Based on 1603.05387, 1607.07651, 1611.00112

Collaboration with Jin-beom BAE (KIAS) and Shailesh LAL (Paris6)

More on

Higher Spin One Loops

Euihun JOUNG (SNU & CFGS)

AdS/CFT Duality

CFT$_d$ with N	AdS$_{d+1}$

- $\mathcal{N}$=4 Super Yang-Mills with SU(N)

't Hooft coupling: $\lambda = N g_{YM}^2$

Free limit $\lambda \to 0$

Exact Operator Spectrum

- Type IIB Strings in $AdS_5 \times S^5$

$$\frac{R_{AdS}^2}{\alpha'} = \sqrt{\lambda} \qquad g_s = \frac{\lambda}{N}$$

Tensionless Limit $\alpha' \to \infty$

- ∞ **massless** fields in **1st** Regge Trajectory
- **massive** fields in **higher** RT

- U(N)/O(N) (free scalar) Vector Models

Simple Single-Trace Operator Spectrum

- Higher-Spin Gravity [Vasiliev]

∞ **massless** fields: spin 0,1,2,3,…
(effectively, 1st RT)

[Sezgin, Sundell, Klebanov, Polyakov] [Giombi, Yin]

1-loop Free Energy of Free Vector Model / Higher Spin duality

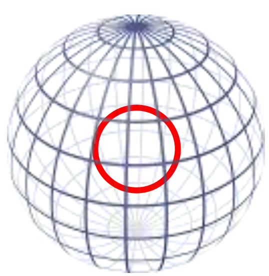
- U(N) Free Scalar Vector Model $F_{U(N)} = N\,2F_0 + 0$ **?**
- Vasiliev's Higher-Spin Gravity $\Gamma_{AdS} = g\,S_{HS} + \Gamma^{(1)} + O(g^{-1})$

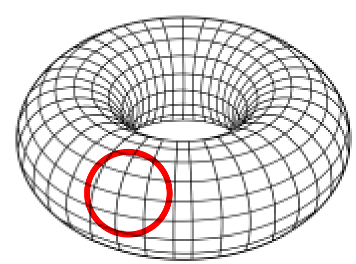
$$\Gamma^{(1)} = \bigcirc = \phi + A_M + G_{MN} + \varphi_{MNL} + \cdots$$

$= 0$ or a finite number (O(N) case) [Giombi, Klebanov, Safidi; Tseytlin]

(using a proper **regularization** like $1 + 2 + \cdots = -\frac{1}{12}$)

(Euclidean) AdS_5
with S^4 boundary



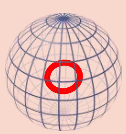
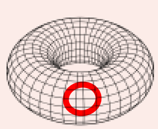


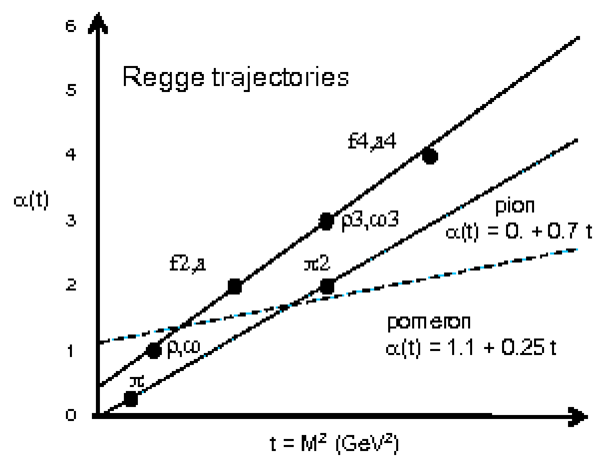
Thermal AdS_5
with $S^1 \times S^3$ boundary

1-loop Free Energy of
Free Adjoint Model / String-like Theory duality

- **Various Stringy SCFTs** (coming soon)
- **Two Toy Models**
  - Free SU(N) Adjoint Scalar / BulkDualAdjointScalar in AdS
  - Free SU(N) Yang-Mills / BulkDualYangMills in AdS

Calculate **trajectory** by **trajectory** (**higher-spin multiplet**)!

	BDAS	BDYM
	I	III
	II	IV



How to calculate

- **Step 1:** Identify the AdS field content (operator spectrum)
  - Group theoretically, tensor product decomposition

$$\mathrm{Rac}^{\otimes 2} = \bigoplus_{s=0}^{\infty} \mathcal{D}(s+1,s) \qquad \mathrm{Rac}^{\otimes 3} = \bigoplus_{s=0}^{\infty} (s+1)\left[\mathcal{D}(s+\tfrac{3}{2},s) \oplus \mathcal{D}(s+\tfrac{7}{2},s+1)\right]$$

$$\mathrm{Rac}^{\otimes 4} = \bigoplus_{s=0}^{\infty} \frac{(1+s)(2+s)}{2}\mathcal{D}(s+2,s) \oplus \bigoplus_{s=0}^{\infty}\bigoplus_{n=1}^{\infty} \frac{(2n+2s+1)(2s+1)+3(-1)^n}{4}\mathcal{D}(s+n+2,s)$$

- **Step 2:** Resum AdS free energies over all fields

  - Calculate free energy of the field (m^2, s) [Camporesi, Higuchi; Beccaria, Tseyltin]
  - 1^{st} RT : **same** as VM/HS case [Giombi, Klebanov, Safidi; Tseytlin]
  - (unprojected) 2^{nd} & 3^{rd} RT : much more **complicated** but doable
  - Properly projected 2^{nd} & 3^{rd} RT : much$^{\text{much}}$ more **complicated** and not even clear how to **regularize** it

- **Two steps at once ?**
  - Step 1: decomposition
  - Step 2: resummation

- **Introduce Character Integral Representation for Free Energy**
  - For any spectra $\mathcal{H}$
  - Identify the corresponding character $\chi_{\mathcal{H}}(\beta, \alpha)$
  - Total free energy of $\mathcal{H}$ given by a certain integral of $\chi_{\mathcal{H}}(\beta, \alpha)$

Great simplification !

since we usually identify the spectra using character

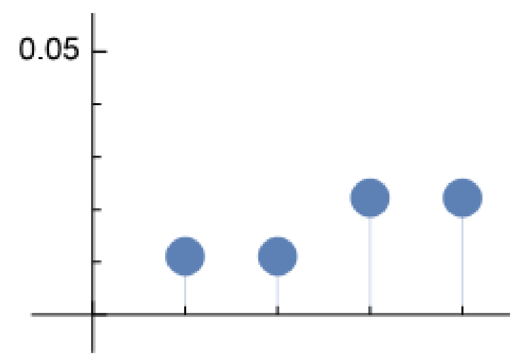
Bulk Dual Adjoint Scalar Theory in AdS

- A boundary Scalar : $\dfrac{\log R}{90}$
- 1^{st} RT : $\dfrac{\log R}{90}$ [Giombi, Klebanov, Safidi; Tseytlin]

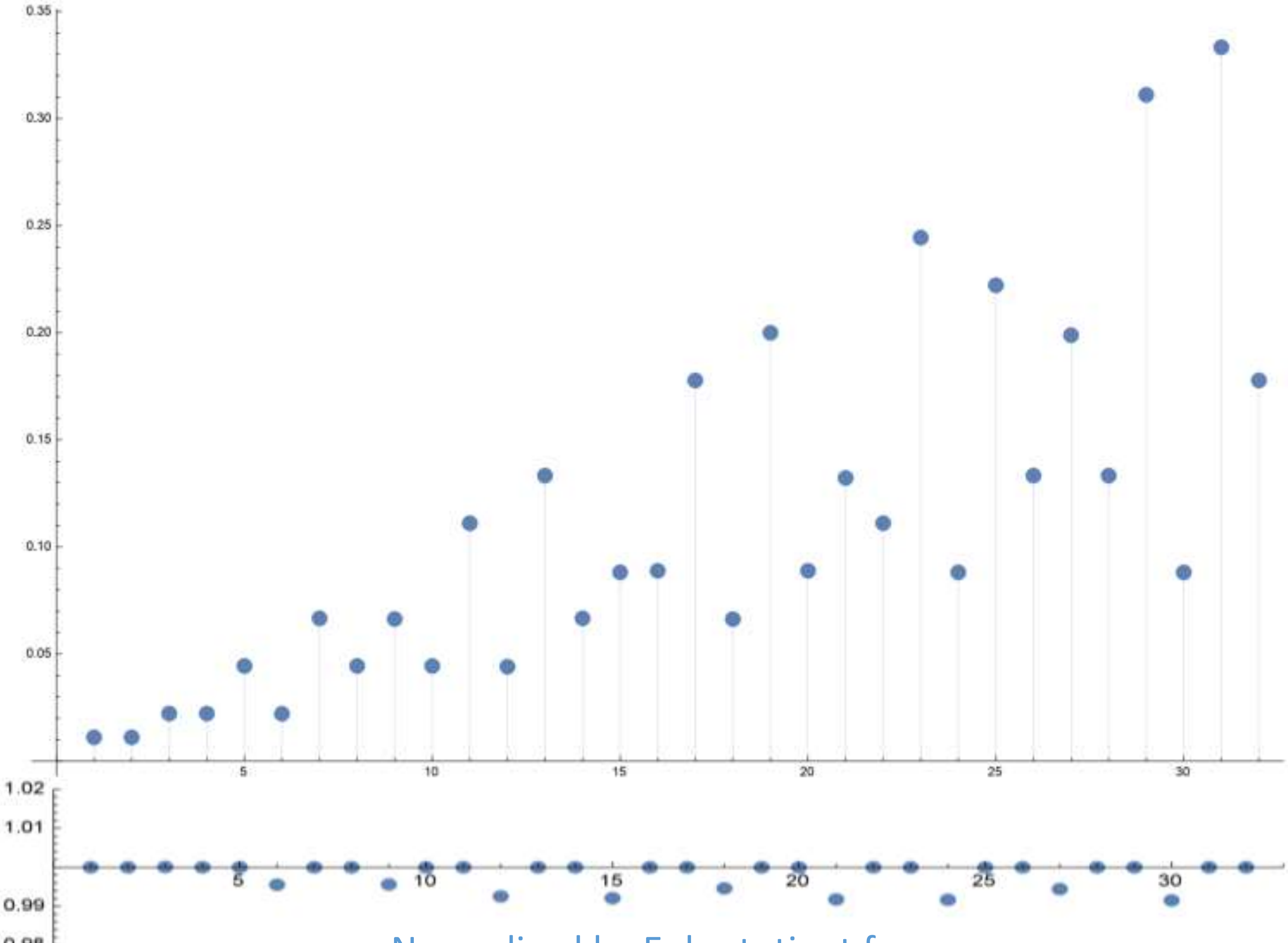
Previous Result

NEW!

- 2^{nd} RT : $\dfrac{362911}{16329600} \log R$
- 3^{rd} RT : $\dfrac{\log R}{45}$

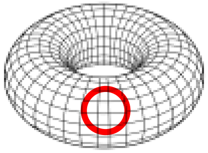


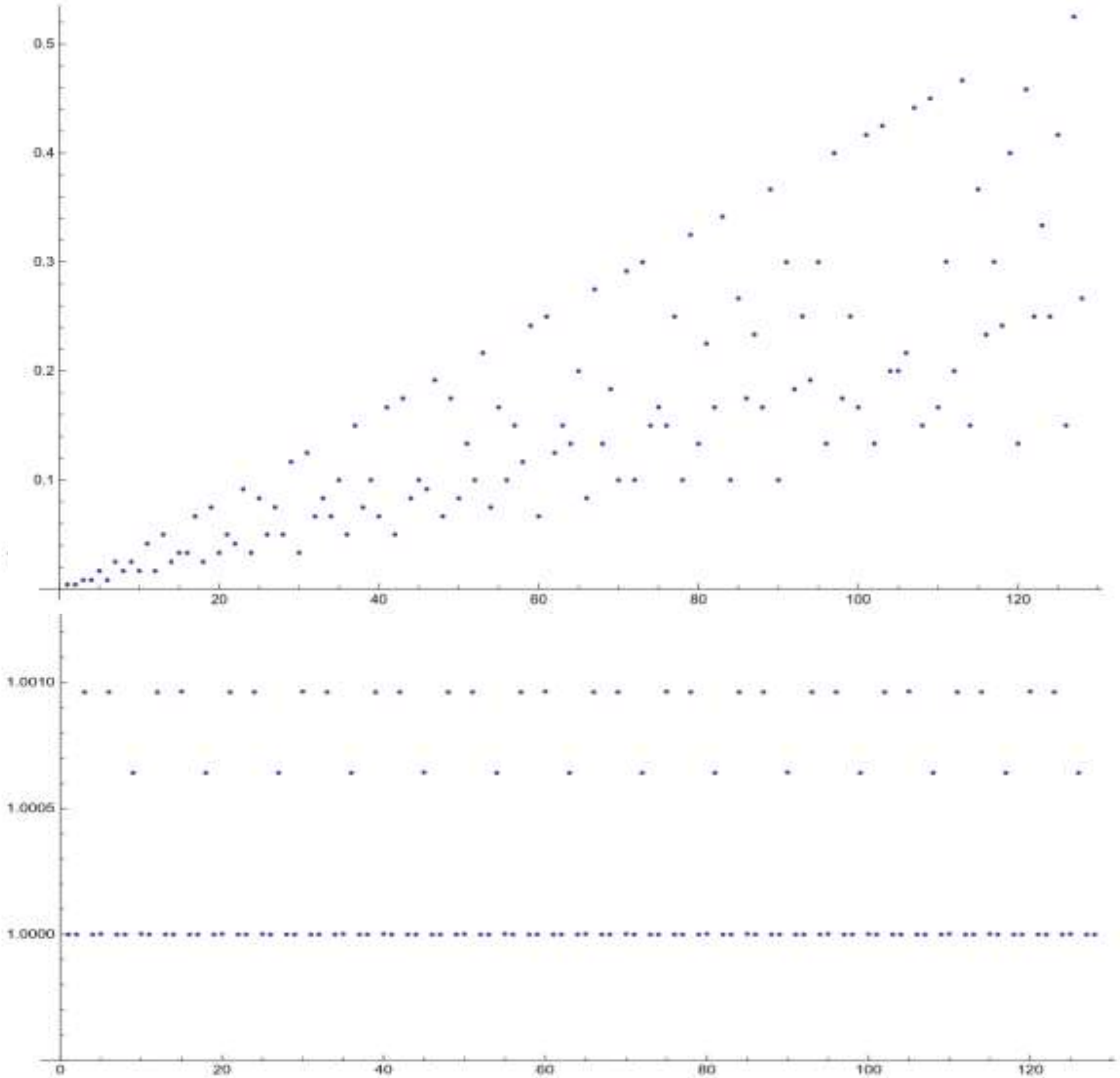
Bulk Dual Adjoint Scalar Theory in AdS_5



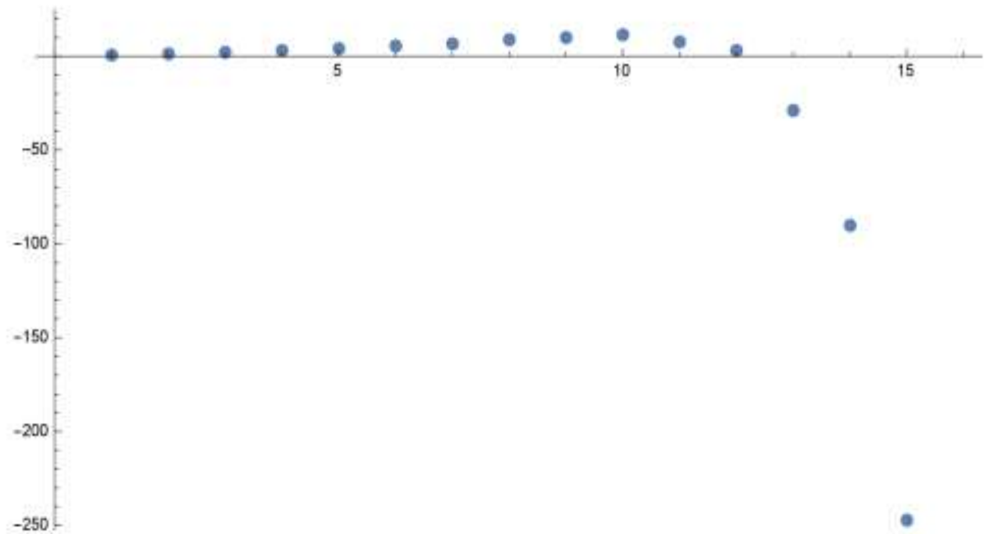
Normalized by Euler totient fn

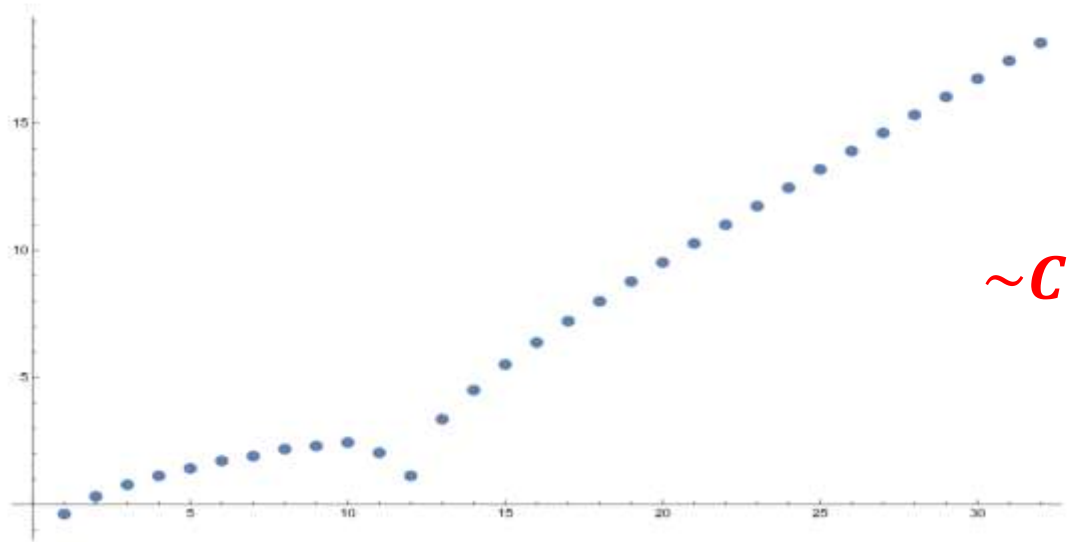
II Bulk Dual Adjoint Scalar Theory in TAdS5





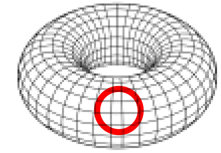
III Bulk Dual Yang Mills Theory in AdS5

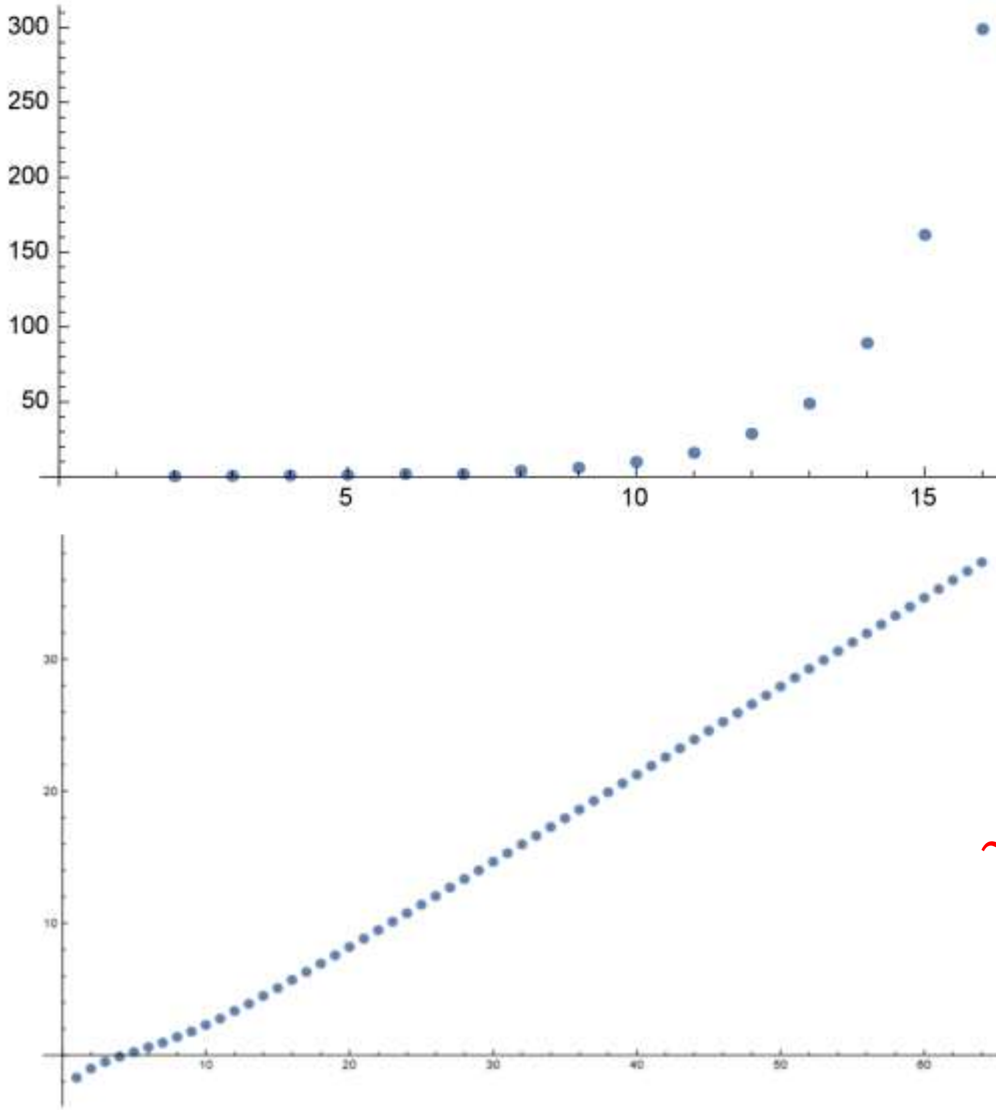




$\sim C\, 2^n$

IV Bulk Dual Yang Mills Theory in TAdS5





$\sim C\ 2^n$

Summation over Trajectories?

Sum over Trajectories

$$\chi_{\mathrm{adj}} = \sum_{n=2}^{\infty} \chi_{\mathrm{cyc}^n}$$

Sum over 'log slice's

$$\chi_{\mathrm{adj}} = -\chi_{\mathrm{Rac}} - \sum_{k=1}^{\infty} \frac{\varphi(k)}{k} \chi_{\log,k}$$

- F_0

$$\chi_{\log,k}(\beta,\alpha_1,\alpha_2) = -\log[1-\chi_{\mathrm{Rac}}(k\,\beta,k\,\alpha_1,k\,\alpha_2)]$$

Full Free Energy gets contribution only from $-F_0$

suggests $g^{-1} = (N^2-1)+1 = N^2$

Back to Vector Model Dualities

Type-j Theory

- Type A (spin 0), B (spin ½), C (spin 1) HS theories in AdS5
  [Beccaria, Tseytlin]
- What about **spin j** ? Flato-Fronsdal thm [Dolan; Beccaria, Tseytlin]

$$\mathcal{H}^{\mathrm{Sym}}_{j,U(N)} = 2\bigoplus_{s=2j}^{\infty}\mathcal{D}(s+2,[\tfrac{s}{2},\tfrac{s}{2}]) \qquad \mathcal{H}^{\mathrm{Sym}}_{j,O(N)} = \bigoplus_{s=2j}^{\infty}\mathcal{D}(s+2,[\tfrac{s}{2},\tfrac{s}{2}]),$$

$$\mathcal{H}^{\mathrm{MixSym}}_{j,U(N)} = \bigoplus_{s=2j+1}^{\infty}\mathcal{D}(s+2,[\tfrac{s}{2}+j,\tfrac{s}{2}-j]_{\mathrm{PI}}) \qquad \mathcal{H}^{\mathrm{MixSym}}_{j,O(N)} = \bigoplus_{\text{even } s\geq 2j+1}\mathcal{D}(s+2,[\tfrac{s}{2}+j,\tfrac{s}{2}-j]_{\mathrm{PI}}),$$

$$\mathcal{H}^{\mathrm{Massive}}_{j,U(N)} = 2\,\mathcal{D}(2j+2,[0,0])\oplus\bigoplus_{r=1}^{2j}\mathcal{D}(2j+2,[r,0]_{\mathrm{PI}}) \qquad \mathcal{H}^{\mathrm{Massive}}_{j,O(N)} = 2\,\mathcal{D}(2j+2,[0,0])\oplus\bigoplus_{2\leq\text{even } r\leq 2j}\mathcal{D}(2j+2,[r,0]_{\mathrm{PI}})$$

No energy momentum tensor in this game!

- HS theories in **RIGID AdS5**
- HS algebras : the **ideal** part of $hs_{\lambda\to j}(su(2,2))$

Type-j Theory

- One-loop Free Energy in AdS5 (with S^4 boundary)

$$\Gamma^{(1)\,\mathrm{ren}}_{j,\mathrm{non\text{-}min}} = (-1)^{2j}\, n_j\, 2\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{j,\mathrm{min}} = \left[(-1)^{2j}\, n_j + 1\right] \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$$

$$n_j = \frac{(2\,j-1)\,2\,j\,(2\,j+1)}{6}\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j} = (-1)^{2j}\,\frac{60\,j^4 - 30\,j^2 + 1}{45}\,\log R$$

- One-loop Free Energy in TAdS5 (with $S^1 \times S^3$ boundary)

[Gunaydin, Skvortsov, Tran]

$$\mathcal{E}_{j,\mathrm{non\text{-}min}} = n_j\,\frac{288\,j^4 - 208\,j^2 - 3}{420} \qquad \mathcal{E}_{j,\mathrm{min}} = n_j\,\frac{288\,j^4 - 208\,j^2 - 3}{840} + (-1)^{2j}\,\frac{30\,j^4 - 20\,j^2 + 1}{120}$$

$$\mathcal{E}_{\mathcal{S}_j} = (-1)^{2j}\,\frac{30\,j^4 - 20\,j^2 + 1}{120}$$

spin-j on $S^1 \times S^3$: ill-defined for j>1

Type-j Theory

- AdS5 Free Energy of spin-j doubleton
  - No UV, but IR divergence

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j} = (-1)^{2j}\,\frac{60\,j^4 - 30\,j^2 + 1}{45}\,\log R \qquad \left(\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_0}\,,\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_{\frac{1}{2}}}\,,\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_1}\right) = \left(\frac{1}{90}\,,\,\frac{11}{180}\,,\,\frac{31}{45}\right)\log R$$

- S^4 Free Energy of spin-j
  - UV divergence (formulation dependent)
  - In Fronsdal,

$$F_j = \frac{75j^4 - 15j^2 + 2}{90}\,\log \Lambda_{\mathrm{CFT}} \qquad \left(F_0\,,\,F_{\frac{1}{2}}\,,\,F_1\right) = \left(\frac{1}{90}\,,\,\frac{11}{180}\,,\,\frac{31}{45}\right)\log \Lambda_{\mathrm{CFT}}$$

  - Maybe other formulation?

Type-AZ Theory

- Boundary : collection of **free massless** spin **0, 1, 2, …**
- Bulk : **all** possible AdS5 fields with **multiplicities**

➢ One-loop Free Energy in AdS5 (with S^4 boundary)

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,non\text{-}min}} = 0 \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,min}} = 0$$

➢ One-loop Free Energy in TAdS5 (with $S^1 \times S^3$ boundary)

$$\mathcal{E}_{\mathrm{AZ,non\text{-}min}} = 0\,, \qquad \mathcal{E}_{\mathrm{AZ,min}} = 0$$

Conclusion / Outlook

- ❖ A lot to play with & a lot to understand
- ❖ Stringy Models ($\mathcal{N}$=4 SYM, ...)
- ❖ SO(N) & Sp(N) Adjoint, Bivector Models
- ❖ Higgs mechanism?

Thank you for the attention!