Emergent Lorentz Invariance with chiral fermions

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1 December 2016

Outline

1. Motivation and history
2. Holographic model in AdS
3. Holographic model in LS
4. Counterexample (CPT)

arXiv:1505.04130
arXiv:1305.00110

Motivation

- Emergent symmetries
  - Common phenomena in condensed physics:
    Vafec, Tesanovic, Franz: cond-mat/0203047
  - Perturbative Lorentz violation
    Nielsen, Ninomiya, Chadha: Nucl.Phys.B 141, 153 & 217, 125
- Improving UV behaviour of the gravity
- The role of discrete symmetries?

General argument

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + k_{LV}\mathcal{O}_{\mu_1..\mu_n} \to \mathcal{L}_{CFT} + k_{lV}\mathcal{O}_{\mu_1...\mu_n}u^{\mu_1}...u^{\mu_n}$$

Unitarity:

$$\dim \mathcal{O}_{\mu_1..\mu_n} \geq d - 2 + n$$

General argument

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Unitarity:

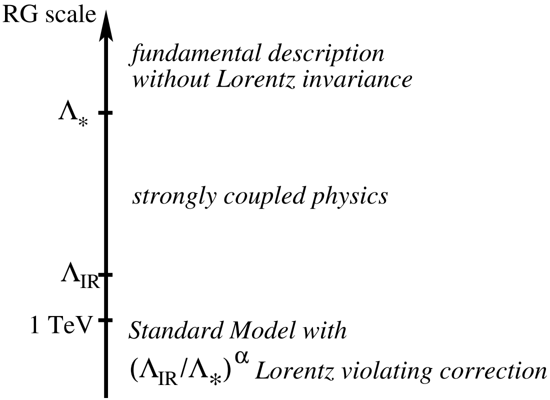
$$\dim\ \mathcal{O}_{\mu_1..\mu_n} \geq d-2+n$$

$$\Downarrow$$

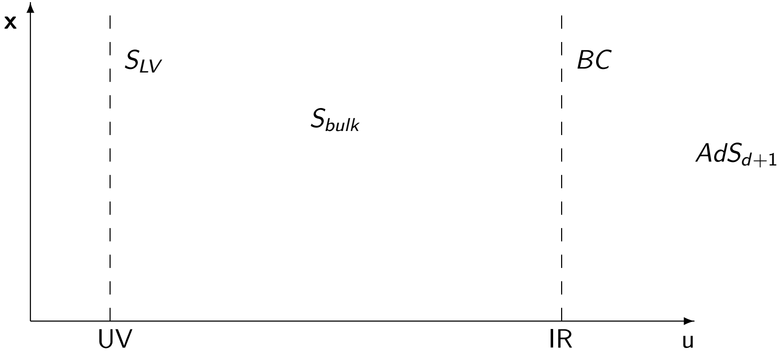
Dangerous only

$$d-1 \leq dim\mathcal{O}_{\mu} \leq d$$

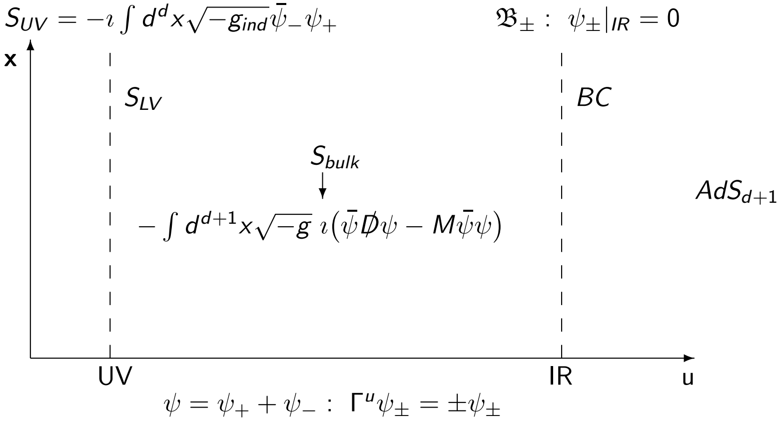
Idea


RG scale
fundamental description
without Lorentz invariance
Λ_*
strongly coupled physics
Λ_{IR}
1 TeV
Standard Model with
$(\Lambda_{IR}/\Lambda_*)^{\alpha}$ Lorentz violating correction

Holographic model



Holographic model



Holography for fermions

$$\psi_-(\vec{p},u) = (pu)^{(d+1)/2}\big(\chi_1(\vec{p})J_{Ml+1/2}(pu) + \chi_2(\vec{p})Y_{Ml+1/2}(pu)\big)\ ,$$

$$\psi_+(\vec{p},u) = (pu)^{(d+1)/2}\frac{ip_\mu\Gamma^\mu}{p}\big(\chi_1(\vec{p})J_{Ml-1/2}(pu) + \chi_2(\vec{p})Y_{Ml-1/2}(pu)\big)\ ,$$

$$Ml > -\tfrac{1}{2}, \quad \psi_- \text{ as a source.}$$

Boundary conditions $\mathfrak{B}_+$: no massless mode

Violating Lorentz invariance

From CFT side :

$$S = S_{CFT} + S_\chi + \Lambda_*^{-Ml} \int d^d x \, \bar{\chi} \, \mathcal{O}_\psi \ ,$$

$$\Downarrow$$

$$\dim \mathcal{O}_\psi = -Ml + \frac{d}{2} \quad \longmapsto \quad -\frac{1}{2} < Ml < \frac{1}{2}$$

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From AdS side:

$$S_{UV} = -b \int d^d x\, \imath(\bar{\psi}_- \gamma^0 \partial_0 \psi_- + v\, \bar{\psi}_- \gamma^i \partial_i \psi_-)\ .$$

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$$S_{UV} = -b \int d^d x \, \imath (\bar{\psi}_- \gamma^0 \partial_0 \psi_- + v \, \bar{\psi}_- \gamma^i \partial_i \psi_-) \ .$$

$$\omega = \pm |\mathbf{k}| \left[1 + (v-1)(1-2Ml)\frac{b}{l} \left(\frac{l}{L} \right)^{1-2Ml} \right], \quad 2^{\frac{d}{2}-2} \text{ components}$$

Interpretation

$$\mathcal{L}_{eff} = -\bar{\chi}_1 \gamma^\mu \partial_\mu \chi_1 + (1-2Ml)\frac{b}{l}\left(\frac{l}{L}\right)^{1-2Ml} \bar{\chi}_1(\gamma^0\partial_0 + v\gamma^i\partial_i)\chi_1$$

- RG towards alternative quantization:

$$\delta\mathcal{L}_{CFT} \sim l^{1-2Ml}\bar{\mathcal{O}}'_\psi \gamma^0 \partial_0 \mathcal{O}'_\psi, \quad \dim \mathcal{O}'_\psi = -Ml + \tfrac{d}{2}$$

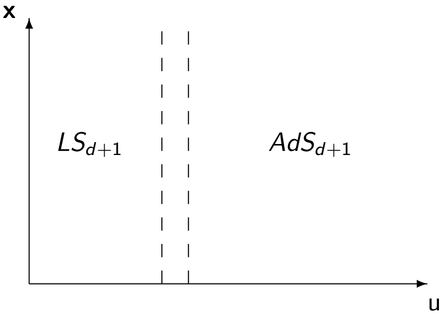
- Generalizes to arbitrary kinetic term:

$$\mathcal{L}_{\chi'} = \bar{\chi}(\gamma^0\partial_0 + \gamma^0\Omega(-\Delta))\chi$$

Anisotropic background

- Definition of LS:
  $t \mapsto \lambda^z t$
  $\mathbf{x} \mapsto \lambda \mathbf{x}, \quad u \mapsto \lambda u$

- Dual viewpoint:
  $\dim \mathcal{O}^\mu_\alpha = d + \alpha,$
  $\alpha = \alpha(d, M, L).$



$$S_V = \frac{1}{16\pi\kappa} \int d^{d+1}x \left(R - 2\Lambda_c - \frac{1}{4} F_{MN} F^{MN} - \frac{M_V^2}{2} V_N V^N \right)$$

Results and interpretation (LS)

$$S = -\int d^{d+1}x\sqrt{-g}\; i\bar{\psi}(\not{D} - \xi\not{D}^{(V)} - M)\psi,$$

$$\Downarrow$$

$$\omega = |\mathbf{k}|\left[1 + a_1(1+2Ml)\left(\frac{l_*}{L}\right)^{2\alpha_V} + a_2(1+2Ml)\left(\frac{l_*}{L}\right)^{1+2Ml}\right].$$

$$\Updownarrow$$

$$\delta S_{CFT} = \int d^d x\big(c_1\; l_*^{\alpha_V}\mathcal{O}_V^0 + c_2\; l_*^{1+2Ml}\bar{\mathcal{O}}_\psi\gamma^0\partial_0\mathcal{O}_\psi\big)$$

Counter-example

$$SO(d-1) \mapsto 2^{\frac{d-3}{2}} \text{ components}$$

$$\not\Updownarrow$$

$$SO(d-1,1) \mapsto 2^{\frac{d-1}{2}} \text{components}$$

Counter-example

$$\mathcal{O}_\psi = \begin{pmatrix} \mathcal{O}_U \\ \mathcal{O}_D \end{pmatrix}$$

$$S = S_{CFT} + l^{Ml} \int d^3x \, \chi_U^\dagger \mathcal{O}_U + b \int d^3x \, \chi_U^\dagger \big(i\partial_0 - \Omega(-\nabla) \big) \chi_U \ ,$$

- consistent with spatial isotropy!

$$S_{UV} = b \int_{u=l} d^3x \, \psi_{-,U}^\dagger \big(i\partial_0 - \Omega(-\nabla) \big) \psi_{-,U} \ ,$$

$$\Downarrow$$

$$\omega = \Omega(k^2)(1 - 2Ml) \frac{b}{l} \left(\frac{l}{L} \right)^{1-2Ml}$$

Conclusions

- LI can be emergent
- Weyl fermions are reproduced
- Expanding theory to gauge fields
- The role of discrete symmetries

Dirac equations

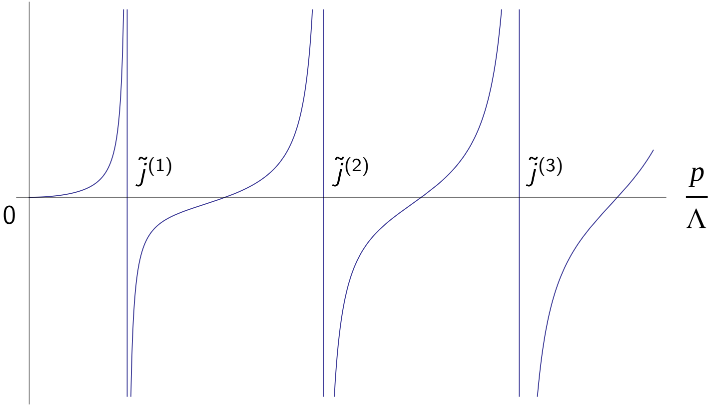
$$S = -\int d^{d+1}x\sqrt{-g}\,\imath(\bar{\psi}\not{D}\psi - M\bar{\psi}\psi) + S_\partial \quad \Rightarrow \quad (\not{D} - M)\psi = 0$$

$$\not{D} = e_A^M \Gamma^A D_M \ , \quad D_M = \left(\partial_M + \frac{1}{2}\omega_{ABM}\Sigma^{AB}\right) \ , \quad \Sigma^{AB} = \frac{1}{4}[\Gamma^A, \Gamma^B]$$

$$\omega_M^{AB} = \frac{1}{2} e_N^A \nabla_M e^{BN}, \quad e_A^M = (u/l)\,\delta_A^M \ , \quad \omega_{u\beta\mu} = \eta_{\beta\mu}/u$$

$$\not{D} = \frac{u}{l}\left(\Gamma^u\partial_u + \Gamma^\mu\partial_\mu\right) - \frac{d}{2l}\Gamma^u$$

Massive states and high momenta



$$\frac{(pL)^{1+2Ml}}{\mathcal{J}(pL)} = 2(v-1)\frac{b}{l}\left(\frac{l}{L}\right)^{1-2Ml}\mathbf{k}^2 L^2$$

Domain-wall geometry

$$S_V = \frac{1}{16\pi\kappa}\int d^{d+1}x\left(R - 2\Lambda_c - \frac{1}{4}F_{MN}F^{MN} - \frac{M_V^2}{2}V_N V^N\right),$$

$$ds^2 = \left(\frac{l}{u}\right)^2(-f^2(u)dt^2 + dx_i dx_i + g^2(u)du^2),$$

$$V_t = \frac{2}{M_V u}f(u)j(u), \quad V_i = V_u = 0\,, \quad l^2 = -\frac{d(d-1)}{2\Lambda_c}$$

$$f = f_0(u/l_*)^{1-z}\,, \quad g = g_0\,, \quad j = j_0\,, \qquad u \to 0\,,$$

$$f = 1 + f_\infty(u/l_*)^{-2\alpha_V},\ g = 1 + g_\infty(u/l_*)^{-2\alpha_V},\ j = j_\infty(u/l_*)^{-\alpha_V}\,,\ u \to \infty\,.$$

$$\alpha_V = -\frac{d}{2} + \sqrt{\left(\frac{d}{2} - 1\right)^2 + (M_V l)^2}\,.$$