# Properties of black hole + scalar solutions to Vasiliev's 4D equations

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# Summary

- Basics of the 4D bosonic Vasiliev's equations
- Particle modes + HS black hole solutions in various "gauges"
  - W=0 gauge, L-gauge and variations
  - Expansion of the initial data on various $\mathfrak{so}(3,2)$-modules
  - Projectors and twisted projectors
- Z-space connection for particle modes and HS black holes
  - Singularities in various gauges
  - Vasiliev gauge
- Conclusions

# Kinematics

- Master-fields living on *correspondence space*, locally $\mathcal{X}$ x $\mathcal{Z}$ x $\mathcal{Y}$:

$$\widehat{W} = dx^\mu \widehat{W}_\mu(Y,Z|x)$$ → gauge fields of all spins + auxiliary

$$\widehat{\Phi} = \widehat{\Phi}(Y,Z|x)$$ → Weyl tensors and their derivatives → local dof

$$\widehat{S} = dz^\alpha \widehat{S}_\alpha(Y,Z|x) + d\bar{z}^{\dot{\alpha}} \widehat{\bar{S}}_{\dot{\alpha}}(Y,Z|x)$$ → Z-space connection, no extra local dof

- Commuting oscillators $Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot{\alpha}})\ ,\quad Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}})$ → $\mathfrak{sp}$(4,R) quartets

$$[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i \begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}\ , \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}}\ , \quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Star-product:

$$\widehat{F}(Y,Z) \star \widehat{G}(Y,Z) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4}\, e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}}\, \widehat{F}(Y+U, Z+U)\, \widehat{G}(Y+V, Z-V)$$

- Inner kleinian operator $\widehat{\kappa}$:

$$\widehat{\kappa} = e^{iy^\alpha z_\alpha}\ , \qquad \widehat{\kappa} \star \widehat{f}(z,y) = \widehat{f}(-z,-y) \star \widehat{\kappa}\ , \qquad \widehat{\kappa} \star \widehat{\kappa} = 1$$

$$\widehat{\kappa} = \kappa_y \star \kappa_z\ , \quad \kappa_y \star \widehat{f}(z,y) = \widehat{f}(z,-y) \star \kappa_y\ , \quad \kappa_y \star \kappa_y = 1\ ,$$

$$\kappa_y = 2\pi\delta^2(y) = 2\pi\delta(y_1)\delta(y_2)$$

# 4D bosonic Vasiliev equations

- Full equations:

$$
\begin{aligned}
d\widehat{W} + \widehat{W} \star \widehat{W} &= 0 \\
d\widehat{\Phi} + \widehat{W} \star \widehat{\Phi} - \widehat{\Phi} \star \pi(\widehat{W}) &= 0 \\
d\widehat{S}_\alpha + [\widehat{W}, \widehat{S}_\alpha]_\star &= 0 \\
\widehat{S}_\alpha \star \widehat{\Phi} + \widehat{\Phi} \star \pi(\widehat{S}_\alpha) &= 0 \\
[\widehat{S}_\alpha, \widehat{S}_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1 - \mathcal{B} \star \widehat{\Phi} \star \widehat{\kappa}) \\
[\widehat{S}_\alpha, \widehat{\bar{S}}_{\dot\beta}]_\star &= 0 \ ,
\end{aligned}
$$

*(Vasiliev)*

$$
\widehat{S}_\alpha = z_\alpha - 2i\widehat{V}_\alpha
$$

- Manifestly consistent ←→ gauge invariant.

- *Z*-oscillators → auxiliary, non-commutative coordinates. Equations fix the evolution along Z in such a way that it gives rise to consistent interactions to all orders among physical fields.
  The latter are contained in the (Z-independent) initial conditions for the Z-evolution,

$$
W = \widehat{W}_{|Z=0} \ , \qquad \Phi = \widehat{\Phi}_{|Z=0} \ .
$$

# Exact solutions: gauge function method

- $\mathcal{X}$ x $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:
- $\mathcal{Y}$ x $\mathcal{Z}$-space eqns:

$$\widehat{W} = \widehat{g}^{-1} \star d\widehat{g}$$

$$\widehat{\Phi} = \widehat{g}^{-1} \star \widehat{\Phi}' \star \pi(\widehat{g}) \,, \qquad d\widehat{\Phi}' = 0$$

$$\widehat{S}_\alpha = \widehat{g}^{-1} \star \widehat{S}'_\alpha \star \widehat{g} \,, \qquad d\widehat{S}'_\alpha = 0$$

$$\widehat{S}'_\alpha \star \widehat{\Phi}' + \widehat{\Phi}' \star \pi(\widehat{S}'_\alpha) = 0$$

$$[\widehat{S}'_\alpha, \widehat{S}'_\beta]_\star = -2i\epsilon_{\alpha\beta}(1 - \mathcal{B} \star \widehat{\Phi}' \star \widehat{\kappa})$$

$$[\widehat{S}'_\alpha, \widehat{\bar{S}}'_{\dot\beta}]_\star = 0$$

- Solve locally all equations with at least one spacetime component via some gauge function.

- With $\hat{g}$(x|Y,Z) = $L$(x|Y) = $AdS_4$ coset element, the vacuum solution $AdS_4$ arises as

$$\widehat{\Phi} = 0 \,, \quad \widehat{S}_\alpha = \widehat{S}^{(0)}_\alpha = z_\alpha \,, \quad \widehat{S}_{\dot\alpha} = \widehat{S}^{(0)}_{\dot\alpha} = \bar{z}_{\dot\alpha} \,, \quad \widehat{W}_\mu = \Omega^{(0)}_\mu = L^{-1} \star \partial_\mu L$$

$$L(x; y, \bar{y}) = e_\star^{i\lambda\tilde{x}^\mu(x)\delta^a_\mu P_a} \; : \; \mathcal{R}^{3,1} \longrightarrow \frac{SO(3,2)}{SO(3,1)} \longrightarrow \quad ds^2_{(0)} = \frac{4dx^2}{(1-x^2)^2}$$

- Solve the twistor-space equations, then "dress" all fields with *x*-dependence by performing star-products with the gauge function.

# Various gauges

- "Nothing gauge", "W=0 gauge" or "internal gauge":

$$\widehat{W} = 0 \,, \quad \widehat{\Phi} = \widehat{\Phi}'(Y,Z) \,, \quad \widehat{S}_\alpha = \widehat{S}'_\alpha(Y,Z) \,, \quad \widehat{S}_{\dot\alpha} = \widehat{S}'_{\dot\alpha}(Y,Z)$$

  Easier to build solutions in this gauge, the equations are algebraic.
  However, in order to read any spacetime feature of the fields (correlation functions, asymptotic charges, …), other gauges should be considered.

- L-gauge:

$$\widehat{W} = \Omega^{(0)} = L^{-1} \star dL \,, \qquad \widehat{\Phi} = L^{-1} \star \widehat{\Phi}' \star \pi(L) \,, \qquad \widehat{S}_\alpha = L^{-1} \star \widehat{S}'_\alpha \star L$$

- $\hat{L}$-gauge:

$$\widehat{W} = \widehat{L}^{-1} \star d\widehat{L} \,, \qquad \widehat{\Phi} = \widehat{L}^{-1} \star \widehat{\Phi}' \star \pi(\widehat{L}) \,, \qquad \widehat{S}_\alpha = \widehat{L}^{-1} \star \widehat{S}'_\alpha \star \widehat{L}$$

$$\widehat{L} = L(Y|x) \star \widetilde{L}(Z|x) \,, \qquad \widetilde{L}(Z|x) : \mathbf{R}^4 \to SL(2;\mathbf{C})$$

$$\widehat{W} = L^{-1} \star dL + \widetilde{L}^{-1} \star d\widetilde{L} \,, \qquad \widehat{W}\Big|_{Z=0} = L^{-1} \star dL$$

  [Implements a specific gauge choice on the Z-space connection → on the gauge fields. Affects the black-hole singularities. However, changes the asymptotics except on Z=0.]

# Solutions in W=0 gauge: factorized Ansatz

- Ansatz:

$$\widehat{\Phi}'(Y,Z) = \Phi'(Y) = F'(Y)\star\kappa_y = \bar{F}'(Y)\star\bar{\kappa}_{\bar{y}} \ ,$$

$$\widehat{V}'_\alpha(Y,Z) = \widehat{V}'_\alpha(Y,z) = \widehat{V}'_\alpha(F'(Y),z) = \sum_{k=1}^{\infty}(F'(Y))^{\star k}\star V^{(k)}_\alpha(z) \ ,$$

$$\widehat{\bar{V}'}_{\dot\alpha}(Y,Z) = \widehat{\bar{V}'}_{\dot\alpha}(Y,\bar{z}) = \widehat{\bar{V}'}_{\dot\alpha}(\bar{F}(Y),\bar{z}) = \sum_{k=1}^{\infty}(\bar{F}'(Y))^{\star k}\star \bar{V}^{(k)}_{\dot\alpha}(\bar{z})$$

$$F' := \Phi'\star\kappa_y \ , \quad \bar{F}' := \Phi'\star\bar{\kappa}_{\bar{y}} \ , \qquad [F',\bar{F}']_\star = 0$$

- Holomorphicity in z + $[F',\bar{F}']=0$ → $[S',\bar{S}'] = 0$.
- $\pi(V') = - V'$ solves $\{S',\Phi'\}_\pi = 0$ .

- Remain:

$$\partial_{[\alpha}\widehat{V}'_{\beta]} + \widehat{V}'_{[\alpha}\star\widehat{V}'_{\beta]} = -\frac{i}{4}\,\epsilon_{\alpha\beta}\,bF'\star\kappa_z \ ,$$

$$\partial_{[\dot\alpha}\widehat{\bar{V}}'_{\dot\beta]} + \widehat{\bar{V}}'_{[\dot\alpha}\star\widehat{\bar{V}}'_{\dot\beta]} = -\frac{i}{4}\,\epsilon_{\dot\alpha\dot\beta}\,\bar{b}\bar{F}'\star\bar{\kappa}_{\bar{z}}$$

But F' is Z-constant → eqs. analogue to deformed oscillator problem with a δ-function deformation (just like for HS bh). With basis spinors $u^{\pm}_\alpha$ , $(u^{+\alpha} u^{-}_\alpha = 1)$

$$z^\pm := u^{\pm\alpha}z_\alpha \ , \quad w_z := z^+z^- \ , \quad [z^-,z^+]_\star = -2i \quad \rightarrow \quad \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}e^{-i\frac{1}{\varepsilon}z^+z^-} = \kappa_z$$

$$\widehat{V}'_\alpha = \sum_{k=1}^{\infty}V'^{(k)}_\alpha\star(F')^{\star k} = 2i\sum_{k=1}^{\infty}\binom{1/2}{k}\left(-\frac{b}{2}\right)^k\int_{-1}^{1}\frac{dt}{(t+1)^2}\frac{(\log(1/t^2))^{k-1}}{(k-1)!}z_\alpha e^{i\frac{t-1}{t+1}w_z}\star(F')^{\star k}$$

# Expansion in so(3,2)-representations

- Idea: decompose Φ' into (enveloping-algebra realization of) $\mathfrak{so}(3,2)$-modules . Consider non-polynomial functions of Y with definite eigenvalues of under two commuting generators (E, J) of the compact subalgebra of $\mathfrak{so}(3,2)$,

$$\widehat{\Phi}'(Y,Z) \;=\; \Phi'(Y) \;\in\; \mathcal{M} \;=\; \bigoplus_{n_1,n_2,n'_1,n'_2} \mathbf{C}\otimes P_{n_1,n_2|n'_1,n'_2}$$

→ generalized projectors

$$P_{\mathbf{n}|\mathbf{n}'}\star P_{\mathbf{m}|\mathbf{m}'} \;=\; \delta_{\mathbf{n}',\mathbf{m}}P_{\mathbf{n}|\mathbf{m}'}\;, \qquad P_{\mathbf{n}|\mathbf{m}'} \;\sim\; |\mathbf{n}><\mathbf{m}|$$

obeying proper reality conditions.

- On expanding the (internal) master fields as

$$\widehat{\mathcal{O}}(Y,Z) \;=\; \sum_{\mathbf{n},\mathbf{n}'}\sum_{k=0,1} P_{\mathbf{n}|\mathbf{n}'}(Y)\star\kappa_y^{\star k}\star\check{\mathcal{O}}_{k;\mathbf{n}|\mathbf{n}'}(Z)$$

Vasiliev's equations are turned into matrix equations, and we can construct a solution by known methods.

# Massless scalar modes projectors

- Focus on certain kinds of generalized projectors: consider those encoding massless particle modes. For simplicity, consider only the $\ell = 0$ line of the scalar.

$$\Phi'(Y) \;=\; \sum_n \widetilde{\nu}_n \,\mathcal{P}_n(E) \;, \qquad (\widetilde{\nu}_n)^* \;=\; \widetilde{\nu}_{-n}$$

with the projectors

$$\mathcal{P}_n(E) \;=\; 4(-)^{n-\frac{1+\varepsilon}{2}}\, e^{-4E} L^{(1)}_{n-1}(8E) \;=\; 2(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta E} \;, \qquad n \in \mathbb{Z}$$

precisely encodes the free massless scalar states | e, 0 > of $\mathcal{D}$ (1,0) and $\mathcal{D}$(2,0).

- In the W=0 gauge this is evident from the action of $\mathfrak{so}$(3,2) on them:

$$\begin{aligned} \mathcal{P}_1 \star \mathcal{P}_1 \;&=\; \mathcal{P}_1 \\ E \star \mathcal{P}_1 \;&=\; \mathcal{P}_1 \star E \;=\; \frac{1}{2}\mathcal{P}_1 \;, \\ L^-_r \star \mathcal{P}_1 \;&=\; 0 \;=\; \mathcal{P}_1 \star L^+_r \;, \\ M_{rs} \star \mathcal{P}_1 \;&=\; 0 \end{aligned} \qquad \Longrightarrow \qquad \boxed{\mathcal{P}_1(E) \;\simeq\; |1/2;0\rangle\langle 1/2;0| \;\in\; \mathcal{D}_0 \otimes \mathcal{D}_0^*}$$

and from the point of view of the two-sided, twisted-adjoint action $K \star \mathcal{P}_1 - \mathcal{P}_1 \star \pi(K)$,

$$\boxed{\mathcal{P}_1(E) \;\simeq\; |1;0\rangle \;\in\; \mathcal{D}(1,0)}$$

# Weyl 0-form for massless scalar

- In the L-gauge we reconstruct exactly the Breitenlohner-Freedman scalar modes.

$$\Phi(x|Y) \;=\; L^{-1}(x)\star\Phi'\star\pi(L)(x) \;=\; L^{-1}\star\sum_n \widetilde{\nu}_n\mathcal{P}_n\star\kappa_y\star L\star\kappa_y$$

- Define the "twisted projectors":

$$\widetilde{\mathcal{P}}_n \;:=\; \mathcal{P}_n\star\kappa_y \;=\; 4\pi(-)^{n-\frac{1+\varepsilon}{2}}\oint_{C(\varepsilon)}\frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y-i\eta\sigma_0\bar{y})$$

- Weyl zero-form only contains a scalar (modes of an AdS massless scalar):

$$\Phi(x|Y) \;=\; L^{-1}(x)\star\Phi'\star\pi(L)(x) \;=\; \sum_n \tilde{\nu}_n\mathcal{N}_n\oint_{C(\varepsilon)}\frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x)\star\delta^2(y-i\eta\sigma_0\bar{y})\star L(x)\star\kappa_y}$$

$$\Phi(x|Y) \;=\; (1-x^2)\sum_n \mathcal{N}_n\widetilde{\nu}_n\oint_{C(\epsilon)}\frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \frac{e^{iy^\alpha M_\alpha{}^{\dot\beta}(x,\eta)\bar{y}_{\dot\beta}}}{1-2i\eta x_0+\eta^2x^2}$$

$$4\widetilde{\nu}_1\frac{1-x^2}{1-2ix_0+x^2} \;\sim\; \widetilde{\nu}_1\frac{e^{-it}}{(1+r^2)^{1/2}}$$

*(C.I., P. Sundell, to appear)*

- Differently from bhs, one does not expect a free scalar to solve the full equations. However, the completion to full solutions is precisely given by the bh sector!

# Twisted projectors

- The Z-space connection (and the gauge fields) receive non-linear corrections of all orders. In the W=0 gauge, they appear as powers of $F' = \Phi' \star \kappa_y$ .
  → Injecting massless particles into $\Phi'$ results in the appearance of the twisted projectors $\mathcal{P}_n \star \kappa_y$ in $F'$, and all non-linear corrections can be expanded over this basis, due to the generalized projector algebra

$$
\begin{aligned}
\mathcal{P}_n \star \mathcal{P}_m &= \delta_{nm}\mathcal{P}_n \ , &\qquad \widetilde{\mathcal{P}}_n \star \widetilde{\mathcal{P}}_m &= \delta_{n,-m}\mathcal{P}_n \\
\mathcal{P}_n \star \widetilde{\mathcal{P}}_m &= \delta_{nm}\widetilde{\mathcal{P}}_n \ , &\qquad \widetilde{\mathcal{P}}_n \star \mathcal{P}_m &= \delta_{n,-m}\widetilde{\mathcal{P}}_n \ ,
\end{aligned}
$$

$$\Longrightarrow$$

$$\boxed{\widetilde{\mathcal{P}}_n \ \simeq \ |n/2;0\rangle\langle -n/2;0| \ \in \ \mathcal{D}_0 \otimes \widetilde{\mathcal{D}}_0^*}$$

$$(F')^{\star n} \ \sim \ \widetilde{\mathcal{P}}^{\star n} \ = \ \begin{cases} \mathcal{P} \ , & n = 2k \ , \\ \widetilde{\mathcal{P}} \ , & n = 2k+1 \end{cases}$$

- Projectors and twisted projectors form a subalgebra of the star-product algebra. Due to the change of sign of the E-eigenvalue, the twisted projector correspond to states with zero energy, static → *soliton*-like solutions.
- Indeed, in the L-gauge the spacetime behaviour of individual fields suggests that they are spherically-symmetric HS black-holes!

# Weyl 0-form for HS black holes

- Indeed, if Φ’(Y) is expanded in twisted projectors,

$$\Phi'(Y) \;=\; \sum_n \nu_n\, \mathcal{P}_n(E) \star \kappa_y \;=\; \sum_n \nu_n\, \widetilde{\mathcal{P}}_n(Y)$$

$$\Longrightarrow \qquad \Phi(x|Y) \;=\; L^{-1}(x) \star \Phi' \star \pi(L)(x) \;=\; \sum_n \widetilde{\nu}_n L^{-1} \star \mathcal{P}_n \star L \star \kappa_y$$

resulting in a generating function of Schwarzschild-like Weyl tensors,

$$\Phi(x|Y) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta E} \star L(x) \star \kappa_y}$$

a tower of type-D Weyl tensors of all spins:

$$\Phi^{(n)}_{\alpha(2s)} \;\sim\; \frac{i^{n-1}\mu_n}{r^{s+1}} \,(u^+u^-)^s_{\alpha(2s)}$$

*(Didenko-Vasiliev, ‘09, C.I.-Sundell, ‘11)*

- Note that the massless particle modes and the black-hole ones are related via a twistor-space Fourier transform. This implies that black holes appear already at second order in perturbation theory in L-gauge!

# Deformed oscillators for HSBH

- Inserting $F^{\star k} = L^{-1} \star (F')^{\star k} \star L = \left( \sum_n \widetilde{\nu}_n \widetilde{\mathcal{P}}_n^L \right)^{\star k} = \sum_n \left( \nu_n^{(k)} \, \mathcal{P}_n^L + \widetilde{\nu}_n^{(k)} \, \widetilde{\mathcal{P}}_n^L \right)$

  into

$$\widehat{V}_\alpha = L^{-1} \star \widehat{V}'_\alpha \star L = 2i \sum_{k=1}^{\infty} \binom{1/2}{k} \left( -\frac{b}{2} \right)^k \int_{-1}^{1} \frac{dt}{(t+1)^2} \frac{(\log(1/t^2))^{k-1}}{(k-1)!} z_\alpha e^{i\frac{t-1}{t+1} w_z} \star F^{\star k}$$

  and computing the star-products, one can read off the Z-space connection in the bh and particle sectors.

- For simplicity, let us only look at the lowest-weight states n=1 . In L-gauge:

$$\widehat{V}^{\pm}_{\rm bh} = \frac{ib|\widetilde{\nu}_1|}{2} \mathcal{P}_1^L \, a^{\pm} \int_{-1}^{1} \frac{dt}{(t+1+i(t-1)r\cos\theta)^2} F^{-} \left[ \frac{b|\widetilde{\nu}_1|}{2} \log t^2 \right] \; e^{\frac{i(t-1)}{t+1+i(t-1)r\cos\theta} a^+ a^-}$$

$$a^{\pm} := u^{\alpha\pm} a_\alpha \,, \qquad a_\alpha = z_\alpha + i(\varkappa_\alpha{}^\beta y_\beta + v_\alpha{}^{\dot\beta} \bar{y}_{\dot\beta}) \,, \quad z_\alpha \star \mathcal{P}_1 = a_\alpha \mathcal{P}_1 \,, \quad [a_\alpha, a_\beta]_\star = -2i\epsilon_{\alpha\beta}$$

$$F^{\pm}[x] := \frac{1}{2} \left( {}_1F_1\left[\frac{1}{2}; 2; x\right] \pm {}_1F_1\left[\frac{1}{2}; 2; -x\right] \right)$$

# Deformed oscillators for HSBH

- The original singularity of the purely z-dependent coefficients $V_\alpha^{(k)}(z)$ has been pushed outside the integration domain, and remains only along the equatorial plane $\theta = \pi/2$.

- $\cos\theta$ appears because the L-transformation induces a rotation of the spin-frame in $\mathcal{P}_1$ wrt the spin-frame in $V_\alpha^{(k)}(z)$. In other words, this is a result of the pointwise non-collinearity of the two spin-frames after the L-rotation.

- One can remedy with a compensating local-Lorentz rotation acting only on the z-variables, i.e., adding a purely z-dependent factor to the gauge function → $\hat{L}$-gauge:

$$\widehat{V}^{\pm}_{\rm bh} = \frac{ib|\widetilde{\nu}_1|}{2}\,\mathcal{P}^L_1\,\tilde{a}^{\pm}\int_{-1}^{1}\frac{dt}{(t+1+ir(t-1))^2}\,F^-\left[\frac{b|\widetilde{\nu}_1|}{2}\log t^2\right]\,e^{\frac{i(t-1)}{t+1+ir(t-1)}\,\tilde{a}^+\tilde{a}^-}$$

→ singular only in $r = 0$. (however, $\tilde{L}(z)$ alters the asymptotics…)

# Deformed oscillators for scalar modes

- For the particle modes, the relevant star product gives a different result:

$$V_{\rm pt}^{\pm} = \widetilde{\mathcal{P}}_n^L \star \widetilde{V}_n^{\pm} = \pm \frac{ib\widetilde{\nu}_1}{2} \frac{1-x^2}{1-2ix_0+x^2} \widetilde{y}^{\pm} e^{i(\widetilde{y}^+ z^- - \widetilde{y}^- z^+)} \int_{-1}^{1} \frac{dt}{(t-1)^2} F^+ \left[ \frac{b|\widetilde{\nu}_1|}{2} \log t^2 \right] e^{i\frac{t+1}{t-1}\widetilde{y}^+ \widetilde{y}^-}$$

$$\widetilde{y}_\alpha := y_\alpha + M_\alpha{}^{\dot\beta}(x) \bar{y}_{\dot\beta}$$

- Interesting to check what it gives at the linear order (up to a Y-dependent term):

$$\widehat{V}_{\rm pt}^{(1)\pm} = \pm \frac{ib\widetilde{\nu}_1}{2} \frac{1-x^2}{1-2ix_0+x^2} \widetilde{y}^{\pm} e^{i(\widetilde{y}^+ z^- - \widetilde{y}^- z^+)} \int_{-1}^{1} \frac{dt}{(t-1)^2} e^{i\frac{t+1}{t-1}\widetilde{y}^+ \widetilde{y}^-}$$

→ indeed solves the equations but contains a pole in Y-space at $\tilde{y} = 0$.
To be clarified. Effects on observables involving the Z-space connection or W should be examined.

# Vasiliev gauge

- For various reasons it is interesting to look at the particle-like solutions in Vasiliev gauge (it is the gauge in which the usual perturbation theory over AdS is formulated, interesting to see whether free Fronsdal field modes coalesce into black-holes also here, and check whether interaction vertices expressed in this basis also give rise to infinities or not )

- The latter is defined by the condition $$z^\alpha \widehat{V}^{(V)}_\alpha = z^+ \widehat{V}^{(V)-} - z^- \widehat{V}^{(V)+} = 0$$

- At first order, the gauge transformation reads ($\tilde{u} := \tilde{y}^\alpha z_\alpha$)

$$H^{(1)} = -\frac{ib\widetilde{\nu}_1}{4} \frac{1-x^2}{1-2ix_0+x^2} \frac{1}{\widetilde{y}^+\widetilde{y}^-} \frac{\widetilde{y}^+ z^- + \widetilde{y}^- z^+}{\widetilde{u}} \left(e^{i\widetilde{u}} - 1\right)$$

→ well-behaved at the spacetime boundary and in Z, but inherits the pole in Y -- not a proper gauge transformation. Likely to induce redefinitions of initial data at higher orders.

- The transformed Z-space connection is regular everywhere, and coincides with the form coming from usual perturbation theory,

$$\widehat{V}^{(V)}_\alpha = -\frac{b\widetilde{\nu}_1}{2} \frac{1-x^2}{1-2ix_0+x^2} \frac{z_\alpha}{\widetilde{u}} \left[e^{i\widetilde{u}} - \frac{e^{i\widetilde{u}-1}}{i\widetilde{u}}\right] = z_\alpha \int_0^1 dt\, t\, \Phi(-tz, \bar{y}) e^{ity^\alpha z_\alpha}$$

# Conclusions and outlook

- Found a wide solution space in W=0 gauge by means of a factorized ansatz.

- Analyzed the spacetime behaviour of the Weyl tensors in L-gauge for a specific choice of generalized projector algebra carrying the Y-dependence. The individual fields coincide with massless scalar field modes and higher-spin black-hole modes. Different properties may be more or less difficult to see in different gauges.

- In L-gauge the generalized projector algebra implies that free scalar modes interactions produce HSbh already at second order. Interesting to see whether there are signs of such organization of the perturbative expansion also in different gauges, particularly Vasiliev gauge.

- Important related issues concerning functional classes of twistor-space elements, and related questions of the admissibility of gauge parameters, regularity under star-product, regularity of observables,…
  If the massless modes basis is a good expansion basis for the master fields in a given superselection sector, this would enable to better control their interactions and extract non-linear Fronsdal theory from the Vasiliev equations.