Generalized type II equations and deformed $AdS \times S$ models

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• search for integrable/solvable deformations of $AdS_n \times S^n$ supercoset string models with no susy but (q-deformed) hidden symmetries

• integrability – key to exact solution for spectrum

• gauge theory dual? some deformation of $N = 4$ SYM ?

• two recent examples:

η-model : JJ (Yang-Baxter) type deformation of supercoset model [Klimcik; Delduc, Magro, Vicedo]

λ-model : gWZW type deformation of non-abelian dual of supercoset model [Sfetsos; Hollowood, Miramontes, Schmidtt]

• both give classically integrable and κ-symmetric GS actions

• UV-finite (scale invariant) sigma models

• but are they Weyl-invariant and define critical string model? corresponding 10d backgrounds solve IIB sugra eqs?

• String in curved space:

sigma model $\int d^2z\,\sqrt{g}g^{ab}\,G_{mn}(x)\,\partial_a x^m \partial_b x^n$

• Scale invariance:

$\beta^G_{mn} = R_{mn} + ... = 0$

$\int d^2z\sqrt{g}\,T^a_a = 0$

• consistency of string theory: decoupling of conf factor of g_{ab}

conformal invariance: $T^a_a = 0$ (for ghost-free spectrum, etc.)

requires adding $\int d^2z\sqrt{g}R^{(2)}\phi(x)$ term to cancel Weyl anomaly

stronger Weyl-invariance condition: $R_{mn} + 2D_m D_n\phi = 0$

effective action: $S = \int d^Dx\sqrt{G}e^{-2\phi}(R + 4\partial^m\phi\partial_m\phi)$

• scale invariance $\neq 0$ conformal invariance

in 2d string model context

(non-compact sigma-models; non-unitary time-like directions)

• Green-Schwarz superstring in curved background:

kappa-symmetry vs supergravity equations

• GS superstring in curved superspace $Z = (x, \theta)$ background

$$L = (\sqrt{g} g^{ab} E^r_M E^s_N \eta_{rs} + \epsilon^{ab} B_{MN}) \partial_a Z^M \partial_b Z^N$$

κ-symmetry $\to$ constraints $\to$ component fields
no explcit dilaton coupling of classical GS string –
dilaton superfield $\phi(Z) = \phi(x) + ...$ via solution of constraints
Weyl invariance requires adding $\int d^2 z \sqrt{g} R^{(2)} \phi(Z)$

• component form of classical GS action

$$S = -T \int d^2 z \sqrt{g} \Big[\tfrac{1}{2} (g^{ab} G_{mn} + \epsilon^{ab} B_{mn}) \partial_a x^m \partial_b x^n \\ + i \bar{\theta} \Gamma_m \nabla \theta \, \partial x^m + \bar{\theta} \Gamma_m \, \mathcal{F} \cdot \Gamma \, \Gamma_n \theta \, \partial x^m \partial x^n + ... \Big]$$

• bosonic sigma model:

$$S = -\tfrac{1}{2} T \int d^2 z \, \sqrt{g} (g^{ab} G_{mn} + \epsilon^{ab} B_{mn}) \partial_a x^m \partial_b x^n$$

• scale invariance or on-shell UV finiteness: $\int d^2z\sqrt{g}T^a_a = 0$

$$\beta^G_{mn} \equiv R_{mn} - \tfrac{1}{4}H_{mkl}H_n{}^{kl} = -\nabla_m X_n - \nabla_n X_m$$
$$\beta^B_{mn} \equiv \tfrac{1}{2}\nabla^k H_{kmn} = X^k H_{kmn} + \partial_m Y_n - \partial_m Y_n$$

X_m and Y_m: freedom of reparams and B-field gauge transf
drop out on eqs of motion or absorbed in $x^m \to x^m + X^m \log\epsilon$

• Weyl invariance condition is stronger: $T^a_a = 0$
can be satisfied adding $\int d^2z\sqrt{g}R^{(2)}\phi(x)$ and if

$$X_m = \partial_m\phi\ , \qquad\qquad dY = 0 \text{ or } Y_m = \partial_m\psi$$

Weyl invariance conditions

$$\bar\beta^G_{mn} \equiv R_{mn} - \tfrac{1}{4}H_{mkl}H_n{}^{kl} + 2\nabla_m\nabla_n\phi = 0$$
$$\bar\beta^B_{mn} \equiv \tfrac{1}{2}\nabla^k H_{kmn} - \partial^k\phi H_{kmn} = 0$$

imply “central charge” identity $\partial_m \bar{\beta}^\phi = 0$ or dilaton equation

$$\bar{\beta}^\phi \equiv R - \frac{1}{12} H^2_{mnk} + 4\nabla^2\phi - 4\partial^m\phi\partial_m\phi = \text{const}$$

equivalent to string eqs from $S \sim \int d^d x \sqrt{G} e^{-2\phi} \bar{\beta}^\phi$

• full set of 1-loop Weyl inv conds of GS sigma model:

follows from type II supergravity action ($\mathcal{F} \equiv e^\phi F$)

$$\int d^d x \, \sqrt{G} \, \left(e^{-2\phi} \, \bar{\beta}^\phi + FF\right) = \int d^d x \, \sqrt{G} \, e^{-2\phi} \, \left(\bar{\beta}^\phi + \mathcal{F}\mathcal{F}\right)$$

• which constraints κ-symm of GS superstring

places on target space (super) geometry ?

Some history:

- kappa-symmetry of superparticle $\to$ constraints of 10d SYM

similar relation expected for GS string and sugra [Witten 1985]

- GS action in type II supergeometry:

κ-symm holds if type II eqs (constraints) are satisfied;
conjectured that converse is also true

[Grisaru, Howe, Mezincescu, Nilsson, Townsend 85]

- true in 11d case: κ-symmetry of supermembrane action

is equivalent to 11d sugra constraints [Howe 97]

- usually assumed that this is also true for 10d superstring
- κ-symmetry of GS superstring implies, in fact, weaker set of

generalized equations equivalent to scale invariance conditions
type IIB supergravity eqs are equivalent to stronger
Weyl invariance conditions

- main issue: existence of dilaton superfield (absent in 11d) type I theory: standard constraints following from κ-symmetry

$$T_{\alpha\beta}{}^{a} = -i\gamma^{a}_{\alpha\beta}\,, \qquad H_{\alpha\beta\gamma} = 0\,, \qquad H_{a\alpha\beta} = -i(\gamma_a)_{\alpha\beta}$$

T – superspace torsion, $H = dB$ – superspace three-form

- dim 1/2 torsion contains spinor ("dilatino") superfield χ_α
- if one assumes that it is expressed in terms of scalar $\phi(Z)$

$$\chi_\alpha = \nabla_\alpha \phi$$

then standard type I sugra eqs follow [Nilsson 1981]

- but this assumption does not directly follow just from κ-symmetry:

it implies χ_α is expressed in terms of vector X_m instead of $\partial_m \phi$

• then generalized eqs follow:

$$
\begin{aligned}
&R_{ab} + 2\nabla_{(a}X_{b)} - \tfrac{1}{4}H_{acd}H_b{}^{cd} = 0\ , \\
&\nabla^c H_{abc} - 4\nabla_{[a}X_{b]} - 2X^c H_{abc} = 0 \\
&\nabla^a X_a - 2X^a X_a + \tfrac{1}{12}H^{abc}H_{abc} = 0
\end{aligned}
$$

• equivalent to scale invariance conditions of type I GS action on flat 2d background

• no reason to expect Weyl-inv eqs to follow from κ-symm as it applies to classical GS action (without dilaton $R\phi$ term)

Type II theory: GS action

$$S = \int d^2\xi \sqrt{-G} + \int B\,, \qquad G = \det\, G_{IJ}\,, \quad I = 1,2$$

$$G_{IJ} = E_I{}^a E_J{}^b \eta_{ab}\,, \qquad E_I{}^A = \partial_I Z^M E_M{}^A(Z)\,, \qquad Z^M = (x^m,\, \theta^\mu)$$

κ-symmetry transformation: $\delta\theta \sim \Gamma\kappa + ...,\quad \delta x^m \sim \bar{\theta}\Gamma^m\delta\theta + ...$

$$\delta_\kappa Z^M E_M{}^a = 0\,, \qquad \delta_\kappa Z^M E_M{}^{\alpha i} = \tfrac{1}{2}(1+\Gamma)^{\alpha i}{}_{\beta j}\kappa^{\beta j}$$

$$\Gamma = \frac{1}{2\sqrt{-G}}\varepsilon^{IJ}E_I{}^a E_J{}^b \gamma_{ab}\,\sigma_3\,, \qquad \mathrm{tr}\,\Gamma = 0,\;\; \Gamma^2 = 1$$

$i, j = 1,2;\;\; i = 1$ in type I; type IIA: $\sigma_3 \to \Gamma_{11}$

requiring κ-symmetry gives constraints

on torsion $T^A = dE^A + E^B\Omega_B{}^A$ and $H = dB$

$$T_{\alpha i\beta j}{}^a = -i\delta_{ij}\gamma^a_{\alpha\beta}\,, \qquad H_{a\alpha i\beta j} = -i\sigma^3_{ij}(\gamma_a)_{\alpha\beta}\,, \qquad H_{\alpha i\beta j\gamma k} = 0$$

next step: combine these with solution of Bianchi identities

$$dT^A + E^B\Omega_B{}^A = E^B R_B{}^A\,, \qquad dH = 0$$

${R_B}^A$: components ${R_a}^b$ and ${R_{\alpha i}}^{\beta j} = -\frac{1}{4} R^{ab} \delta_{ij} (\gamma_{cd})^\beta{}_\alpha$

dim 1/2: get $H_{\alpha i bc} = 0, \quad {T_{\alpha i b}}^c = 0$ and

$$ {T_{\alpha i \beta j}}^{\gamma k} = \delta^{\gamma k}_{(\alpha i} \chi_{\beta j)} + (\sigma^3 \delta)^{\gamma k}_{(\alpha i} (\sigma^3 \chi)_{\beta j)} - \frac{1}{2} \delta_{ij} \gamma^a_{\alpha\beta} (\gamma_a \chi)^{\gamma k} $$

with some spinor superfield $\chi_{\alpha i}$

dim 1:

$$ {T_{a \alpha i}}^{\delta j} = \frac{1}{8} (\gamma^{bc} \sigma^3)_{\alpha i}{}^{\delta j} H_{abc} + \frac{1}{8} (\gamma_a \mathcal{S})_{\alpha i}{}^{\delta j} $$

$$ \begin{aligned} \nabla_{\alpha i} \chi_{\beta j} = & \frac{1}{2} \chi_{\alpha i} \chi_{\beta j} + \frac{1}{2} (\sigma^3 \chi)_{\alpha i} (\sigma^3 \chi)_{\beta j} + \frac{i}{2} (\delta_{ij} \mathrm{X}_a + \sigma^3_{ij} I_a) \gamma^a_{\alpha\beta} \\ & - \frac{i}{24} \sigma^3_{ij} \gamma^{abc}_{\alpha\beta} H_{abc} - \frac{i}{16} (\gamma_a \mathcal{S} \gamma^a)_{\alpha i \beta j} \end{aligned} $$

X_a and I_a are some vector superfields

$\mathcal{S}^{\alpha i \beta j}$ is bi-spinor superfield that can be expanded as

$$ \mathcal{S} = -i \sigma^2 \gamma^a \mathcal{F}_a - \frac{1}{3!} \sigma^1 \gamma^{abc} \mathcal{F}_{abc} - \frac{1}{2 \cdot 5!} i \sigma^2 \gamma^{abcde} \mathcal{F}_{abcde} $$

dim 2: generalized type IIB equations (set fermions to 0)

$$R_{ab} + 2\nabla_{(a}\mathrm{X}_{b)} - \tfrac{1}{4}H_{ade}H_b{}^{de} + \tfrac{1}{128}\mathrm{Tr}(\mathcal{S}\gamma_b\mathcal{S}\gamma^c) = 0$$
$$\nabla^a H_{abc} - 2\mathrm{X}^a H_{abc} - 4\nabla_{[b}I_{c]} - \tfrac{1}{64}\mathrm{Tr}(\sigma^3\gamma_c\mathcal{S}\gamma_b\mathcal{S}) = 0$$

analog of RR eq

$$\gamma^a\nabla_a\mathcal{S} - \gamma^a(\mathrm{X}_a + \sigma^3 I_a)\mathcal{S} + \tfrac{1}{8}\gamma^a\sigma^3\mathcal{S}\gamma^{bc}H_{abc} + \tfrac{1}{24}\gamma^{abc}\sigma^3\mathcal{S}H_{abc} = 0$$

analog of dilaton eq

$$\nabla^a X_a - 2X^a X_a + \tfrac{1}{12}H^{abc}H_{abc} - \tfrac{1}{256}\mathrm{Tr}(\mathcal{S}\gamma^a\mathcal{S}\gamma_a) = 0$$

$$X_a = \mathrm{X}_a + I_a$$
$$\nabla_{(a}I_{b)} = 0\,, \qquad \mathrm{X}^a I_a = 0$$
$$2\nabla_{[a}\mathrm{X}_{b]} + I^c H_{abc} = 0$$

$$
\begin{aligned}
&R_{mn} - \tfrac{1}{4} H_{mkl} H_n{}^{kl} - \mathrm{T}_{mn} + \nabla_m X_n + \nabla_n X_m = 0 \\
&\tfrac{1}{2} \nabla^k H_{kmn} + \mathcal{I}_{mn} - X^k H_{kmn} - \partial_m X_n + \partial_n X_m = 0 \\
&\mathrm{T}_{mn} \equiv \tfrac{1}{2} \mathcal{F}_m \mathcal{F}_n + \tfrac{1}{4} \mathcal{F}_{mpq} \mathcal{F}_n{}^{pq} + \tfrac{1}{4\times 4!} \mathcal{F}_{mpqrs} \mathcal{F}_n{}^{pqrs} \\
&\qquad\quad -\tfrac{1}{2} G_{mn} (\tfrac{1}{2} \mathcal{F}_k \mathcal{F}^k + \tfrac{1}{12} \mathcal{F}_{kpq} \mathcal{F}^{kpq}) \\
&\mathcal{I}_{mn} \equiv \tfrac{1}{2} \mathcal{F}^k \mathcal{F}_{kmn} + \tfrac{1}{12} \mathcal{F}_{mnklp} \mathcal{F}^{klp} \\
&R - \tfrac{1}{12} H^2_{mnk} + 4 \nabla^m X_m - 4 X^m X_m = 0
\end{aligned}
$$

$$
\nabla^m \mathcal{F}_m - \mathrm{X}^m \mathcal{F}_m - \tfrac{1}{6} H^{mnp} \mathcal{F}_{mnp} = 0 \,, \qquad\qquad I^m \mathcal{F}_m = 0
$$

$$
\begin{aligned}
&(d\mathcal{F}_1 - \mathrm{X} \wedge \mathcal{F}_1)_{mn} - I^p \mathcal{F}_{mnp} = 0 \\
&\nabla^p \mathcal{F}_{pmn} - \mathrm{X}^p \mathcal{F}_{pmn} - \tfrac{1}{6} H^{pqr} \mathcal{F}_{mnpqr} - (I \wedge \mathcal{F}_1)_{mn} = 0 \\
&(d\mathcal{F}_3 - \mathrm{X} \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{mnpq} - I^r \mathcal{F}_{mnpqr} = 0 \\
&\nabla^r \mathcal{F}_{rmnpq} - \mathrm{X}^r \mathcal{F}_{rmnpq} + \tfrac{1}{36} \varepsilon_{mnpqrstuvw} H^{rst} \mathcal{F}^{uvw} - (I \wedge \mathcal{F}_3)_{mnpq} = 0 \\
&(d\mathcal{F}_5 - \mathrm{X} \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{mnpqrs} + \tfrac{1}{6} \varepsilon_{mnpqrstuvw} I^t \mathcal{F}^{uvw} = 0
\end{aligned}
$$

generalized type II equations:

up to "measure zero" set of solutions same as type II sugra eqs

- choosing simplest $I_a = 0$ solution:

then $\mathrm{X}_a = \partial_a \phi$

and get back to type IIB sugra eqs

- if metric G admits a Killing vector:

one may choose I_a = Killing and get new background

corresponding to κ-symmetric, scale-invariant

but not Weyl-invariant GS superstring sigma model

- deeper meaning of these generalized eqs? Action?

- non-trivial solutions of generalized eqs are

T-dual to type II sugra solutions with non-isometric dilaton

Example of background with non-isometric linear dilaton

$$ds^2 = e^{2a(x)}[dy + A_\mu(x)dx^\mu]^2 + g_{\mu\nu}(x)dx^\mu dx^\nu$$
$$\phi = -b\,y + f(x)$$

suppose it is string background: solves $R_{mn} + 2\nabla_m\nabla_n\phi = 0$

• dilaton breaks y-isometry: T-duality cannot be applied to get another consistent string background

• but can still be applied to sigma model on flat 2d space

$\rightarrow$ gives T-dual scale-invariant sigma model

but no dilaton exists to make it Weyl-invariant:

it corresponds to generalized background

• T-dual (G, B) background

$$ds^2 = e^{-2a(x)}d\hat{y}^2 + g_{\mu\nu}(x)dx^\mu dx^\nu \;, \qquad B = A_\mu(x)d\hat{y} \wedge dx^\mu$$

• solves generalized eqs (equiv to scale invariance)

$$R_{mn} - \tfrac{1}{4} H_{mkl} H_n{}^{kl} + \nabla_m X_n + \nabla_n X_m = 0$$

$$\tfrac{1}{2} \nabla^k H_{kmn} - X^k H_{kmn} - \partial_m X_n + \partial_m X_n = 0$$

$$R - \tfrac{1}{12} H^2_{mnk} + 4 \nabla^m X_m - 4 X^m X_m = 0$$

$$X_m = \mathrm{X}_m + I_m \,, \qquad I^m \mathrm{X}_m = 0$$

$$\nabla_m I_n + \nabla_n I_m = 0 \,, \qquad \nabla^m I_m = 0$$

$$\partial_m \mathrm{X}_n - \partial_n \mathrm{X}_m + I^k H_{kmn} = 0 \,, \qquad \mathrm{X}_m = \partial_m \widehat{\phi} + B_{km} I^k$$

I_m= Killing vector , X_m= “generalized” dilaton gradient solving “generalized dilaton Bianchi identity”

• in the present example

$$I_{\widehat{y}} = b\, e^{-a} \,, \qquad I_\mu = 0 \,,$$

$$\mathrm{X}_{\widehat{y}} = 0, \qquad \mathrm{X}_\mu = \partial_\mu \widehat{\phi} + b\, A_\mu \,, \qquad \widehat{\phi} = \phi - a$$

origin of I_m in non-isometric dilaton term: $I^{\widehat{y}} = b = -\partial_y \phi$

$AdS_n \times S^n$ type IIB backgrounds:

$AdS_5 \times S^5$ + F_5: limit of D3

$AdS_3 \times S^3 \times T^4$ + F_3, H_3: limit of D5-D1 + NS5-NS1

$AdS_2 \times S^2 \times T^6$ + F_5: limit of D3-D3-D3-D3

most symmetric superstrings:

$AdS_n \times S^n = G/H \to$ supercosets $\widehat{G}/H$

$\widehat{G}$ = super-isometries

$AdS_5 \times S^5 :\quad \widehat{G} = PSU(2,2|4)$

$H = SO(1,4) \times SO(5)$

$AdS_3 \times S^3 :\quad \widehat{G} = PSU(1,1|2) \times PSU(1,1|2)$

$H = SO(1,2) \times SO(3)$

$AdS_2 \times S^2 :\quad \widehat{G} = PSU(1,1|2)$

$H = SO(1,1) \times SO(2)$

GS superstrings in $AdS_n \times S^n$:

alg($\widehat{G}$) = alg(H) + odd + alg(G/H) + odd

$J = \widehat{g}^{-1} d\widehat{g} = J^{(0)} + J^{(1)} + J^{(2)} + J^{(3)}$

$$I = h \int d^2\xi \ \mathrm{STr}\Big[\sqrt{g} g^{ab} J_a^{(2)} J_b^{(2)} + \epsilon^{ab} J_a^{(1)} J_b^{(3)}\Big]$$

- classical integrability; κ-symmetry; Weyl invariance
- quantum integrability $\rightarrow$ solution for spectrum
- critical $d = 10$: symmetries if $AdS_n \times S^n \times T^{10-2n}$

- integrable deformations? solvable models with no susy?
- known examples: orbifolds; T-duality

e.g. β-deformation and generalizations

• recent example: η-model [Delduc, Magro, Vicedo 13,14]
integrable deformation of $AdS_5 \times S^5$ supercoset σ-model
with q-deformed $U_q[\mathfrak{psu}(2,2|4)]$ symmetry
and q-deformed $AdS_5 \times S^5$ l.c. S-matrix

[Arutyunov, Borsato, Frolov 13]

• defines a quantum-consistent solvable string model?

Integrable deformation of group space σ-model

"Yang-Baxter" σ-model [Klimcik 02,08]

PCM: $L_0 = \mathrm{Tr}\,(J_+ J_-), \quad J = g^{-1} dg, \quad g \in G$

$$L = \mathrm{Tr}(J_+ K J_-) \ , \qquad K = \frac{1}{1 - \kappa R_g}$$

$$R_g(M) = g^{-1} R(g M g^{-1}) g \ , \quad M \in \mathrm{alg}(G)$$

R=const – solution of classical modified YBE

$[R(M), R(N)] - R([R(M), N] + [M, R(N)]) = [M, N]$
e.g. $R = \pm 1$ on positive/negative roots and 0 on Cartans

- classical integrability for any κ: Lax pair
- manifest $G \times G$ symmetry broken to $[U(1)]^r \times G$
- hidden non-local conserved charges:
  Hopf-Poisson $U_q[\mathrm{alg}(G)], \quad q = q(\kappa)$
- simplest example: "squashed" S^3 [Cherednik 81]

Integrable deformed G/H σ-models [Delduc, Magro, Vicedo 13]

$$L = \mathrm{Tr}\left(J_+ K J_-\right), \qquad K = \frac{P_2}{1 - \kappa R_g \cdot P_2}$$

$\mathrm{alg}(G) = \mathrm{alg}(H) +$ coset, $\quad J = J^{(0)} + J^{(2)}, \quad P_2(J) = J^{(2)}$

- local charges: G broken to $[U(1)]^r$ – Cartans of $\mathrm{alg}(G)$

• non-local charges: for simple roots $q = e^{-\kappa/h}$

$$\{Q_{+\alpha_n}, Q_{-\alpha_m}\} = i\delta_{nm}[C_n]_q \ , \qquad [C]_q = \frac{q^C - q^{-C}}{q - q^{-1}}$$

• simplest case: deformed S^2 = "sausage" model

[Fateev, Onofri, Zamolodchikov 93]

Deformed $AdS_5 \times S^5$ superstring σ-model:

$$\kappa = 0 : \qquad \frac{\widehat{G}}{H} = \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

Z_4 of $\mathfrak{psu}(2,2|4)$: projectors $P_0 + P_2 + P_1 + P_3 = \mathbf{1}$

$$L_0 = h\,\mathrm{STr}\big(J_+\,K_0\,J_-\big)$$
$$J_a = g^{-1}\partial_a g \ , \qquad K_0 \equiv P_2 + \tfrac{1}{2}(P_1 - P_3)$$

"η-deformation": $\kappa = \frac{2\eta}{1-\eta^2}$ [Delduc, Magro, Vicedo 13,14]

$$L_\kappa = h\,\mathrm{STr}\big(J_+\,K\,J_-\big)$$

$$K = \frac{P_\kappa}{1 - \kappa R_g \cdot P_\kappa}\,, \qquad P_\kappa = P_2 + \frac{1}{1+\sqrt{1+\kappa^2}}(P_1 - P_3)$$

- deformation preserves integrability
- classical $U_q[\mathfrak{psu}(2,2|4)]$ with real $q = e^{-\kappa/h}$
- kappa-symmetry of GS action preserved:

expect UV finiteness to be preserved

- corresponding type IIB background?

puzzle: 10d background extracted from GS action

is not type IIB sugra solution [Arutyunov, Borsato, Frolov 13,15]

GS superstring σ-model: extract background fields

$$I = h \int d^2\sigma \, \left[(G_{mn} + B_{mn}) \partial_+ x^m \partial_- x^n + \bar{\theta} (\nabla + \mathcal{F}_5 + \ldots) \theta \, \partial x \, + \ldots . \right]$$

undeformed $AdS_5 \times S^5$ metric:

$$ds_A^2 + ds_S^2 = -(1+\rho^2) dt^2 + \tfrac{d\rho^2}{1+\rho^2} + \rho^2 dS_3$$
$$+(1-r^2) d\varphi^2 + \tfrac{dr^2}{1-r^2} + r^2 dS_3'$$

deformed $AdS_5 \times S^5$ metric: [Arutyunov, Borsato, Frolov 13,15]

$SO(2,4) \times SO(6)$ broken to $[U(1)]^3 \times [U(1)]^3$, no susy

$$ds_A^2 = -h(\rho) dt^2 + f(\rho) d\rho^2 + \rho^2 \left[v(\rho, \zeta) \left(d\zeta^2 + \cos^2 \zeta \, d\psi_1^2 \right) + \sin^2 \zeta \, d\psi_2^2 \right]$$
$$h = \tfrac{1+\rho^2}{1-\kappa^2 \rho^2} \,, \quad f = \tfrac{1}{(1+\rho^2)(1-\kappa^2 \rho^2)} \,, \quad v = \tfrac{1}{1+\kappa^2 \rho^4 \sin^2 \zeta}$$
$$ds_S^2 = \widetilde{h}(r) d\varphi^2 + \widetilde{f}(r) dr^2 + r^2 \left[\widetilde{v}(r, \theta) \left(d\theta^2 + \cos^2 \theta \, d\phi_1^2 \right) + \sin^2 \theta \, d\phi_2^2 \right]$$
$$\widetilde{h} = \tfrac{1-r^2}{1+\kappa^2 r^2} \,, \quad \widetilde{f} = \tfrac{1}{(1-r^2)(1+\kappa^2 r^2)} \,, \quad \widetilde{v} = \tfrac{1}{1+\kappa^2 r^4 \sin^2 \theta}$$

$$B_{\psi_1\zeta} = \tfrac{1}{2}\kappa\,\rho^4 v(\rho,\zeta)\sin 2\zeta\ \ , \qquad B_{\phi_1\theta} = -\tfrac{1}{2}\kappa\, r^4 \widetilde{v}(r,\theta)\sin 2\theta$$

$$\mathcal{F}_1 = \kappa^2 \mathrm{F}\left[\rho^4\sin^2\zeta\, d\psi_2 - r^4\sin^2\xi\, d\phi_2\right]$$

$$\begin{aligned}
\mathcal{F}_3 = \kappa\,\mathrm{F}\Big[& \frac{\rho^3\sin^2\zeta}{1-\kappa^2\rho^2}dt\wedge d\psi_2\wedge d\rho + \frac{r^3\sin^2\xi}{1+\kappa^2 r^2}d\varphi\wedge d\phi_2\wedge dr \\
& + \frac{\rho^4\sin\zeta\cos\zeta}{1+\kappa^2\rho^4\sin^2\zeta}d\psi_2\wedge d\psi_1\wedge d\zeta + \frac{r^4\sin\xi\cos\xi}{1+\kappa^2 r^4\sin^2\xi}d\phi_2\wedge d\phi_1\wedge d\xi \\
& + \frac{\kappa^2\rho r^4\sin^2\xi}{1-\kappa^2\rho^2}dt\wedge d\rho\wedge d\phi_2 - \frac{\kappa^2\rho^4 r\sin^2\zeta}{1+\kappa^2 r^2}d\psi_2\wedge d\varphi\wedge dr \\
& + \frac{\kappa^2\rho^4 r^4\sin\zeta\cos\zeta\sin^2\xi}{1+\kappa^2\rho^4\sin^2\zeta}d\psi_1\wedge d\zeta\wedge d\phi_2 + \frac{\kappa^2\rho^4 r^4\sin^2\zeta\sin\xi\cos\xi}{1+\kappa^2 r^4\sin^2\xi}d\psi_2\wedge d\phi_1\wedge d\xi\Big]\ ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_5 = \mathrm{F}\Big[& \frac{\rho^3\sin\zeta\cos\zeta}{(1-\kappa^2\rho^2)(1+\kappa^2\rho^4\sin^2\zeta)}dt\wedge d\psi_2\wedge d\psi_1\wedge d\zeta\wedge d\rho \\
& - \frac{r^3\sin\xi\cos\xi}{(1+\kappa^2 r^2)(1+\kappa^2 r^4\sin^2\xi)}d\varphi\wedge d\phi_2\wedge d\phi_1\wedge d\xi\wedge dr \\
& - \frac{\kappa^2\rho r}{(1-\kappa^2\rho^2)(1+\kappa^2 r^2)}(\rho^2\sin^2\zeta\, dt\wedge d\psi_2\wedge d\rho\wedge d\varphi\wedge dr + r^2\sin^2\xi\, dt\wedge d\rho\wedge d\varphi\wedge d\phi_2
\end{aligned}$$

$$+ \frac{\kappa^2 \rho^4 r^4 \sin\zeta \cos\zeta \sin\xi \cos\xi}{(1+\kappa^2\rho^4 \sin^2\zeta)(1+\kappa^2 r^4 \sin^2\xi)} (d\psi_2 \wedge d\psi_1 \wedge d\zeta \wedge d\phi_1 \wedge d\xi - d\psi_1 \wedge d\zeta \wedge d\phi_2 \wedge d\phi$$

$$+ \frac{\kappa^2 \rho r^4 \sin\xi \cos\xi}{(1-\kappa^2\rho^2)(1+\kappa^2 r^4 \sin^2\xi)} (\rho^2 \sin^2\zeta \, dt \wedge d\psi_2 \wedge d\rho \wedge d\phi_1 \wedge d\xi - dt \wedge d\rho \wedge d\phi_2 \wedge d\phi_1$$

$$- \frac{\kappa^2 \rho^4 r \sin\zeta \cos\zeta}{(1+\kappa^2 r^2)(1+\kappa^2\rho^4 \sin^2\zeta)} (r^2 \sin^2\xi \, d\psi_1 \wedge d\zeta \wedge d\varphi \wedge d\phi_2 \wedge dr + d\psi_2 \wedge d\psi_1 \wedge d\zeta \wedge d$$

$$- \frac{\kappa^4 \rho^5 r^4 \sin\zeta \cos\zeta \sin^2\xi}{(1-\kappa^2\rho^2)(1+\kappa^2\rho^4 \sin^2\zeta)} dt \wedge d\psi_1 \wedge d\zeta \wedge d\rho \wedge d\phi_2$$

$$- \frac{\kappa^4 \rho^4 r^5 \sin^2\zeta \sin\xi \cos\xi}{(1+\kappa^2 r^2)(1+\kappa^2 r^4 \sin^2\xi)} d\psi_2 \wedge d\varphi \wedge d\phi_1 \wedge d\xi \wedge dr \Big] ,$$

$$\mathrm{F} \equiv \frac{4\sqrt{1+\kappa^2}}{\sqrt{1-\kappa^2\rho^2}\sqrt{1+\kappa^2\rho^4\sin^2\zeta}\sqrt{1+\kappa^2 r^2}\sqrt{1+\kappa^2 r^4 \sin^2\xi}}$$

• $\mathcal{F}_p \equiv e^{\phi} F_p$ – effective p-form IIB R-R strengths
from GS action interpreted as action in type IIB background

• no dilaton ϕ exists to promote this to type IIB solution

but is, in fact, solution of generalized type IIB equations!

• Applying T-duality to $(G, B, \mathcal{F}_p)$ in 6 isometries (including t) gives much simpler $(G, \mathcal{F}_5)$ background for which exists a dilaton giving (complex) type IIB supergravity solution with only G, ϕ and $\mathcal{F}_5$ being non-trivial [Hoare, AT 15]

$$\widehat{ds}^2 = -\frac{1-\kappa^2\rho^2}{1+\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1+\rho^2)(1-\kappa^2\rho^2)} + \frac{d\hat{\psi}_1^2}{\rho^2\cos^2\zeta} + (\rho\, d\zeta + \kappa\rho\tan\zeta\, d\hat{\psi}_1)^2 + \frac{d\hat{\psi}_2^2}{\rho^2\sin^2\zeta}$$
$$+ \frac{1+\kappa^2 r^2}{1-r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1-r^2)(1+\kappa^2 r^2)} + \frac{d\hat{\phi}_1^2}{r^2\cos^2\xi} + (r\, d\xi - \kappa r\tan\xi\, d\hat{\phi}_1)^2 + \frac{d\hat{\phi}_2^2}{r^2\sin^2\xi}\,,$$

$$\hat{B} = 0\,, \qquad \hat{\mathcal{F}}_1 = \hat{\mathcal{F}}_3 = 0\,,$$

$$\hat{\mathcal{F}}_5 = \frac{4i\sqrt{1+\kappa^2}}{\sqrt{1+\rho^2}\sqrt{1-r^2}}\Big[(d\hat{t} + \frac{\kappa\rho d\rho}{1-\kappa^2\rho^2}) \wedge \frac{d\hat{\psi}_2}{\rho\sin\zeta} \wedge \frac{d\hat{\psi}_1}{\rho\cos\zeta} \wedge (rd\xi - \kappa r\tan\xi\, d\hat{\phi}_1) \wedge (\frac{dr}{1+\kappa^2 r^2} +$$
$$- (d\hat{\varphi} - \frac{\kappa r dr}{1+\kappa^2 r^2}) \wedge \frac{d\hat{\phi}_2}{r\sin\xi} \wedge \frac{d\hat{\phi}_1}{r\cos\xi} \wedge (\rho d\zeta + \kappa\rho\tan\zeta\, d\hat{\psi}_1) \wedge (\frac{d\rho}{1-\kappa^2 r^2} + \kappa\rho d\hat{t})\Big]$$

$$\hat{\phi} = \phi_0 - 4\kappa(\hat{t} + \hat{\varphi}) - 2\kappa(\hat{\psi}_1 - \hat{\phi}_1) + \log\frac{(1-\kappa^2\rho^2)^2(1+\kappa^2 r^2)^2}{\rho^2 r^2\sqrt{1+\rho^2}\sqrt{1-r^2}\,\sin 2\zeta\sin 2\xi}$$

• linear dilaton – breaking 4 isometries of $(G, \mathcal{F}_5)$

• $\mathcal{F}_5$ takes simple form in "rotated" vielbein basis

$$\widehat{ds}^2 = \eta_{MN} e^M e^N$$

$$e^0 = \frac{1}{\sqrt{1+\rho^2}}\big(d\widehat{t} + \frac{\kappa\rho d\rho}{1-\kappa^2\rho^2}\big)\,, \qquad e^1 = \frac{d\widehat{\psi}_2}{\rho\sin\zeta}\,, \qquad e^2 = \frac{d\widehat{\psi}_1}{\rho\cos\zeta}\,,$$
$$e^3 = \rho\, d\zeta + \kappa\rho\tan\zeta\, d\widehat{\psi}_1\,, \qquad e^4 = \frac{1}{\sqrt{1+\rho^2}}\big(\frac{d\rho}{1-\kappa^2\rho^2} + \kappa\rho d\widehat{t}\big)\,,$$
$$e^5 = \frac{1}{\sqrt{1-r^2}}\big(d\widehat{\varphi} - \frac{\kappa r dr}{1+\kappa^2 r^2}\big)\,, \qquad e^6 = \frac{d\widehat{\phi}_2}{r\sin\xi}\,, \qquad e^7 = \frac{d\widehat{\phi}_1}{r\cos\xi}\,,$$
$$e^8 = r\, d\xi - \kappa r\tan\xi\, d\widehat{\phi}_1\,, \qquad e^9 = \frac{1}{\sqrt{1-r^2}}\big(\frac{dr}{1+\kappa^2 r^2} + \kappa r d\widehat{\varphi}\big)\,,$$

$$\widehat{\mathcal{F}}_5 = 4i\sqrt{1+\kappa^2}\big(e^0\wedge e^1\wedge e^2\wedge e^8\wedge e^9 - e^3\wedge e^4\wedge e^5\wedge e^6\wedge e^7\big)$$

• T-dual IIB background also defines integrable GS sigma model but now also Weyl-invariant: consistent string model

• its origin: limit of λ-model background [Hoare, AT 15]

λ-deformation and η-deformation

• "η-deformation": real $q = e^{-\kappa/h}$
deformation of $AdS_5 \times S^5$ supercoset model
[Delduc, Magro, Vicedo 13]

• "λ-deformation": root of unity $q = e^{-i\pi/k}$
[Hollowood, Miramontes, Schmidtt 14]
based on deformed $\widehat{F}/\widehat{F}$ gauged WZW model
may be interpreted as integrable deformation of
non-abelian T-dual of $AdS_5 \times S^5$ supercoset action

• closely related – form pair of Poisson-Lie dual models:
two "faces" of single interpolating theory [Vicedo 15; Hoare, AT 15]

• λ-model has no isometries; special limit generates isometries

• η-model can be obtained from λ-model in this limit
+ analytic continuation + T-duality [Hoare, AT 15]

• why need T-duality: remnant of non-ab T-dual in the limit

Bosonic models: non-abelian T-duality

• simplest example: $SU(2)$ PCM

$L = \mathrm{tr}(J_a J_a)\ , \qquad J_a = g^{-1}\partial_a g$

interpolating action

$L = \mathrm{tr}(v\, \epsilon^{ab} F_{ab} + A_a A_a)\ , \qquad F = dA + AA$

integrate out $v \in su(2)$ – get back to S^3 model

integrate out A – get dual model $L(v)$

• Explicitly: S^3 with $g = (\alpha, \beta, \gamma)$

$ds^2 = d\alpha^2 + \sin^2\alpha\, (d\beta^2 + \sin^2\beta\, d\gamma^2)$

dual background with $v = (r, \phi, \psi)$

$ds^2 = dr^2 + \frac{r^2}{1+r^2}(d\phi^2 + \sin^2\phi\, d\psi^2)$

$B = \frac{r^3}{1+r^2}\ \sin\phi\, d\phi \wedge d\psi\ , \qquad e^{-2\phi} = 1 + r^2$

• Similarly for cosets F/G: in general, dual has no isometries

but integrability preserved: hidden symmetries

λ-model :

interpolating between F/G and its non-ab. dual

- special JJ deformation of WZW model

or AA deformation of F/F gauged WZW model

- parameters (k, λ): $\quad \gamma \equiv \lambda^{-2} - 1$

$$I_{k,\lambda}(f, A, C) = k\big[I_{\rm gWZW}(f, A) - \gamma \int d^2\xi \ {\rm Tr}(A_a P A_a)\big]$$

$$I_{\rm gWZW} = \int d^2\xi \ \ {\rm Tr}\left[\tfrac{1}{2}f^{-1}\partial_+ f f^{-1}\partial_- f + A_+\partial_- f f^{-1} - A_- f^{-1}\partial_+ f \right.$$
$$\left. - f^{-1}A_+ f A_- + A_+ A_-\right] - \tfrac{1}{3}\int d^3x \ \epsilon^{abc}\, {\rm Tr}\left[f^{-1}\partial_a f f^{-1}\partial_b f f^{-1}\partial_c f\right]$$

$f \in F, \quad A_a \in {\rm alg}(F); \qquad PA$ - coset projection of A: to F/G

- $\gamma \to \infty$: gives $A_a \in {\rm alg}(G)$ – get F/G gauged WZW model
- $\gamma \to 0$ and $k \to \infty$: $\quad \lambda = 1 - \frac{\pi}{k}h + \ldots, \ \gamma = \frac{2\pi}{k}h + \ldots$

$f = e^{-v/k} = 1 - \frac{1}{k}v + \ldots, \qquad v \in {\rm alg}(F)$ [Sfetsos:2013]

$$I_{k\to\infty,\lambda\to 1} = \int d^2\xi \ {\rm Tr}\big[\, v\, \epsilon^{ab} F_{ab}(A) - h A_a P A_a\big]$$

Supercoset-based λ-model [Hollowood, Miramontes, Schmidtt 14]

$\widehat{F} = PSU(2,2|4)$ in $AdS_5 \times S^5$ case, $\frac{\widehat{F}}{G_1 \times G_2} \supset \frac{F_1}{G_1} \times \frac{F_2}{G_2}$

F_i and G_i bosonic subgroups; $f \in \widehat{F}, \; A_\pm \in \mathrm{alg}(\widehat{F})$

$$\widehat{I}_{k,\lambda}(f,A) = k\Big[I_{\rm gWZW}(f,A) + \gamma \int d^2\xi \;\mathrm{STr}\left(A_+ P_\lambda A_-\right)\Big]$$

$\gamma = \lambda^{-2} - 1\,, \qquad P_\lambda = P_2 + \frac{\lambda}{\lambda+1}(P_1 - \lambda P_3)$

- no global symmetry: $G_1 \times G_2$ gauge symm. to be fixed
- limit $k \to \infty, \; \lambda \to 1$ combined with scaling $f \to 1$

$$f = \exp(-\tfrac{4\pi}{k}\, v) = 1 - \tfrac{4\pi}{k} v + \dots\,, \qquad \lambda = 1 - \tfrac{\pi}{k} h + \dots$$

$$\widehat{I}_{k\to\infty,\lambda\to 1} = \int d^2\xi \;\mathrm{STr}\left[\, v\, F_{+-}(A) + h\, A_+ P A_- \right]$$

$$P = P_\lambda\Big|_{\lambda=1} = P_2 + \tfrac{1}{2}(P_1 - P_3)$$

- λ-model is a deformation of interpolating action between $AdS_5 \times S^5$ supercoset model and non-abelian T-dual
- classically integrable; κ-symmetric; scale-inv [Appadu, Hollowood]

λ-model background

• choose coset parametrization of $f \to (x, \theta)$

– integrate out A_a (as in gWZW model)

– interpret as GS sigma model: get metric and RR field

$$L = (G + B)\partial x \partial x + \bar{\theta}(\nabla + H + \mathcal{F})\theta \partial x + \ldots$$

– dilaton from sdet from integrating out $A_a \in \mathrm{alg}(\widehat{F})$

– kappa-symmetry and no isometries: must be type IIB solution

– indeed, background solves IIB sugra eqs [Borsato, Wulff, AT 16]

• "bosonic" version of the model: $\theta = 0, \quad A_a \in \mathrm{alg}(F)$

same metric but dilaton is different (no fermionic part of sdet)

– can also be promoted to IIB sugra solution (adding RR flux)

[Sfetsos, Thompson 14,15; Demulder, Sfetsos, Thompson 15]

• full (supergroup) and "bosonic" type IIB backgounds:

have no isometries; become same in limit producing isometries

• special limit: enhances bosonic Cartan directions [Hoare, AT 15]
– gives linear isometry-breaking terms in dilaton
– effectively suppresses role of fermionic components of A:
– both backgrounds become same in the limit
• get type IIB background related by analytic continuation
and formal T-duality to η-model background
that solves only generalized type II eqs [AFHRT 15]

Example: $AdS_2 \times S^2 \times T^6$

Metric: $(\varkappa \equiv \frac{1-\lambda^2}{1+\lambda^2})$

bosonic coset space is $\frac{SO(2,1)}{SO(1,1)} \times \frac{SO(3)}{SO(2)}$

$$f = \big[\exp(it\sigma_3)\exp(\xi\sigma_1)\big]\oplus\big[\exp(i\varphi\sigma_3)\exp(i\zeta\sigma_1)\big]$$

$$ds^2 = -\,dt^2 + \cot^2 t\, d\xi^2 - (\varkappa^{-2} - 1)(\cosh\xi\, dt - \cot t \sinh\xi\, d\xi)^2$$
$$+\, d\varphi^2 + \cot^2\varphi\, d\zeta^2 + (\varkappa^{-2} - 1)(\cos\zeta\, d\varphi + \cot\varphi \sin\zeta\, d\zeta)^2$$

algebraic coordinates $(x, y; p, q)$

$$t = \arccos \sqrt{\varkappa x^2 - \varkappa^{-1} y^2}\,, \quad \xi = \operatorname{arccosh} \frac{\varkappa^{1/2} x}{\sqrt{\varkappa x^2 - \varkappa^{-1} y^2}}\,,$$

$$\varphi = \arccos \sqrt{\varkappa p^2 + \varkappa^{-1} q^2}\,, \quad \zeta = \arccos \frac{\varkappa^{1/2} p}{\sqrt{\varkappa p^2 + \varkappa^{-1} q^2}}$$

metric takes simple diagonal form

$$ds^2 = \frac{-dx^2 + dy^2}{1 - \varkappa x^2 + \varkappa^{-1} y^2} + \frac{dp^2 + dq^2}{1 - \varkappa p^2 - \varkappa^{-1} q^2}$$

Dilaton:

• bosonic λ-model

$$e^{\phi_B} = \frac{e^{\phi_0}}{\sin t \ \sin \varphi} = \frac{e^{\phi_0}}{\sqrt{(1 - \varkappa x^2 + \varkappa^{-1} y^2)(1 - \varkappa p^2 - \varkappa^{-1} q^2)}}$$

• supergroup λ-model

$$e^{\phi} = e^{\phi_B} M \ , \qquad\qquad M \equiv \varkappa - x^2 + y^2 - p^2 - q^2 + 2\sqrt{1-\varkappa^2}\, xp$$

does not factorize due to fermion factor M

RR background

from λ-model action get same as in type IIB solution $M^4 \times T^6$
for above metric and full dilaton

$$F_5 = \tfrac{1}{2}\big(F \wedge \mathrm{Re}\,\Omega_3 - F^* \wedge \mathrm{Im}\,\Omega_3\big)$$
$$F \equiv \tfrac{1}{2} F_{mn}(x) dx^m \wedge dx^n \ , \qquad \Omega_3 \equiv dz_1 \wedge dz_2 \wedge dz_3$$

effective action and eqs

$$S = \int d^4x\, \sqrt{-g}\Big[e^{-2\phi}\big(R + 4(\nabla\phi)^2\big) - \tfrac{1}{4} F_{mn}F^{mn}\Big]$$
$$R_{mn} + 2\nabla_m\nabla_n\phi = \tfrac{1}{2} e^{2\phi}(F_{mp}F_n{}^p - \tfrac{1}{4} g_{mn}F_{kl}F^{kl}) \ ,$$

$$R + 4\nabla^2\phi - 4(\nabla\phi)^2 = 0\ , \qquad \nabla^2 e^{-2\phi} = 0$$

$$\partial_n(\sqrt{-g}\, F^{mn}) = 0\ , \qquad \partial_{[m}F_{nk]} = 0\ .$$

• there may be several solutions for ϕ and F for the same metric

[Lunin, Roiban, AT 14]

• solution for "bosonic" dilaton ϕ_B: [Sfetsos, Thompson 14]

$$A_B = c\sqrt{1-\varkappa^2}\, p\, dy\ , \qquad F = dA_B \sim dp \wedge dy\ , \qquad c = \tfrac{1}{2}\varkappa^{-1/2}e^{-\phi_0}$$

this "bosonic" solution may be related to alternative λ-model with only bosonic subgroup of supergroup $\widehat{F}$ gauged

• solution for full dilaton ϕ: [Borsato, Wulff, AT 2016]

$$A = c\, M^{-1}\left[y\, dx + (\sqrt{1-\varkappa^2}\, p - x)\, dy\right]$$

• involves "non-factorized" $M(x, y, p, q)$ part of dilaton and thus $F = dA$ is more complicated

Scaling limit

$$(x,y)\to\gamma_1(x,y)\ ,\qquad (p,q)\to\gamma_2(p,q)\ ,\qquad\qquad \gamma_1,\gamma_2\to\infty$$

generates scaling isometries in the metric

$$ds^2=\frac{1}{-\varkappa x^2+\varkappa^{-1}y^2}\big(-dx^2+dy^2\big)+\frac{1}{-\varkappa p^2-\varkappa^{-1}q^2}\big(dp^2+dq^2\big)$$

• supergroup λ-model : in "asymmetric" limit with $\frac{\gamma_1}{\gamma_2}\to 0$
and after $p'=\frac{p}{p^2+q^2}\ ,\quad q'=\frac{q}{p^2+q^2}$ same as "bosonic" solution

$$ds^2=\frac{1}{-\varkappa x^2+\varkappa^{-1}y^2}\big(-dx^2+dy^2\big)+\frac{1}{-\varkappa p'^2-\varkappa^{-1}q'^2}\big(dp'^2+dq'^2\big)$$

$$e^{\phi}=\frac{e^{\phi_0'}}{\sqrt{(\varkappa x^2-\varkappa^{-1}y^2)(-\varkappa p'^2-\varkappa^{-1}q'^2)}}\ ,\qquad A=c\,e^{-\phi_0'}\sqrt{1-\varkappa^2}\,p'\,dy$$

• limit is still type IIB solution and gives T-dual of η-model background (with $\kappa=i\varkappa$) which solves only generalized type II eqs

Conclusions

• kappa-symmetry of GS string model $\to$ generalized eqs

– weaker than type II supergravity eqs

– generalise scale-invariance eqs to GS superstring sigma model

– "extra" solutions exist only if there is an isometry and are T-dual to type II sugra solutions with non-isometric dilaton

• integrable deformations of $AdS_n \times S^n$:

provide examples of kappa-symmetric backgrounds

that may (λ-model) or may not (η-model) be type II sugra solns

• relation between λ-model and η-model :

– classically equivalent in the phase space description (PL dual)

– backgrounds related by a limit, analytic contn and T-duality

• full implications remain to be understood

e.g. origin of q-deformed symmetries, gauge dual, etc.

$AdS_2 \times S^2 \times T^6$ $\qquad \frac{PSU(1,1|2)}{SO(1,1)\times SO(2)}$

[Berkovits,Bershadsky,Hauer,Zhukov,Zwiebach 99;Sorokin, AT, Wulff,Zarembo 11]

embedding to IIB string: D3-D3-D3-D3 [Klebanov, AT 96]

$$ds^2 = -(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} + (1-r^2)d\varphi^2 + \frac{dr^2}{1-r^2} + dT^6$$

$$F_5 = \Omega_2(AdS_2) \wedge \mathrm{Re}\,\Omega_3(T^6) \ + \ * \,, \qquad \Omega_3 = dz^1 \wedge dz^2 \wedge dz^3$$

effective 4d theory:

$$L = e^{-2\Phi}\left[R + 4(\partial_m \Phi)^2\right] - \tfrac{1}{4} F_{mn} F^{mn} + \ldots$$

Bertotti-Robinson: $AdS_2 \times S^2$ with $\Phi = 0$ and

$$F_2 = \sqrt{2}(d\rho \wedge dt + dr \wedge d\varphi)$$

Deformed $AdS_2 \times S^2$ metric

deformed supercoset action leads to 4d metric

$$ds^2_A + ds^2_S = \frac{1}{1-\varkappa^2\rho^2}\Big[-(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2}\Big] + \frac{1}{1+\varkappa^2 r^2}\Big[+(1-r^2)d\varphi^2 + \frac{dr^2}{1-r^2}\Big]$$

• ds^2_S = "sausage" model [Fateev, Onofri, Zamolodchikov 93]

deformation of S^2 stable under RG flow:

$$ds^2 = f(y)(d\varphi^2 + dy^2), \qquad \frac{\partial f}{\partial t} \sim R(f) \sim e^{-f}\partial_y^2 f$$

$$ds^2 = \frac{dy^2+d\varphi^2}{\cosh^2 y + \varkappa^2 \sinh^2 y} = \frac{1}{1+\varkappa^2 r^2}\Big[(1-r^2)d\varphi^2 + \frac{dr^2}{1-r^2}\Big]$$

• Full η-model background: $M^4 \times T^6$ metric, $\mathcal{F}_3,\ \mathcal{F}_5 \neq 0$

$$\mathcal{F}_3 = \kappa\,\mathrm{F}\Big[(-\rho + r)\omega_r + (\rho + r)\omega_i\Big]\,, \quad \mathrm{F} \equiv \frac{\sqrt{2}\sqrt{1+\kappa^2}}{\sqrt{1-\kappa^2\rho^2}\sqrt{1+\kappa^2 r^2}}\,, \quad \Omega_3 = \omega_i + i\omega_r$$

$$\mathcal{F}_5 = \mathrm{F}\Big[\frac{1-\kappa^2\rho r}{1-\kappa^2\rho^2}dt\wedge d\rho\wedge\omega_r - \frac{1+\kappa^2\rho r}{1-\kappa^2\rho^2}dt\wedge d\rho\wedge\omega_i + \frac{1+\kappa^2\rho r}{1+\kappa^2 r^2}d\varphi\wedge dr\wedge\omega_r + \frac{1-\kappa^2\rho r}{1+\kappa^2 r^2}d\varphi\wedge dr\wedge\omega_i\Big]$$

- T-dual background is type IIB solution: [Hoare, AT 15]

$$\widehat{ds}^2 = -\frac{1-\kappa^2\rho^2}{1+\rho^2}d\widehat{t}^2 + \frac{d\rho^2}{(1-\kappa^2\rho^2)(1+\rho^2)} + \frac{1+\kappa^2 r^2}{1-r^2}d\widehat{\varphi}^2 + \frac{dr^2}{(1+\kappa^2 r^2)(1-r^2)} + dT^6$$

$$\widehat{\mathcal{F}}_5 = \frac{i\sqrt{1+\kappa^2}}{\sqrt{2}\sqrt{1+\rho^2}\sqrt{1-r^2}}\Big[(d\widehat{t} + \frac{\kappa\rho d\rho}{1-\kappa^2\rho^2})\wedge(\frac{dr}{1+\kappa^2 r^2} + \kappa r d\widehat{\varphi})\wedge(\omega_r+\omega_i) + (d\widehat{\varphi} - \frac{\kappa r dr}{1+\kappa^2 r^2})\wedge(\frac{d\rho}{1-\kappa^2\rho^2} + \kappa\rho d\widehat{t})\wedge(\omega_r-\omega_i)\Big],$$

$$\widehat{\phi} = \phi_0 - \kappa(\widehat{t} + \widehat{\varphi}) + \log\frac{(1-\kappa^2\rho^2)(1+\kappa^2 r^2)}{\sqrt{1+\rho^2}\sqrt{1-r^2}}$$

limit of λ-model type IIB background (+ analytic cont)