Higher Spin Theory
and Holography-5
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Continuous spin field in (A)dS

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Plan

1) **Massless and massive continuous spin field in $R^{d,1}$**

2) **Continuous spin field in AdS_{d+1}**

3) **Computation of partition functions**

of continuous spin fields

Continuous spin field via deformation of tower decoupled Fronsdal fields

for continuous massless spin

in $R^{3,1}$ field by Schuster and Toro 2014

Totally symmetric double-traceless field in $R^{d,1}$

$$\phi^{a_1 \dots a_n}, \qquad n = 0, 1, \dots \infty$$

Lagrangian for decoupled Fronsdal fields

$$\mathcal{L} = \sum_{n=0}^{\infty} \mathcal{L}_n$$

$$\mathcal{L}_n - Fronsdal \quad action \quad for \quad spin \quad n - field$$

$$[\bar{\alpha}^a, \alpha^b] = \eta^{ab}, \qquad [\bar{v}, v] = 1$$

$$\bar{\alpha}^a|0\rangle = 0 \qquad \bar{v}|0\rangle = 0$$

$$|\phi\rangle = \sum_{n=0}^{\infty} \upsilon^n \alpha^{a_1} \dots \alpha^{a_n} \phi^{a_1 \dots a_n} |0\rangle$$

$$(N_\alpha - N_\upsilon)|\phi\rangle = 0$$

$$N_\alpha \equiv \alpha^a \bar{\alpha}^a \qquad N_\upsilon = \upsilon \bar{\upsilon}$$

gauge transformation parameters

$$\xi^{a_1\dots a_n}\,, \qquad n = 0, 1, \dots \infty$$

$$|\xi\rangle = \sum_{n=0}^{\infty} \upsilon^{n+1}\alpha^{a_1}\dots\alpha^{a_n}\xi^{a_1\dots a_n}|0\rangle\,.$$

gauge transformations

$$\delta|\phi\rangle = \alpha\partial|\xi\rangle$$

Lagrangian for decoupled Fronsdal

massless fields in $R^{d,1}$

$$\mathcal{L} = \langle\phi|\Box|\phi\rangle + \langle\bar{L}\phi|\bar{L}\phi\rangle$$

$$\bar{L} = \bar{\alpha}\partial - \frac{1}{2}\alpha\partial\bar{\alpha}^2$$

$$\bar{\alpha}\partial \equiv \bar{\alpha}^a\partial^a \qquad \bar{\alpha}^2 \equiv \bar{\alpha}^a\bar{\alpha}^a$$

keep in $\Box$ for traces

Consider **massless deformation**

$$\mathcal{L} = \langle \phi | \Box | \phi \rangle + \langle \mathbf{\bar{L}} \phi | \mathbf{\bar{L}} \phi \rangle$$

$$\delta\phi = (\alpha\partial + \boldsymbol{\alpha^2 \bar{e}_1 + e_1}) |\xi\rangle$$

$$\mathbf{\bar{L}} = \bar{\alpha}\partial - \frac{1}{2}\alpha\partial\bar{\alpha}^2 + \boldsymbol{\bar{\alpha}^2 e_2 + \bar{e}_2}$$

$$e_1 = e_2 \qquad \bar{e}_2 = \bar{e}_1$$

$$
\begin{aligned}
\delta\phi^{a_1\ldots a_n} &= \partial^{(a_1}\xi^{a_2\ldots a_n)} \\
&+ \xi^{\mathbf{a_1\ldots a_n}} \\
&+ \eta^{\mathbf{(a_1 a_2}}\xi^{\mathbf{a_3\ldots a_n)}}
\end{aligned}
$$

Solution

$$e_1 = f\bar{v}\,, \qquad \bar{e}_1 = -vf$$

$$f = \left[\frac{\mu_1}{(N_v+1)(2N_v+\mathbf{d}-1)}\right]^{1/2}$$

μ_1 dimensionfull parameter

for **d=3** agrees with Schuster and Toro

massive deformation

$$\mathcal{L} = \langle\phi|(\Box - \mu_{\mathbf{0}})|\phi\rangle + \langle\bar{\mathbf{L}}\phi|\bar{\mathbf{L}}\phi\rangle$$

$$\delta\phi = (\alpha\partial + \alpha^{\mathbf{2}}\bar{\mathbf{e}}_{\mathbf{1}} + \mathbf{e}_{\mathbf{1}})|\xi\rangle$$

$$\bar{\mathbf{L}} = \bar{\alpha}\partial - \frac{1}{2}\alpha\partial\bar{\alpha}^2 + \bar{\alpha}^{\mathbf{2}}\mathbf{e}_{\mathbf{2}} + \bar{\mathbf{e}}_{\mathbf{2}}$$

$$e_1 = f\bar{v}\,, \qquad \bar{e}_1 = -vf$$

$$f = \left[\frac{1}{(N_v+1)(2N_v+d-1)}F(N_v)\right]^{1/2}$$

$$F(N_v) \equiv \mu_1 - N_v(N_v+d-2)\mu_0$$

Classical unitarity

$$\mathcal{L} = \eta \langle \phi | \Box | \phi \rangle + \ldots$$

1) $\eta > 0$

2) $\mathbf{F}(\mathbf{N}_v) \geq 0$

All previously known classically unitary systems turns out to be associated with unitary representations of space-time symmetry algebras

$$F(n) > 0 \qquad n = 0, 1, \ldots, \infty$$

$$F(n) \equiv \mu_1 - n(n+d-2)\mu_0 > 0$$

$$\mu_1 > 0 \qquad \mu_0 < 0$$

conjecture

massive classically unitary continuous spin field

is associated with

tachyonic UIR of Poincaré algebra

reducible case

$$F(s) = 0 \quad \Longrightarrow$$

$$\mu_1 = s(s + d - 2)\mu_0$$

$$F(N_v) = (s - N_v)(s + d - 2 + N_v)\mu_0$$

$$|\phi\rangle = |\phi^{0,s}\rangle + |\phi^{s+1,\infty}\rangle$$

$$|\phi^{M,N}\rangle \equiv \sum_{n=M}^{N} \upsilon^n \alpha^{a_1} \ldots \alpha^{a_n} \phi^{a_1 \ldots a_n} |0\rangle$$

$$\mathcal{L}(\phi) = \mathcal{L}(\phi^{0,s}) + \mathcal{L}(\phi^{s+1,\infty})$$

0-

$$\mu_0 > 0$$

$\phi^{0,s}$ – classically **unitary**

$\phi^{s+1,\infty}$ – classically **non-unitary**

$$\mu_0 < 0$$

$\phi^{0,s}$ – classically **non-unitary**

$\phi^{s+1,\infty}$ – classically **unitary**

Continuous spin field in (A)dS$_{d+1}$

$$\mathcal{L} = \langle\phi|(\Box + \mathbf{m_1})|\phi\rangle + \langle\bar{\mathbf{L}}\phi|\bar{\mathbf{L}}\phi\rangle$$

$$\mathbf{m_1} = -\mu_0 - \rho(\mathbf{N}_v(\mathbf{N}_v + \mathbf{d} - \mathbf{1}) + \mathbf{2d} - \mathbf{4})$$

$$\rho = -1/R^2 \qquad for \quad AdS$$

$$\rho = +1/R^2 \qquad for \quad dS$$

$$e_1 = f\bar{\upsilon}\,, \qquad \bar{e}_1 = -\upsilon f$$

$$f = \left[\frac{1}{(N_\upsilon + 1)(2N_\upsilon + d - 1)} F(N_\upsilon)\right]^{1/2}$$

$$F(N_\upsilon) \equiv \mu_1 - N_\upsilon(N_\upsilon + d - 2)\mu_0$$

$$-\rho \mathbf{N}_\upsilon(\mathbf{N}_\upsilon + \mathbf{1})(\mathbf{N}_\upsilon + \mathbf{d} - \mathbf{2})(\mathbf{N}_\upsilon + \mathbf{d} - \mathbf{3})$$

Analyse $F(n) \geq 0$

For **dS** there are **NO** classically unitary solution

many solutions for **AdS**

Partition function

Apply Faddeev-Popov procedure

Step 1. Introduce

Faddeev-Popov fields

$$\overline{c}^{a_1 \cdots a_n} \qquad c^{a_1 \cdots a_n}$$

$$|\overline{c}\rangle \qquad |c\rangle$$

Nakanishi-Lautrup field

$$b^{a_1 \cdots a_n}$$

$$|b\rangle$$

Step 2.

$$\mathcal{L}^{BRST} = \mathcal{L} + \mathcal{L}_{\mathbf{g.fix}}$$

$$\mathcal{L}_{\mathsf{qu}} = -\langle b|\bar{L}|\phi\rangle + \langle\bar{c}|(\Box + M_{\mathsf{FP}})|c\rangle + \frac{1}{2}\alpha\langle b||b\rangle$$

$$M_{\mathsf{FP}} = -\mu_0 - \rho(N_v(N_v + d - 3) + d - 2)$$

$\alpha = 0$ **Landau gauge**

$\alpha = 1$ **Feynman gauge**

Step 3.

Feynman gauge $\alpha = 1$

Integrate out field $\mathbf{b}$

$$\mathcal{L}_{\text{tot}} = \langle \phi | (\Box + m_1) | \phi \rangle + \langle \bar{c} | (\Box + M_{\text{FP}}) | c \rangle$$

spin-**n** doube-traceless tensor

$=$ spin-**n** traceless

$\oplus$

spin-**(n-2)** traceless tensor

$$|\phi\rangle = |\phi_{\mathrm{I}}\rangle + |\phi_{\mathrm{II}}\rangle$$

0-

$$\mathcal{L} = \mathcal{L}_{\mathrm{I}} + \mathcal{L}_{\mathrm{II}} + \mathcal{L}_{\mathsf{FP}}$$

$$\mathcal{L}_{\mathrm{I}} \equiv \sum_{n=0}^{\infty} \mathcal{L}_{\mathrm{I},n}\,, \qquad \mathcal{L}_{\mathrm{II}} = \sum_{n=0}^{\infty} \mathcal{L}_{\mathrm{II},n}$$

$$\mathcal{L}_{\mathsf{FP}} = \sum_{n=0}^{\infty} \mathcal{L}_{\mathsf{FP},n}$$

$$\mathcal{L}_{\mathrm{I},n} \equiv \phi_{\mathrm{I}}^{a_1 \ldots a_n} (\Box + \mathbf{M_n}) \phi_{\mathrm{I}}^{a_1 \ldots a_n}$$

$$\mathcal{L}_{\mathrm{II},n} \equiv \phi_{\mathrm{II}}^{a_1 \ldots a_n} (\Box + \mathbf{M_n}) \phi_{\mathrm{II}}^{a_1 \ldots a_n}$$

$$\mathcal{L}_{\mathsf{FP},n} \equiv \bar{c}^{a_1 \ldots a_n} (\Box + \mathbf{M_n}) c^{a_1 \ldots a_n}$$

$$\mathbf{M_n} \equiv -\mu_0 - \rho(n(n + d - 1) + 2d - 4)$$

$$\mathbf{Z = 1}$$