Progress in HS theory

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Plan

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)

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- Giombi and Yin tests from equations of motion: substantial piece of evidence that many of $3pt$ functions match.
- Generic structure of $3pt$-correlators (Maldacena, Zhiboedov)

$$\langle JJJ \rangle = \cos^2\phi \langle JJJ \rangle_b + \sin^2\phi \langle JJJ \rangle_f + \frac{1}{2}\sin(2\phi)\langle JJJ \rangle_o$$

Locality

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Vasiliev equation encode HS vertices in a field-redefinition independent way through certain differential equations in a twistor space. The price to pay – one has to specify solution in a proper class of functions.

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$$DC(Y|x) = -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.$$

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If $H(Y) = 0$, then $\Upsilon(C, C)$ is nonlocal.

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- Vasiliev+Gelfond: Unique field redefinition that brings equations into a local form with fixed coefficients

$$C(Y|x)+\eta\int\limits_{[0,1]^3} e^{iu_A v^A}\delta'(1-t_1-t_2-t_3)J(t_3u+t_1y,v-t_2y,\bar{y}+\bar{u},\bar{y}+\bar{v})$$

$$J(y_1,y_2,\bar{y}_1,\bar{y}_2)=C(y_1,\bar{y}_1)C(y_2,\bar{y}_2)$$

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$$J(y_1,y_2,\bar{y}_1,\bar{y}_2)=C(y_1,\bar{y}_1)C(y_2,\bar{y}_2)$$

- Fierz identities at higher orders bring in the new homotopy operator that renders equations local automatically.

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- Calculate three-point correlation functions

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RESULTS:

- The local form of HS equations was shown to be perfectly consistent with AdS/CFT expectations
- Known bosonic truncations of Vasliev equations in four dimensions admit no CFT duals other than free theories
- Parity broken $CFT's$ require different truncation of the full Vasiliev system which is available at least in perturbation theory
- Correlation functions $\langle J_{s_1} J_{s_2} J_{s_3} \rangle$ with $s_3 \geq s_1 + s_2$ were found including in the parity broken case

Vasiliev equations

Vasiliev equations in $d = 4$

$$
\begin{aligned}
&\mathrm{d}W + W * W = 0\,,\\
&\mathrm{d}S + [W, S]_* = 0\,,\\
&\mathrm{d}B + [W, B]_* = 0\,,\\
&S * S = -i\theta_\alpha \wedge \theta^\alpha(1 + \eta B * k\varkappa) - i\bar{\theta}_{\dot{\alpha}} \wedge \bar{\theta}^{\dot{\alpha}}(1 + \bar{\eta} B * \bar{k}\bar{\varkappa})\,,\\
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Note extra Klein operators k and $\bar{k}$

Field-current correspondence

Free HS equations for 0-form

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Using Poincare connection (Vasiliev)

$$C(y, \bar{y}; k, \bar{k}) = z e^{y_\alpha \bar{y}^\alpha} T(w, \bar{w}; k, \bar{k})\,,$$

$$w = \sqrt{z} y\,, \qquad \bar{w} = \sqrt{z} \bar{y}\,.$$

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Gives current conservation

$$\mathrm{d_x} T - \frac{i}{2}\mathrm{dx}^{\alpha\alpha}\partial_\alpha\bar{\partial}_\alpha T = 0\,, \qquad \partial\cdot J = 0$$

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Boundary HS connections

$$D_{\mathrm{x}} \omega_{\mathrm{x}} = \frac{1}{4} H^{\alpha\beta}_{\mathrm{xx}} \frac{\partial^2}{\partial w^\alpha \partial w^\beta} \left(\bar{\eta} T(w, 0) \bar{k} - \eta T(0, iw) k \right)\,.$$

Boundary conditions

- Parity preserving cases $\eta = 1$ or $\eta = i$

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Propagators

0-form $\Delta = 1$ propagators

$$C^{+} = \eta K e^{i f_{\alpha\dot\alpha} y^{\alpha} \bar y^{\dot\alpha} + i\xi^{\alpha} y_{\alpha}}\,, \qquad C^{-} = \bar\eta K e^{i f_{\alpha\dot\alpha} y^{\alpha} \bar y^{\dot\alpha} + i\bar\xi^{\dot\alpha} \bar y_{\dot\alpha}}$$

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$$K = \frac{z}{(\mathrm{x}-\mathrm{x}_0)^2 + z^2}\,,$$

$$f_{\alpha\dot{\alpha}} = -\frac{2z}{(\mathrm{x}-\mathrm{x}_0)^2 + z^2}(\mathrm{x}-\mathrm{x}_0)_{\alpha\dot{\alpha}} - i\,\frac{(\mathrm{x}-\mathrm{x}_0)^2 - z^2}{(\mathrm{x}-\mathrm{x}_0)^2 + z^2}\epsilon_{\alpha\dot{\alpha}}\,,$$

$$\xi_{\alpha} = \Pi_{\alpha}{}^{\beta}\mu_{\beta}\,, \qquad \Pi_{\alpha\beta} = K\left(\frac{1}{\sqrt{z}}(\mathrm{x}-\mathrm{x}_0)_{\alpha\beta} - i\sqrt{z}\epsilon_{\alpha\beta}\right)$$

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$\Delta = 2$ scalar propagator

$$C_{\Delta=2} = K^2(1 + if_{\alpha\dot{\alpha}} y^{\alpha} \bar{y}^{\dot{\alpha}}) \times e^{if_{\alpha\dot{\alpha}} y^{\alpha} \bar{y}^{\dot{\alpha}}}\,.$$

Boundary limit

Current sources

$$T^{+} = \frac{\eta}{|\mathrm{x} - \mathrm{x}_0|^2} e^{-2i(\mathrm{x}-\mathrm{x}_0)^{-1}_{\alpha\dot\alpha} w^{\alpha} \bar{w}^{\dot\alpha} + i(\mathrm{x}-\mathrm{x}_0)_{\alpha\beta}\mu^{\beta} w^{\alpha}},$$

$$T^{-} = \frac{\bar{\eta}}{|\mathrm{x} - \mathrm{x}_0|^2} e^{-2i(\mathrm{x}-\mathrm{x}_0)^{-1}_{\alpha\dot\alpha} w^{\alpha} \bar{w}^{\dot\alpha} + i(\mathrm{x}-\mathrm{x}_0)_{\alpha\beta}\bar{\mu}^{\beta} \bar{w}^{\alpha}}$$

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$\Delta=2$ source

$$T_{\Delta=2}=\frac{w_{\alpha}\bar{w}^{\alpha}}{|\mathrm{x}-\mathrm{x}_0|^4}\times e^{-2i(\mathrm{x}-\mathrm{x}_0)^{-1}_{\alpha\dot\alpha}w^{\alpha}\bar{w}^{\dot\alpha}}.$$

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Boundary condition

$$\bar\eta T^{+}(w,\bar{w}) = \eta T^{-}(-i\bar{w}, iw).$$

Second order

Local form in the 0-form sector

$$DC = \frac{i}{2}\eta e^{\alpha\dot{\alpha}} \int e^{i\bar{u}_{\dot{\alpha}}\bar{v}^{\dot{\alpha}}} y_{\alpha}(t\bar{u}_{\dot{\alpha}} + (1-t)\bar{v}_{\dot{\alpha}}) J(ty, -(1-t)y, \bar{y} + \bar{u}, \bar{y} + \bar{v})k$$

$$J = CC$$

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Vertices in 1-form sector are phase-independent

$$D\omega = \eta\bar{\eta} \int e^{i\bar{u}_{\dot{\alpha}}\bar{v}^{\dot{\alpha}}} (\dots)$$

Boundary limit: currents

Carrying out boundary limit in the 0-form sector $z \to 0$

$$C(y, \bar{y}; k, \bar{k}) = z e^{y_\alpha \bar{y}^\alpha} T(w, \bar{w}; k, \bar{k}), \quad I = TT$$

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$$\mathrm{d_x} T - \frac{i}{2}\mathrm{dx}^{\alpha\alpha}\partial_\alpha \bar{\partial}_\alpha T =$$

$$-\frac{\eta}{4}\mathrm{dx}^{\alpha\alpha} w_\alpha \int_0^1 (t\bar{\partial}_{2\alpha} - (1-t)\bar{\partial}_{1\alpha}) I(tw, -(1-t)w, \bar{w} + i(1-t)w, \bar{w} - itw) k$$

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HS currents do not conserve $\partial \cdot J \neq 0$

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HS currents do not conserve $\partial \cdot J \neq 0$
They do for the free theories boundary conditions

$$T(w, \bar{w})k = \pm T(-i\bar{w}, iw)\bar{k} \qquad \Rightarrow \partial \cdot J = 0$$

Boundary limit: HS connections

Boundary limit in the 1-form sector $z \to 0$

$$D_{\mathrm{x}}\omega_{\mathrm{x}} = \frac{i}{8}\eta\bar{\eta}\int d^2t\delta'(1-t_1-t_2)H_{\mathrm{xx}}^{\alpha\alpha}\left(\frac{\partial}{\partial u^{\alpha}}\right)^2 \times$$
$$\times\Big\{I(t_1(w+u),-t_2(w+u),it_2w,-it_1w)-$$
$$-I(t_1w,-t_2w,it_2(w+u),-it_1(w+u))\Big\}\Big|_{u=0}$$

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For parity broken theory $\eta \neq 1, i$

$$D_{\rm x}\omega_{\rm x} \neq 0$$

Green's function

- Giombi-Yin: Correlators from equations of motion

$$DC = J[C,C]\,, \qquad \langle JJJ\rangle \sim \lim_{z\to 0} z^{-1} G(wz^{-\frac{1}{2}}, \bar{w}z^{-\frac{1}{2}})\Big|_{\bar{w}=0}$$

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- $G(w,\bar{w}|\mathrm{x},z)$ – is the Green's function, w_α – to be associated with outgoing spinor polarization

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- $G(w,\bar{w}|\mathrm{x},z)$ – is the Green's function, w_α – to be associated with outgoing spinor polarization

$$C^{2s}(y|\mathrm{x},z) \to z^{s+1}\int \frac{dz_0 d^3\mathrm{x}_0}{z_0^4} G(y,\partial_{y_0}|\mathrm{x}-\mathrm{x}_0,z_0)J(y_0|\mathrm{x}_0,z_0)$$

Green's function

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$$G_s(y,\lambda|\mathrm{x},z) \sim \int_0^\infty \frac{dt}{(1+t)^{-2s}}\Big\{\frac{(y\lambda\partial)^{2s}}{(2s)!}\Big[\Gamma[2-2s]\mathrm{x}^{2s-3}\sin(2(s-1)\arctan\frac{\mathrm{x}}{z})\Big]\Big\}\Big|_{z\to(2t+1)z}$$

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GY Green's function does not take into account consistency constraint on $J[C, C]$

Homotopy Green's function

Equation to solve

$$DG = \frac{i}{2} e^{\alpha\dot{\alpha}} \int e^{i\bar{u}_{\dot{\alpha}}\bar{v}^{\dot{\alpha}}} y_{\alpha}(t\bar{u}_{\dot{\alpha}} + (1-t)\bar{v}_{\dot{\alpha}}) J(ty, -(1-t)y, \bar{y}+\bar{u}, \bar{y}+\bar{v})k\,.$$

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Equation to solve

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We're looking for solution outside a spin triangle

$$s \geq s_1 + s_2$$

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The Green's function:

$$G = \frac{1}{2} \int_{[0,1]^3\times\mathbf{R}} \delta'(1-t_1-t_2-t_3) e^{iu_A v^A} J(u+t_1 y, t_3 v - t_2 y, \bar y + \bar u, \bar y + \bar v)k$$

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Equation to solve

$$DG = \frac{i}{2} e^{\alpha\dot{\alpha}} \int e^{i\bar{u}_{\dot{\alpha}}\bar{v}^{\dot{\alpha}}} y_\alpha (t\bar{u}_{\dot{\alpha}} + (1-t)\bar{v}_{\dot{\alpha}}) J(ty, -(1-t)y, \bar{y}+\bar{u}, \bar{y}+\bar{v})k\,.$$

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Valid for opposite helicity signs only!

3pt functions

- Recall that propagators get naturally split into positive and negative helicity parts

$$C^{+}=\eta K e^{if_{\alpha\dot{\alpha}}y^{\alpha}\bar{y}^{\dot{\alpha}}+i\xi^{\alpha}y_{\alpha}}\,,\quad C^{-}=\bar{\eta}K e^{if_{\alpha\dot{\alpha}}y^{\alpha}\bar{y}^{\dot{\alpha}}+i\bar{\xi}^{\dot{\alpha}}\bar{y}_{\dot{\alpha}}}$$

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- all different three-point correlators arrange in

$$\langle JJJ\rangle_{boson} \sim G^{++} + G^{--} + G^{+-} + G^{-+}\,,$$
$$\langle JJJ\rangle_{fermion} \sim G^{++} + G^{--} - G^{+-} - G^{-+}\,,$$
$$\langle JJJ\rangle_{odd} \sim G^{++} - G^{--}\,,$$

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$$\langle JJJ\rangle_{fermion} \sim G^{++} + G^{--} - G^{+-} - G^{-+}\,,$$
$$\langle JJJ\rangle_{odd} \sim G^{++} - G^{--}\,,$$

- Simple Gaussian integration gives

$$G^{+-} = \int d^3t \frac{K_1 K_2}{\Delta}\delta'(1-t_1-t_2-t_3)e^{2\frac{t_1 t_2}{\Delta}Q + \frac{t_1}{\Delta}((1-t_3)P_1 + zt_3\tilde{S}_1) - \frac{t_1}{\Delta}((1-t_3)P_2 + zt_3\tilde{S}_2)}$$

3pt functions

$$
\begin{aligned}
G^{++} &= \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|x_{01}||x_{02}||x_{12}|}\int_0^\infty d\tau\,\frac{\tau^{2s}(\tau P_1+S_1)^{2s_1}(-\tau P_2+S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,,\\
G^{--} &= \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|x_{01}||x_{02}||x_{12}|}\int_0^\infty d\tau\,\frac{\tau^{2s}(\tau P_1-S_1)^{2s_1}(-\tau P_2-S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,,\\
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G^{-+} &= \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|x_{01}||x_{02}||x_{12}|}\int_0^\infty d\tau\,\frac{\tau^{2s}(\tau P_1-S_1)^{2s_1}(-\tau P_2+S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,.
\end{aligned}
$$

3pt functions

$$G^{++} = \frac{z}{2} K_{s_1 s_2 s} \frac{Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 + S_1)^{2s_1}(-\tau P_2 + S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}},$$

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$$K_{s_1 s_2 s} = \frac{2^{s-s_1-s_2}(s+s_1+s_2)!}{(2s)!(2s_1)!(2s_2)!}$$

3pt functions

$$G^{++} = \frac{z}{2} K_{s_1 s_2 s} \frac{Q^{s-s_1-s_2}}{|x_{01}||x_{02}||x_{12}|} \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 + S_1)^{2s_1}(-\tau P_2 + S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}},$$

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$$K_{s_1 s_2 s} = \frac{2^{s-s_1-s_2}(s+s_1+s_2)!}{(2s)!(2s_1)!(2s_2)!}$$

$$P_1 = i\frac{(x_{01})_{\alpha\alpha}\mu_0^\alpha \mu_1^\alpha}{|x_{01}|^2}, \qquad P_2 = i\frac{(x_{02})_{\alpha\alpha}\mu_0^\alpha \mu_2^\alpha}{|x_{02}|^2}; \qquad Q = \left(\frac{x_{01}}{|x_{01}|^2} - \frac{x_{02}}{|x_{02}|^2}\right)_{\alpha\alpha} \mu_0^\alpha \mu_0^\alpha,$$

$$S_1 = \frac{(x_{02})^{\beta\alpha}(x_{12})_\alpha{}^\gamma \mu_{1\gamma}\mu_{0\beta}}{|x_{01}||x_{02}||x_{12}|}, \quad S_2 = \frac{(x_{01})^{\beta\alpha}(x_{12})_\alpha{}^\gamma \mu_{2\gamma}\mu_{0\beta}}{|x_{01}||x_{02}||x_{12}|}.$$

3pt functions

- Free theory correlators

$$G^{++}+G^{--}=\frac{K_{s_1 s_2 s}Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|}\int_{-\infty}^{\infty}d\tau\frac{\tau^{2s}(\tau P_1+S_1)^{2s_1}(\tau P_2+S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}$$

$$G^{+-}+G^{-+}=\frac{K_{s_1 s_2 s}Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|}\int_{-\infty}^{\infty}d\tau\frac{\tau^{2s}(\tau P_1+S_1)^{2s_1}(\tau P_2-S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}$$

3pt functions

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Match with free boson and free fermion

3pt functions

- Parity broken correlators $\eta \neq 1, i$

$$\langle J_{s_1} J_{s_2} J_s \rangle_{odd} \sim \frac{1}{2} \frac{K_{s_1 s_2 s}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \int_0^\infty d\tau \frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}} Q^{s-s_1-s_2} \times$$
$$\left((\tau P_1 + S_1)^{2s_1} (\tau P_2 - S_2)^{2s_2} - (\tau P_1 - S_1)^{2s_1} (\tau P_2 + S_2)^{2s_2}\right)$$

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$$\langle J_{s_1} J_{s_2} J_s \rangle \sim \int_0^\infty d\tau \frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}} (\tau a + b)^{2s_1} (\tau c + d)^{2s_2}$$

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- Parity broken correlators $\eta \neq 1, i$

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$$\langle J_{s_1} J_{s_2} J_s \rangle \sim \int_0^\infty d\tau \frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}} (\tau a + b)^{2s_1} (\tau c + d)^{2s_2}$$

$$\int_0^{\pi/2} d\phi \, \sin^{2s}\phi \, \sin^{2s_1}(\phi+\phi_1) \sin^{2s_2}(\phi+\phi_2)\,, \qquad \tau = \tan\phi$$

Examples

- Quasi-bosonic scalar (S. Giombi 1, V. Gurucharan, V. Kirilin, S. Prakash, E.D. Skvortsov)

$$\langle J_1 J_0 J_3 \rangle \sim Q_3^2 S_2 \,,$$
$$\langle J_2 J_0 J_4 \rangle \sim Q_3^2 S_2 (4P_2^2 + Q_1 Q_3) \,,$$
$$\langle J_1 J_0 J_5 \rangle \sim Q_3^4 S_2 \,,$$
$$\langle J_3 J_0 J_5 \rangle \sim Q_3^2 S_2 (2P_2^2 + Q_1 Q_3)(6P_2^2 + Q_1 Q_3) \,,$$
$$\langle J_2 J_0 J_6 \rangle \sim Q_3^4 S_2 (6P_2^2 + Q_1 Q_3) \,,$$
$$\langle J_4 J_0 J_6 \rangle \sim Q_3^2 S_2 (107 P_2^4 Q_1 Q_3 + 40 P_2^2 Q_1^2 Q_3^2 + 102 P_2^6 + 3 Q_1^3 Q_3^3)$$

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All match.

Conclusion

- Vasiliev equations are analyzed in the current interaction sector $s \geq s_1 + s_2$ to the second order. Perfect agreement with CFT expectation is found.
- Boundary limit of equations was investigated. It is shown that known bosonic HS system has no CFT dual other than free theories. Parity broken CFTs result from different truncation of Vasiliev equations. Boundary conditions require nonlinear modifications at higher orders.
- Homotopy Green's function in the current interaction sector is found.
- Parity broken three-point functions $\langle J_{s_1} J_{s_2} J_s \rangle$ were extracted from equations of motion.