\epsilon + \frac{\mathrm{i}}{8}\bar{R}(\bar{\epsilon}\tilde{\sigma}_m)^\alpha + \frac{\mathrm{i}}{16}\left(3G_m\epsilon^\alpha - (\epsilon\sigma_a\tilde{\sigma}_m)^\alpha G^a\right) , \\
\delta\bar{\psi}_m &= \hat{\nabla}\bar{\epsilon} + \frac{\mathrm{i}}{8}\bar{R}(\tilde{\sigma}_m\epsilon)^{\dot{\alpha}} - \frac{\mathrm{i}}{16}\left(3G_m\bar{\epsilon}^{\dot{\alpha}} - (\tilde{\sigma}_m\sigma_a\bar{\epsilon})^{\dot{\alpha}}G^a\right) , \\
\delta R &= -\frac{16}{3}\hat{\nabla}_m\psi_n\sigma^{mn}\epsilon - 2\mathrm{i}\epsilon\sigma_a\bar{\psi}^a R - \mathrm{i}\epsilon\psi_a G^a , \\
\delta\bar{R} &= -\frac{16}{3}\bar{\epsilon}\tilde{\sigma}^{mn}\hat{\nabla}_m\bar{\psi}_n + 2\mathrm{i}\psi^a\sigma_a\bar{\epsilon}\bar{R} + \mathrm{i}\bar{\psi}_a\bar{\epsilon}G^a , \\
\delta G_a &= -\frac{40\mathrm{i}}{3e}e_{ma}\varepsilon^{mnkl}\left(\hat{\nabla}_n\psi_k\sigma_l\bar{\epsilon} - \epsilon\sigma_l\hat{\nabla}_n\bar{\psi}_k\right) - \frac{32}{3}e_a^{[m}\left(\hat{\nabla}_m\psi_n\sigma^{n]}\bar{\epsilon} + \epsilon\sigma^{n]}\hat{\nabla}_m\bar{\psi}_n\right) \\
&\quad + \mathrm{i}G^a\left(\psi_b\sigma^b\bar{\epsilon} - \epsilon\sigma^b\bar{\psi}_b\right) + \frac{\mathrm{i}}{2}\epsilon_{abcd}\left(\psi^b\sigma^c\bar{\epsilon} + \varepsilon\sigma^b\bar{\psi}^c\right)G^d - 2\mathrm{i}R\bar{\psi}_a\bar{\epsilon} + 2\mathrm{i}\bar{R}\epsilon\psi_a ,
\end{aligned}
$$

differ from constrained superfield action, but should be related by non-linear field redefinitions a la Kuzenko & Tyler

$$
\begin{aligned}
\underline{\delta\theta^\alpha = -\epsilon^\alpha} &- \mathrm{i}\left(\theta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\theta}\right)\left[\psi_m^\alpha + \nabla_m\theta^\alpha - \mathrm{i}\left(\theta\sigma^n\bar{\psi}_m - \psi_m\sigma^n\bar{\theta}\right)\left(\psi_n^\alpha + \nabla_m\theta^\alpha\right)\right] \\
&+ \frac{1}{16}\left(\theta\sigma^a\bar{\epsilon} - \epsilon\sigma^a\bar{\theta}\right)\left[2\theta^\alpha G_a + (\theta\sigma_{ab})^\alpha G^b + 2(\bar{\theta}\tilde{\sigma}_a)^\alpha R\right] + \ldots ,
\end{aligned}
$$

non-homogeneous susy transform of the goldstino

CONCLUSION

- Spontaneous susy breaking and appearance of VA goldstini
  - can be triggered by the presence in the theory of brane-like objects
  - or described in the IR effective field theory limit by constrained (nilpotent) superfields

- Spontaneous SUSY breaking generates a positive contribution to the vacuum energy thus allowing for the appearance of de Sitter vacua in supergravity and string theory.

- This makes these supergravity models attractive for inflationary model building

- Realization of the local susy breaking with the use of the space-filling VA 3-brane coupled to gravity and matter multiplets may by useful for finding a more direct connection of these 4d effective field theories to string theory constructions involving anti-D-branes, like the KKLT model and its refinings

$$S_{SG+\text{matter}+VA} = \frac{3}{4\kappa^2}\int d^8z\,\mathrm{Ber}\,E\,e^{-\frac{\kappa^2}{3}K(\bar\Phi, e^V\Phi)} + \frac{m}{2\kappa^2}\int d^6z_L\,\mathcal{E}\Big(W(\Phi) + \mathbf{tr}\,\mathrm{g}(\Phi)\mathbf{W}_\alpha\mathbf{W}^\alpha\Big) + c.c.$$
$$+ f^2\int d^4\xi\,\mathcal{F}\underline{(\Phi, V, \varphi(\xi))}\,\det \mathrm{E}_i^a(z^M(\xi))$$

$$S_{NS} = \frac{3}{4\kappa^2}\int d^8z\,\mathrm{Ber}\,E_M^A\,e^{-\frac{\kappa^2}{3}K(\bar\Phi, e^V\Phi, \underline{S,\bar S})} + \frac{m}{2\kappa^2}\int d^6z_L\,\mathcal{E}\Big(W(\underline{\Phi, S}) + \mathbf{tr}\,\mathrm{g}(\underline{\Phi, S})\mathbf{W}_\alpha\mathbf{W}^\alpha + \Lambda S^2\Big) + c.c.$$

The two actions are equivalent (work in progress)