Local Lorentz symmetry in HS systems

(with N. Misuna and M. Vasiliev)

- Introduction and motivation
- Lorentz covariant HS equations
- Lorentz covariant HS Lagrangian extension
- Conclusion

Question: Is there a way of doing HS AdS/CFT tests at the level of e.o.m?

Proposal: (Vasiliev) On-shell gauge-invariant functional may be a candidate for AdS/CFT generating functions

$$\mathcal{F}\left(\phi(\vec{x}_1, z), \dots, \phi(\vec{x}_n, z)\right)\Big|_{z\to 0} \sim \langle J(\vec{x}_1), \dots, J(\vec{x}_n)\rangle$$

$\mathcal{F}$ is defined uniquely within the unfolded extension of HS equations.

"Lagrangian" extension

HS equations in $d=4$

$W(z,Y)$ – 1-form HS potentials $\qquad S_\alpha(z,Y)$, $\bar{S}_{\dot\alpha}(z,Y)$ –

$B(z,Y)$ – 0-form HS curvatures $\qquad$ – aux. field

space-time sector:

$$
\begin{cases}
dW + W * W = 0 \\
dS_\alpha + [W, S_\alpha] = 0 \\
d\bar{S}_{\dot\alpha} + [W, \bar{S}_{\dot\alpha}] = 0 \\
dB + [W, B]^{tw} = 0
\end{cases}
$$

twistor sector:

$$
\begin{cases}
[S_\alpha, S_\beta] = -2i\,\epsilon_{\alpha\beta}(1 + \eta B * \varkappa) \\
\{S_\alpha, B * \varkappa\} = 0 \\
[S_\alpha, \bar{S}_{\dot\alpha}] = 0
\end{cases}
$$

$$\mathcal{W} = W + S_\alpha \theta^\alpha + \bar{S}_{\dot\alpha} \bar\theta^{\dot\alpha}$$

$$\theta^\alpha dx^{\underline{n}} + dx^{\underline{n}} \theta^\alpha = 0$$

$$\begin{cases} d\mathcal{W} + \mathcal{W} * \mathcal{W} = i\theta^\alpha \theta_\alpha + i\eta B * \gamma + c.c. \\ dB + [\mathcal{W}, B] = 0 \end{cases} \qquad \gamma = e^{iz_\alpha y^\alpha} \theta^\gamma \theta_\gamma$$

A candidate for on-shell action should be $\int (4\text{-form}) d^4x$

$\Rightarrow$ diff. forms extension:

$$W \to 1\text{-forms} \oplus 3\text{-forms}$$

$$B \to 0\text{-forms} \oplus 2\text{-forms}$$

$$\begin{cases} d\mathcal{W} + \mathcal{W} * \mathcal{W} = i\theta^\alpha \theta_\alpha + i\eta B * \gamma + c.c. \\ \qquad + ig\gamma * \delta + \mathcal{L} \\ dB + [\mathcal{W}, B] = 0 \end{cases}$$

$$F = \int \mathcal{L}$$

$$\hat{W} = \Big(\underbrace{W,\ S_\alpha \theta^\alpha,\ \bar{S}_{\dot\alpha} \bar\theta^{\dot\alpha}}_{1-\text{forms}}\ ;\ \underbrace{t_\alpha \theta^\alpha \bar\theta\bar\theta,\ \Omega\theta\theta,\ \Omega_{\alpha\dot\alpha}\theta^\alpha\bar\theta^{\dot\alpha},\ \Omega_\alpha \theta^\alpha,\ \mathcal{W}}_{3-\text{forms}} \Big)$$

twistor sector:

$$\begin{cases} [S_\alpha, S_\beta] = -2i\,\epsilon_{\alpha\beta}\,(1+\eta\, B * \varkappa)\ , \qquad \{S_\alpha, B\varkappa\} = 0 \\ [t_\alpha, S^\alpha] + [\bar{t}_{\dot\alpha}, \bar{S}^{\dot\alpha}] = i g\, \varkappa * \bar\varkappa + i\eta\, \bar{b} * \varkappa + i\bar\eta\, b * \bar\varkappa \\ \{S_\alpha, \bar{b}\varkappa\} + \{t_\alpha, B\varkappa\} = 0 \end{cases}$$

Generalized deformed oscillator

space-time sector:

$$\begin{cases} dW + WW = 0 \\ dS_\alpha + [W, S_\alpha] = 0 \end{cases}$$

$$\begin{cases} dt_\alpha + [W, t_\alpha] - [S_\alpha, \bar{\Omega}] - [\bar{S}_{\dot\alpha}, \Omega_\alpha{}^{\dot\alpha}] = \bar{\eta}\, b_\alpha\, x \\ d\Omega + \{W, \Omega\} - [S_\alpha, \Omega^\alpha] = \eta\, b\, x \\ d\Omega_{\alpha\dot\alpha} + \{W, \Omega_{\alpha\dot\alpha}\} + [S_\alpha, \Omega_{\dot\alpha}] - [\bar{S}_{\dot\alpha}, \Omega_\alpha] = 0 \\ d\Omega_\alpha + [W, \Omega_\alpha] - [S_\alpha, \mathcal{W}] = 0 \\ d\mathcal{W} + \{W, \mathcal{W}\} = 0 \end{cases}$$

Component perturbation theory is very hard.

proper technique is available.

At first order we find

NOT LORENTZ covariant

$$d\mathcal{W}^{(3)} + \{W_0, \mathcal{W}^{(3)}\} = \int (1,1)\, \omega_0^2 \wedge h_0^2\, C + (1,1)\, \omega_0 \wedge h^3\, C$$

Lesson: Non-covariant version of HS equations causes a lot of fake difficulties.

Covariantisation

Original HS equations

$$\begin{cases} dW + WW = 0 \\ dS_\alpha + [W, S_\alpha] = 0 \\ dB + [W, B] = 0 \end{cases}$$

$$W(Z, Y) = \sum \underbrace{W_{A_1 \dots A_n, B_1 \dots B_m}}_{\text{NOT space-time tensors}} Z^{A_1} \dots Z^{A_n} Y^{B_1} \dots Y^{B_m}$$

Lorentz covariant derivative:

$$D^L f_{A_1 \dots A_n} = d f_{A_1 \dots A_n} + n \omega^L{}_A{}^B f_{BA \dots A}$$

Star-product realization: $L^0_{AB} = -\frac{i}{4}(Y_A Y_B - Z_A Z_B)$

$$\rightarrow \quad [L^0_{AB}, f(Z, Y)] = \left(Y_A \frac{\partial}{\partial Y^B} + Z_A \frac{\partial}{\partial Z^B} + \text{sym} \right) f(Z, Y)$$

Seeming way out:

$$W \to W + \omega_L{}^{AB}\, L^0_{AB} \quad ; \quad D^L = d + \omega_L{}^{AB}\, [\, L^0_{AB}, \bullet\,]$$

problem:

$$D^2 S_\alpha = dS_\alpha + \omega_L{}^{AB}\, [L^0_{AB}, S_\alpha] + \underbrace{\left(\omega_{L\alpha}{}^{\beta} S_\beta\right)}_{\text{missed!}}$$

Proper Lorent differential

$$D^L f(z, Y; \theta) = df + \omega_L{}^{AB}[L^0_{AB}, f] + \omega_L{}^{AB}\,\theta_A \frac{\partial}{\partial\theta^B} f$$

$$(D^L)^2 f = R^{AB}[L^0_{AB}, f] + R^{AB}\,\theta_A \frac{\partial}{\partial\theta^B} f$$

$$R_{AB} = d\omega^L_{AB} + \omega^L{}_A{}^C \wedge \omega^L_{CB}$$

Assume Lorentz covariant form exists

$\Rightarrow$ some of the equations are known

$$D^2 S + [W, S] = 0$$

Inspecting consistency $(D^2)^2 \sim R^2$ one constraints the form of other eqs.

$\Rightarrow$ covariant form:

$$\begin{cases} D^2 W + W * W + R^{AB}\left(L^0_{AB} - \frac{i}{4\nu} S_A * S_B\right) = i\nu \theta^\alpha \theta_\alpha + i\eta B * \gamma + c.c. \\ D^2 B + [W, B] = 0 \end{cases}$$

<u>comment:</u> No Lorentz covariance at $\nu = 0$!

Stückelberg symmetry :

$$\delta_\xi \omega^L_{AB} = \xi_{AB} \quad ; \quad \delta_\xi B = \delta_\xi S = 0 \; ; \quad \delta_\xi W = -\xi^{AB}\left(L^0_{AB} - \frac{i}{4} S_A * S_B\right)$$

ω^L_{AB} can be always gauged away. Unless

Physical requirement : No other Lorentz connection in $W(Z,Y)$

$$\Rightarrow \frac{\partial^2}{\partial y^\alpha \partial y^\beta} W(0,Y)\Big|_{Y=0} = 0 \quad ; \quad \frac{\partial^2}{\partial \bar{y}^{\dot\alpha} \partial \bar{y}^{\dot\beta}} W(0,Y)\Big|_{Y=0} = 0$$

Origin of local Lorentz symmetry : twistor sector

$$\begin{cases} [S_\alpha, S_\beta] = -2i\,\epsilon_{\alpha\beta}(1 + B * x) \\ \{S_\alpha, B * x\} = 0 \end{cases}$$

$\rightarrow$ contains field dependent Lorentz generators

$$M_{\alpha\beta} = S_\alpha * S_\beta$$

$$[M, M] \sim M$$

Extended HS system

twistor sector:

$$\begin{cases} [S_\alpha, S_\beta] = -2i\,\epsilon_{\alpha\beta}(1+\eta B*\varkappa), \qquad \{S_\alpha, B\varkappa\} = 0 \\ [t_\alpha, \bar{S}^{\dot\alpha}] + [\bar{t}_{\dot\alpha}, \bar{S}^{\dot\alpha}] = ig\,\varkappa*\bar\varkappa + i\eta\,\bar{b}*\varkappa + i\bar\eta\, b*\bar\varkappa \\ \{S_\alpha, \bar{b}\varkappa\} + \{t_\alpha, B\varkappa\} = 0 \end{cases}$$

gauge symmetry $\qquad \delta S_\alpha = [\varepsilon, S_\alpha]$

$$\delta t_\alpha = [\varepsilon, t_\alpha] + [\varphi, S_\alpha] + [\psi_{\alpha\dot\beta}, \bar{S}^{\dot\beta}]$$

take $\varepsilon = \Lambda^{\alpha\beta} S_\alpha * S_\beta \;\rightarrow\; \delta_\Lambda S_\alpha = \Lambda_\alpha{}^\beta S_\beta \;;\quad \delta_\Lambda t_\alpha - ?$

$$\varphi = \Lambda^{\alpha\beta}\{S_\alpha, t_\beta\} \quad , \quad \psi_{\alpha\dot\beta} = \Lambda_\alpha{}^\gamma\{S_\gamma, \bar t_{\dot\beta}\}$$

$\Rightarrow$ $\delta_\Lambda t_\alpha = \Lambda_\alpha{}^\beta t_\beta$! (strong indication on Lorentz covariantization)

Still, hard to derive the proper form of eqs.

Some if exist are immediately available :

$$\begin{cases} D^L t_\alpha + [W, t_\alpha] - [S_\alpha, \bar\Omega] - [\bar S_{\dot\alpha}, \Omega_\alpha{}^{\dot\alpha}] = b_\alpha * x \\ D^L S_\alpha + [W, S_\alpha] = 0 \end{cases}$$

consistency implies some constraint on other fields

Covariant equations

W, B — set of (1 and 3) and (0, 2) - forms packed with θ^A

fixed!

$$\begin{cases} D^L W + W * W + R^{AB}\left(L^0_{AB} - \frac{i}{4}\partial_A W * \partial_B W\right) = i\theta^\alpha \theta_\alpha + i\eta B * \gamma + i g \gamma * \bar{\gamma} + \mathcal{L} \\ \qquad - \frac{\eta}{4} R^{\alpha\beta} \partial_\alpha B * \partial_\beta \gamma + \frac{i\eta}{32} R^{\alpha\alpha} R^{\beta\beta} \partial_\alpha \partial_\beta B * \partial_\alpha \partial_\beta \gamma \\ D^L B + [W, B] - \frac{i}{4} R^{AB}\{\partial_A B, \partial_B W\} = 0 \end{cases}$$

$$\partial_A = \frac{\partial}{\partial \theta^A}$$

$$\delta_\xi \omega_{AB} = \xi_{AB}, \qquad \delta_\xi B = \frac{i}{4}\xi^{AB}\{\partial_A B, \partial_B W\}$$

$$\delta_\xi W = -\xi^{\alpha\beta}\left(L^0_{\alpha\beta} - \frac{i}{4}\partial_\alpha W \partial_\beta W\right) - \frac{\eta}{4}\xi^{\alpha\beta}\partial_\alpha B * \partial_\beta \gamma + \frac{i\eta}{32}\left(\xi^{\alpha\alpha} R^{\beta\beta} + \xi^{\beta\beta} R^{\alpha\alpha}\right)\partial_\alpha \partial_\beta B * \partial_\alpha \partial_\beta \gamma + c.c.$$

Comment: Lorentz covariance fixes all central terms in eqs. uniquely.

Conclusion and to do

1) Lorentz covariant form of extended HS equations is constructed.

2) Consistency fixes central terms uniquely which otherwise admit some arbitrariness.

- Construct covariant perturbation theory.
- Keep pushing invariant functionals.