Holographic RenormGroup Flow

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Outlook

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- **HRG for one phenomenological model**

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- **HRG for simple models**

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- **Exact HRG for two exp potential**

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- What **is special for**

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- **What is special for $\mu \neq 0$**
- **What is special for anizotropic case**

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- What **is special for anizotropic case**

- **Few remarks on relation with HJ-method&Refs**

Starting point - 5-dim background

5-dim Background

I.A., K. Rannu, JHEP' 18

5-dim Background

Einstein-dilaton-two-Maxwell

I.A., K. Rannu, JHEP' 18

$$S=\int\frac{d^5x}{16\pi G_5}\sqrt{-\det(g_{\mu\nu})}\left[R-\frac{f_1(\phi)}{4}F_{(1)}^2-\frac{f_2(\phi)}{4}F_{(2)}^2-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]$$

$$ds^2=B^2(w)\Big(-g(w)dt^2+dx^2\Big)+R(w)d\vec{y}^{\,2}+\frac{dw^2}{g(w)}$$

$$\phi=\phi(w),\qquad A_\mu^{(1)}=A_t(w)\delta_\mu^0,$$

$$F_{\mu\nu}^{(2)}=q\,\delta_\mu^1\delta_\nu^2 dy^1\wedge dy^2$$

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Ads Black-Brane

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Coupling constant $\lambda=e^\phi$

Energy scale $E\sim B$

5-dim Background: Multiplicity and R-factor in the metric

IA, Golubtsova, JHEP'15

$$\mathcal{M}_{LHC} \sim s^{0.155(4)}$$

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$$S = \int d^5x\sqrt{|g|}\left(R[g] + \Lambda - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}e^{\lambda\phi}F^2_{(2)}\right)$$

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Shock domain walls collision: ENTROPY

$$\mathcal{M} = \frac{\nu}{2G_5}(8\pi G_5)^{1/(1+\nu)} s^{\frac{1}{2+\nu}}$$

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Holographic Anizotropic RenormGroup Flow

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Non-zero Aniz. : $H_2 \neq 0$

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$$\frac{dX}{d\phi} = -\frac{2}{9}\,\mathfrak{K}\left(1 + \frac{2V' - f_1' H_1^2 - 2 f_2 H_2^2 X + f_2' H_2^2}{X\left(2V + f_1 H_1^2 + f_2 H_2^2\right)}\right)$$

$$\frac{dY}{d\phi} = -\frac{2}{9}\,\frac{Y}{X}\,\mathfrak{K}\left(1 + \frac{3 f_1 H_1^2 - 4 f_2 H_2^2 Y}{2Y\left(f_1 H_1^2 + f_2 H_2^2 + 2V\right)}\right)$$

$$\frac{dZ}{d\phi} = -\frac{(1-2Z)}{9X}\,\mathfrak{K}\left(1 + \frac{4Z+1}{1-2Z}\,\frac{f_2 H_2^2}{\left(2V + f_1 H_1^2 + f_2 H_2^2\right)}\right)$$

$$\frac{dH_1}{d\phi} = -\left(\frac{f_1'}{f_1} + \frac{4Z+1}{3X}\right) H_1$$

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$$X = \frac{1}{3}\frac{\dot\phi}{\dot B}B, \quad Y = \frac{1}{4}\frac{\dot g}{g}\frac{B}{\dot B}, \quad H_1 = \frac{\dot A_t}{B}$$

$$H_2(\phi) = \frac{q}{R}$$

$$Z = \frac{BR'}{4RB'}$$

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I.A.,K.Rannu

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Holographic Isotropic RenormGroup Flow

Non-zero chemical potential $\mu \neq 0, \quad H_1 \neq 0$ Non-zero temperature $Y \neq 0$

Suitable for NICA

Einstein-Dilaton-one-Maxwell E.O.M. are equivalent to Isotropic RenormGroup eqs:

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(-\frac{3X^2}{8} + Y + 1\right)\left(1 + \frac{2V' - H_1^2 f_1'}{f_1 H_1^2 X + 2V X}\right)$$

$$\frac{dY}{d\phi} = -\frac{4}{3}\left(-\frac{3X^2}{8} + Y + 1\right)\frac{Y}{X}\left(1 + \frac{3 f_1 H_1^2}{2Y\left(f_1 H_1^2 + 2V\right)}\right)$$

$$\frac{dH_1}{d\phi} = -H_1\left(\frac{f_1'}{f_1} + \frac{1}{X}\right)$$

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I.A., K.Rannu, JHEP'18

Holographic Isotropic zero μ RG Flow

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Zero chemical potential $H_1 = 0$ Non-zero temperature $Y \neq 0$

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Gursoy, Kiritsis, Mazzanti, Nitti, arXiv:0812.0792

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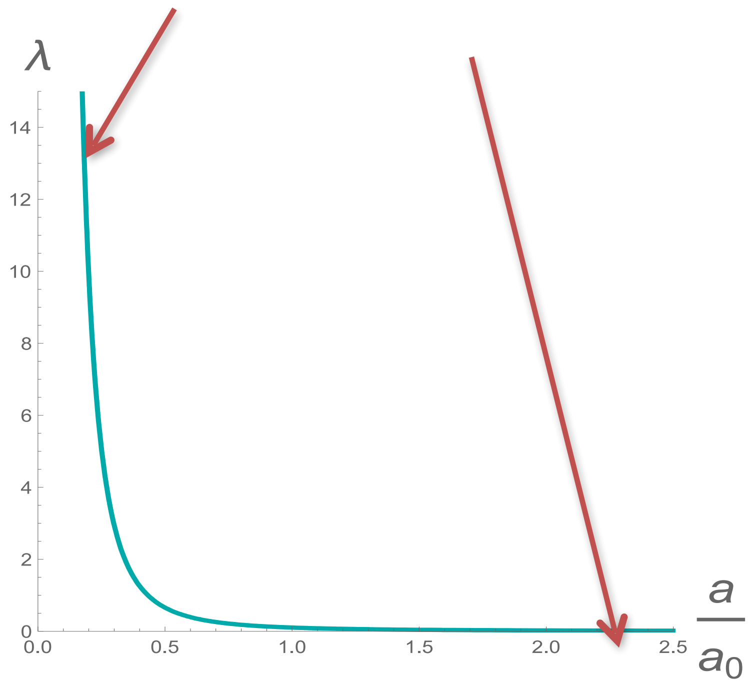
IR Quark confinement + UV Asymptotic freedom



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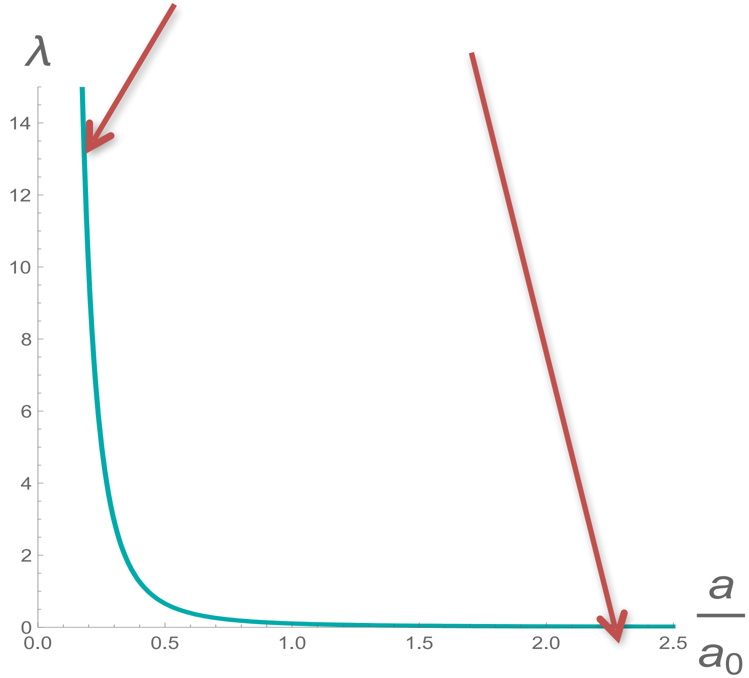

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V ~ const –conformal case,

near conformal deformations

Improved HQCD,

Big activity 08-14

Holographic RenormGroup Flow, T=0

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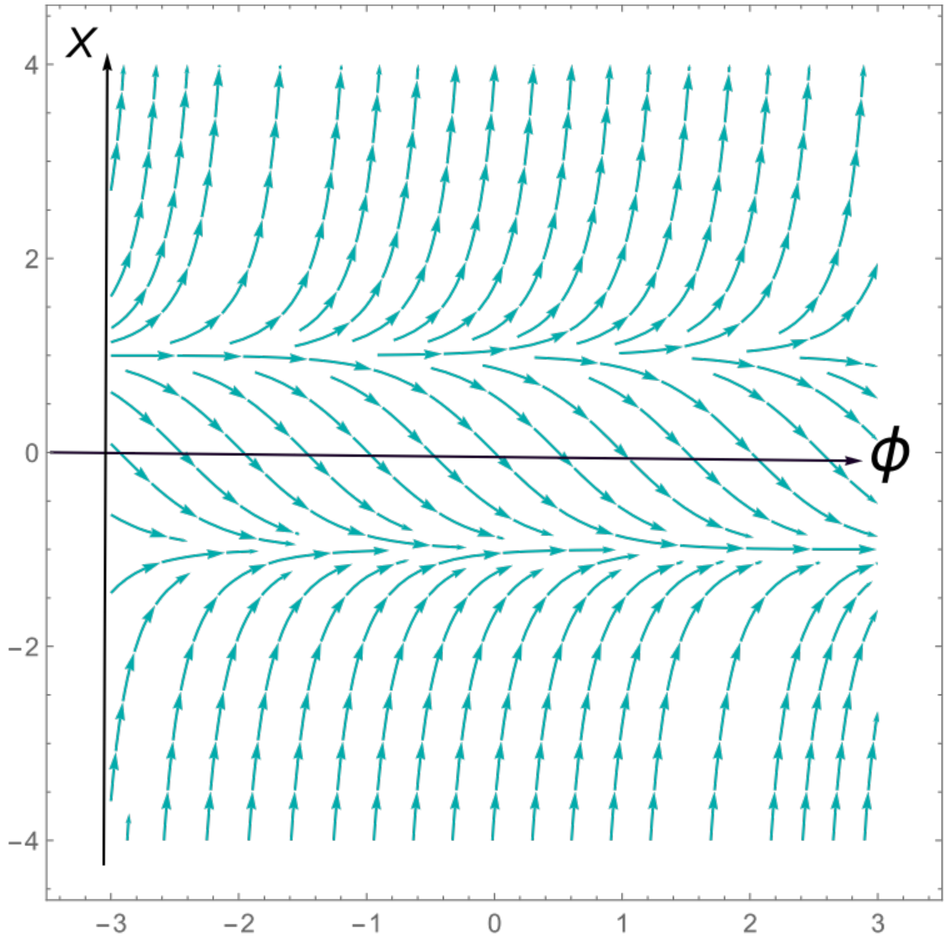

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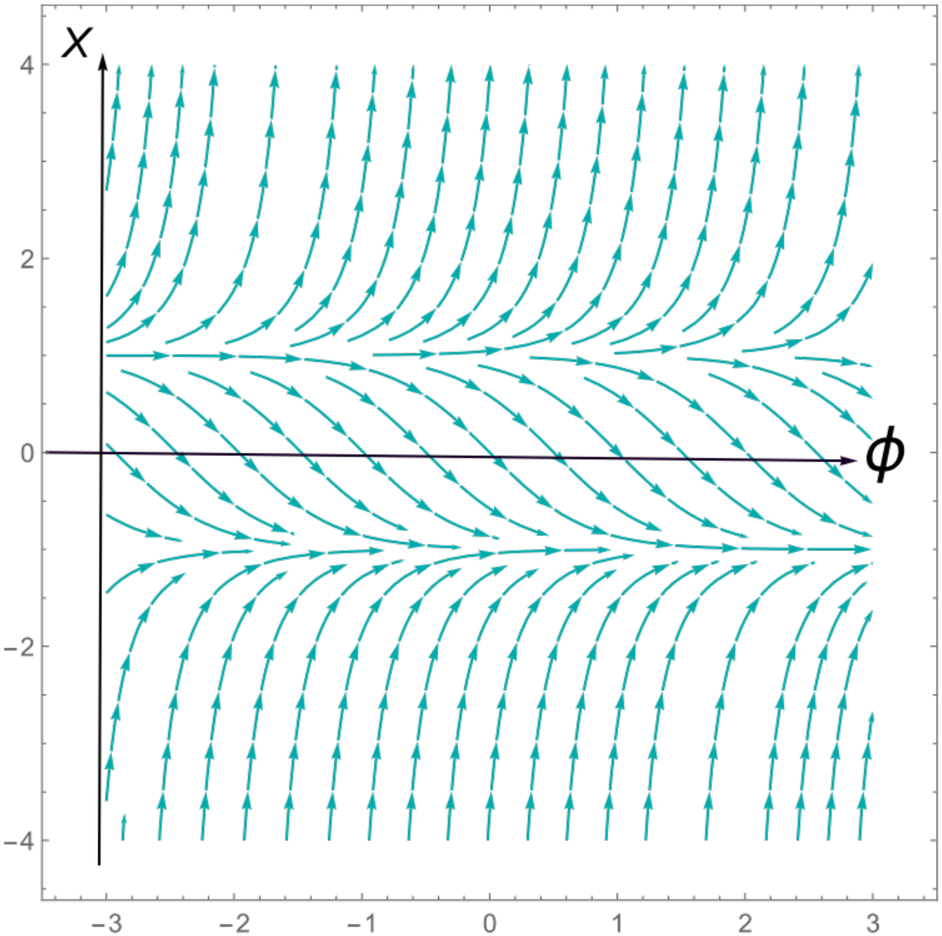

Explicit solutions

$$|X| < 1$$

$$X(\phi) = -\tanh\left(\frac{4}{3}(\phi + \phi_0)\right)$$

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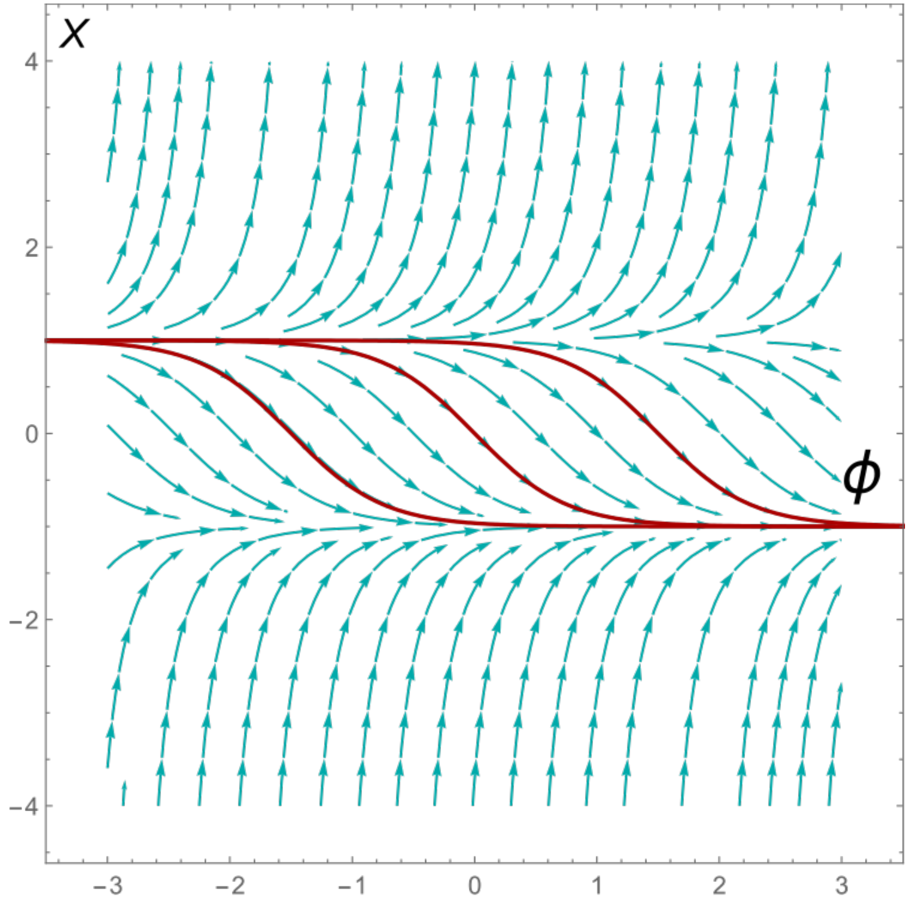

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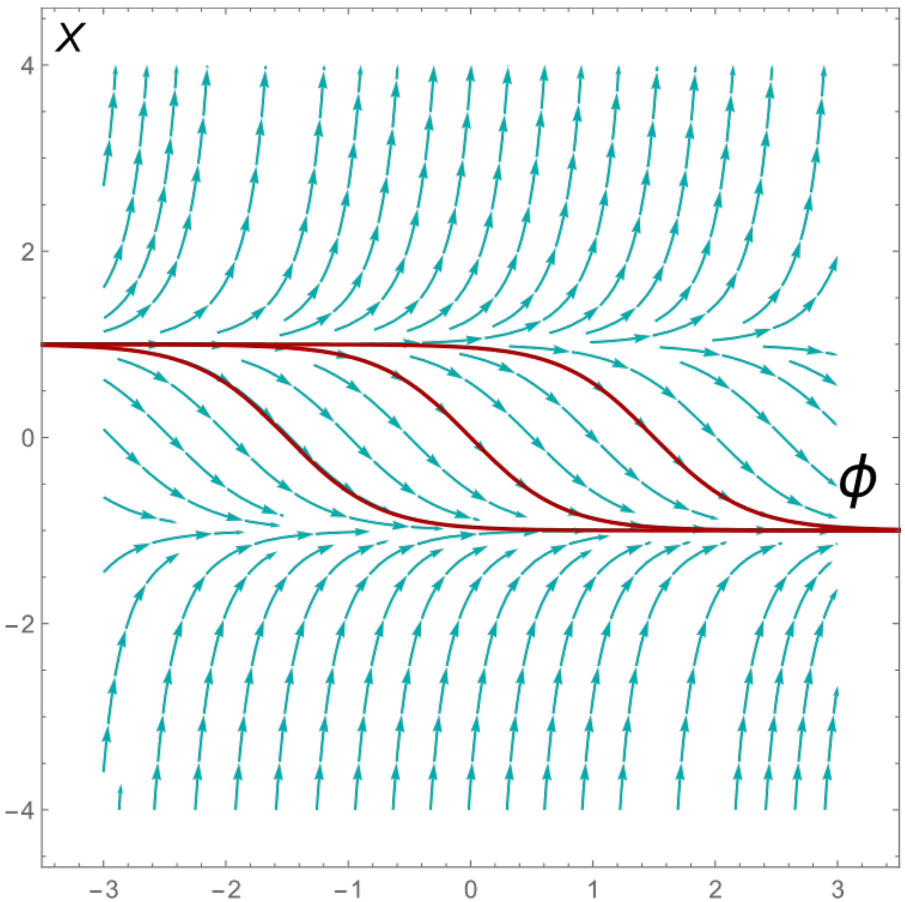
V ~ const –conformal case



Explicit solutions

$$|X| < 1$$

$$-\tanh\left(\frac{4}{3}(\phi + \phi_0)\right)$$

$$X \to \infty$$

$$\frac{dX}{d\phi} = \frac{4}{3}X^2$$

Holographic RenormGroup Flow, T=0

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2\right)$$

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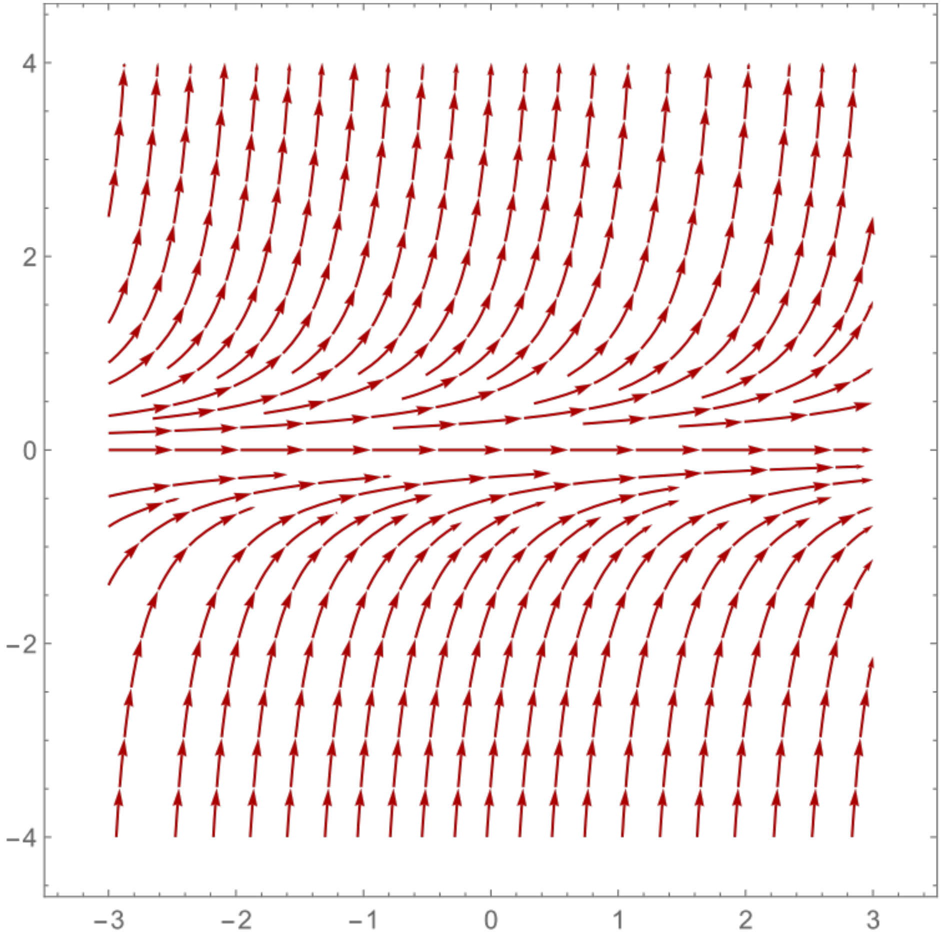

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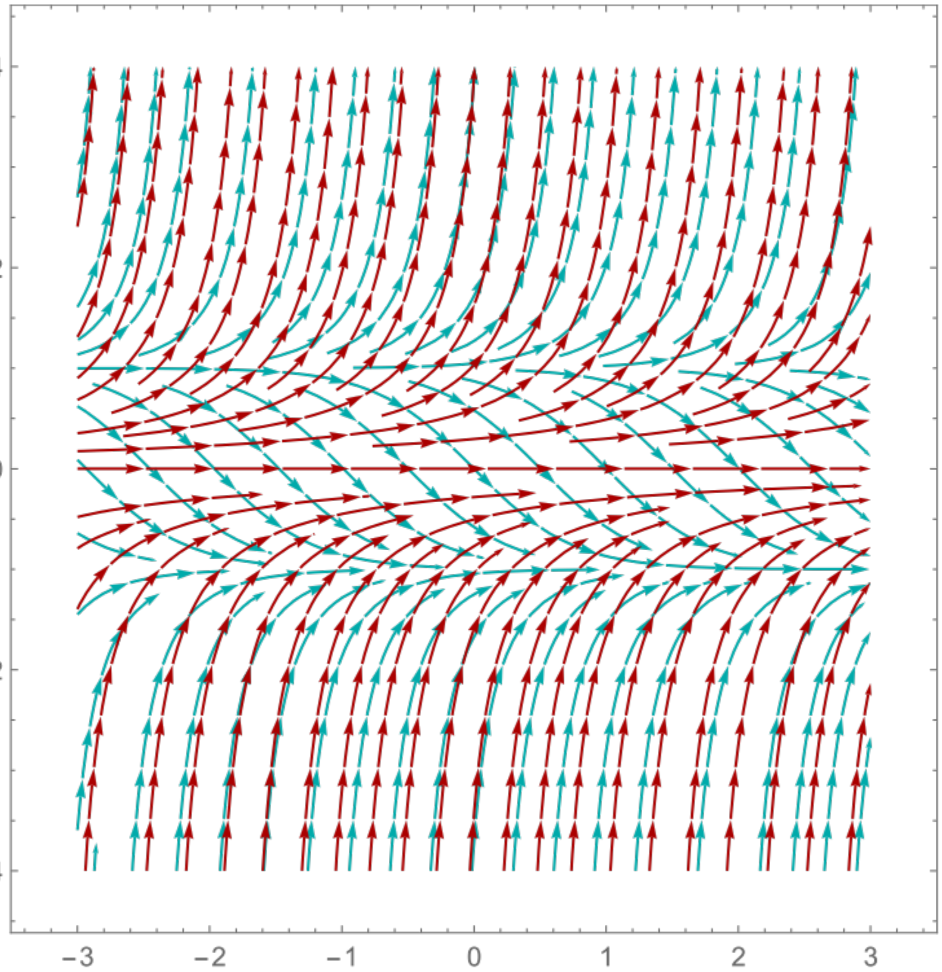

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$$V = \frac{12}{\ell^2}\left(1 + v_0\lambda + \ldots\right) \qquad X(\lambda) = -\frac{b_0}{3}\lambda + \ldots$$

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$$B = \frac{\ell}{r}\left(1 + \frac{4}{9}\frac{1}{\ln(r\Lambda)} + \ldots\right) \sim E$$

$$\lambda = -\frac{1}{b_0}\frac{1}{\ln(r\Lambda)} + \ldots$$

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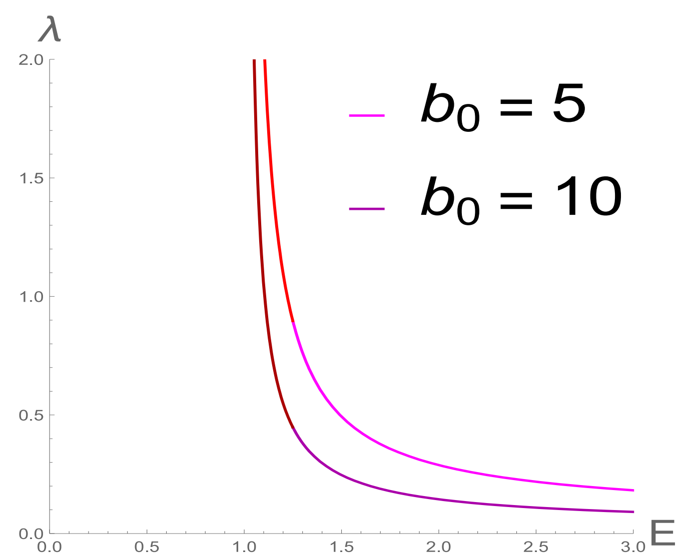

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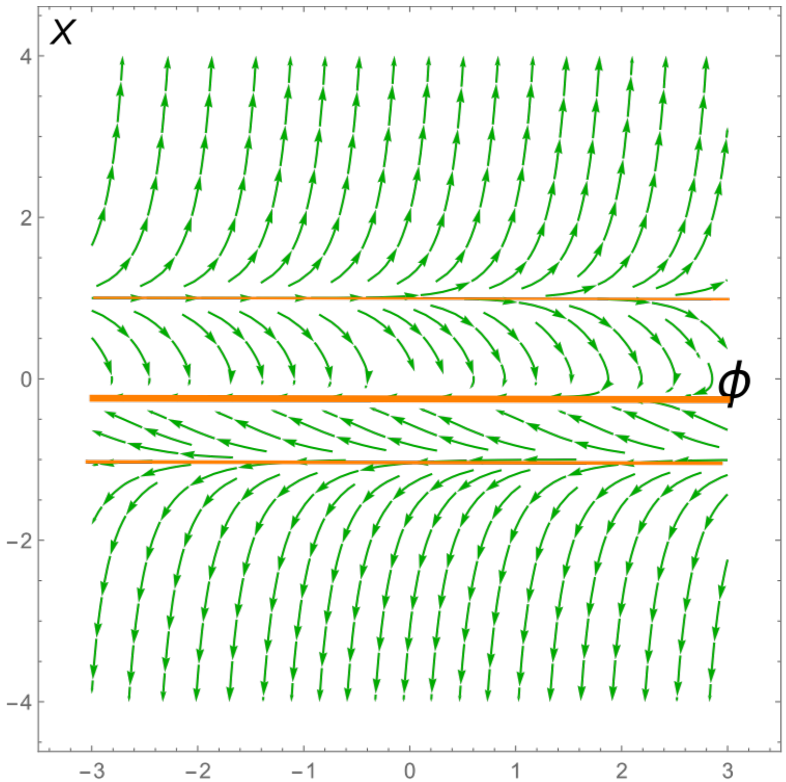

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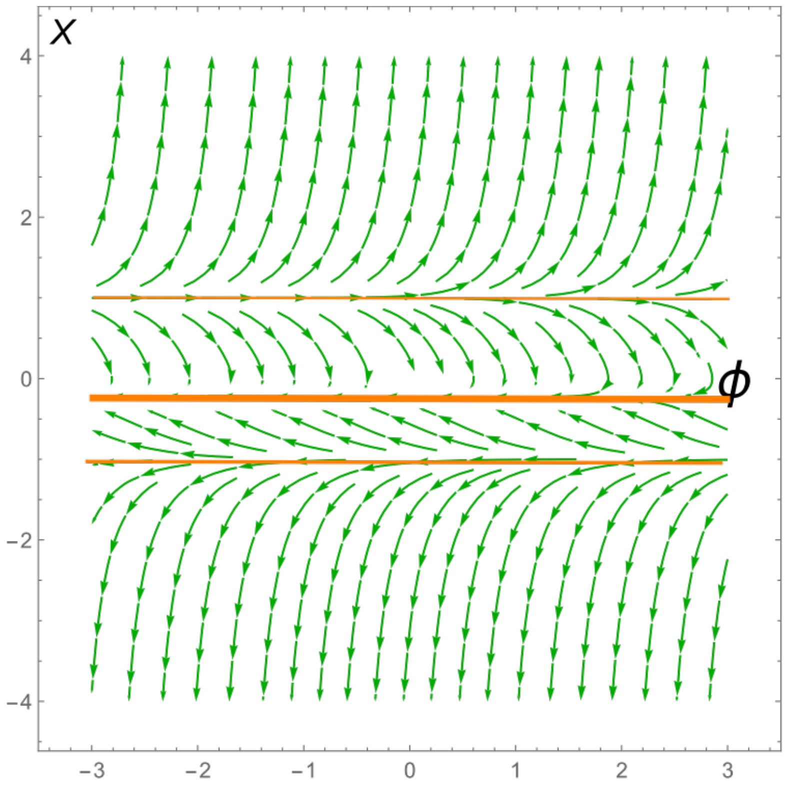

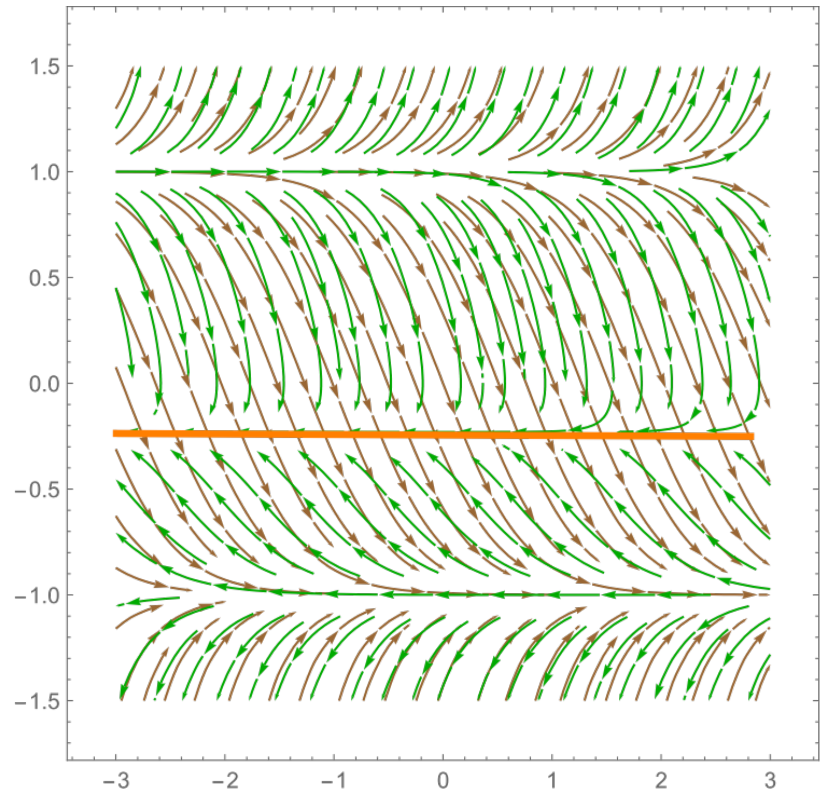

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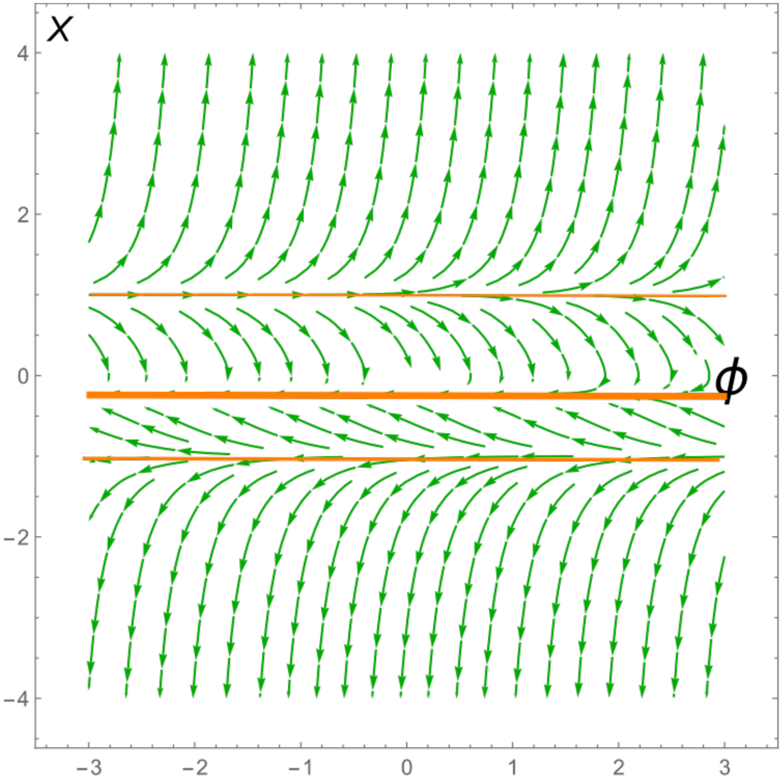
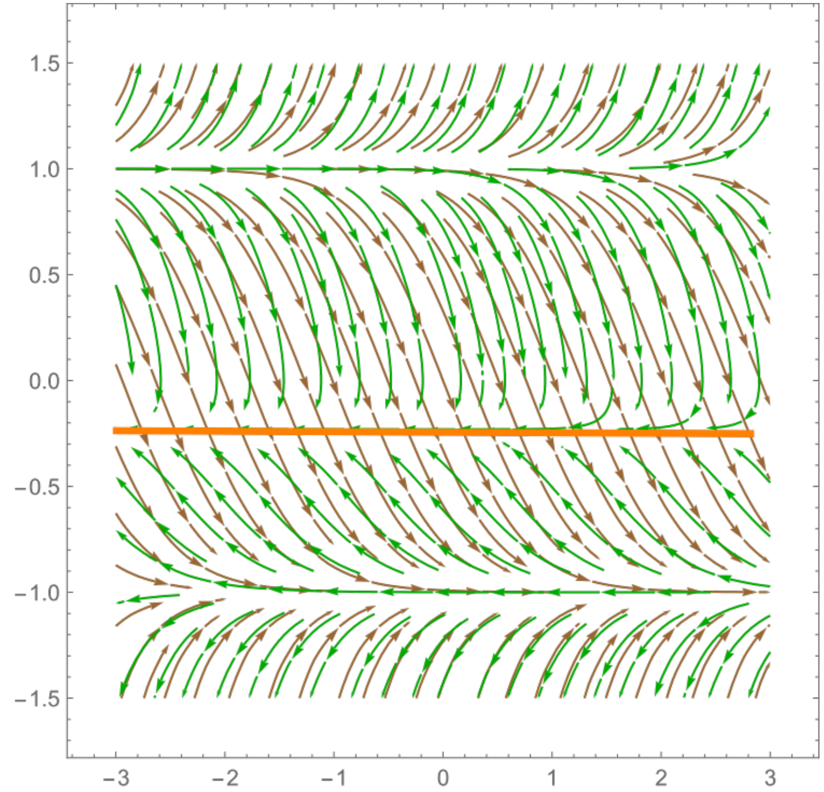


NEW



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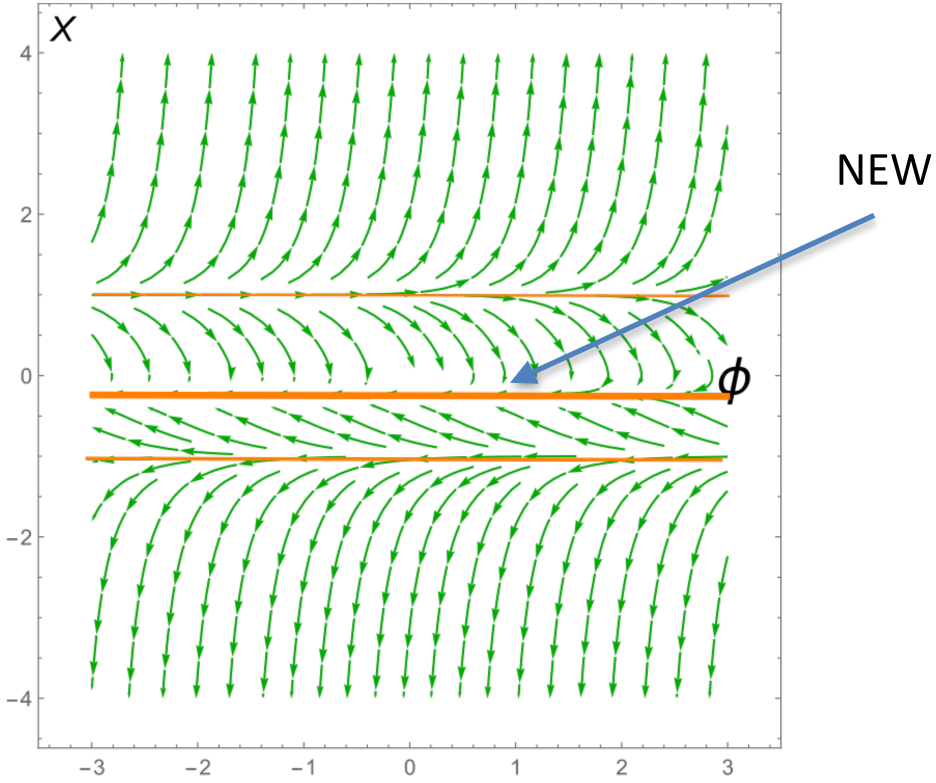

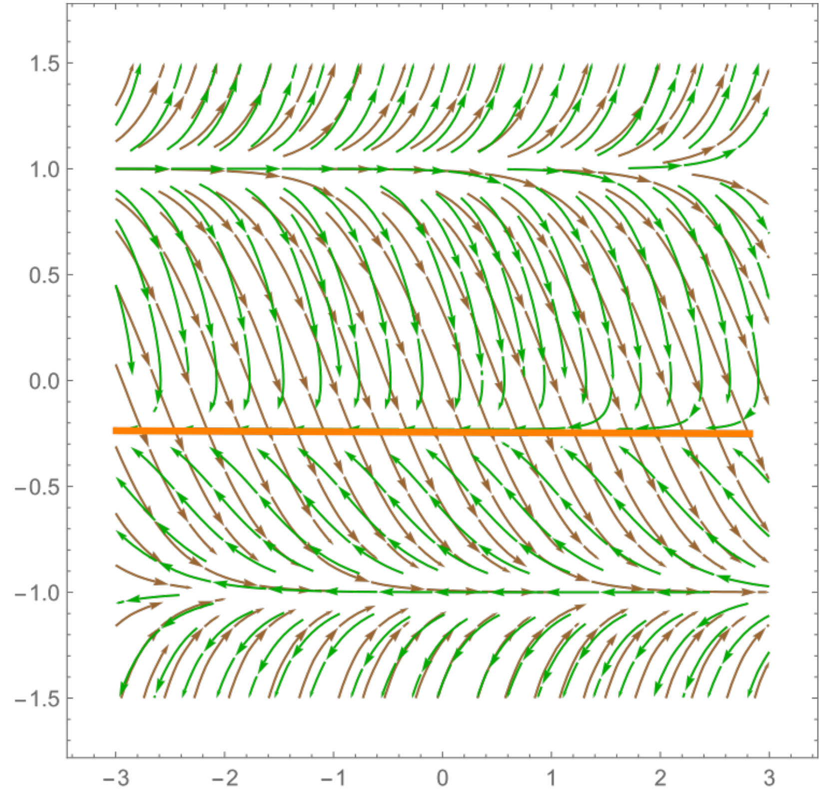

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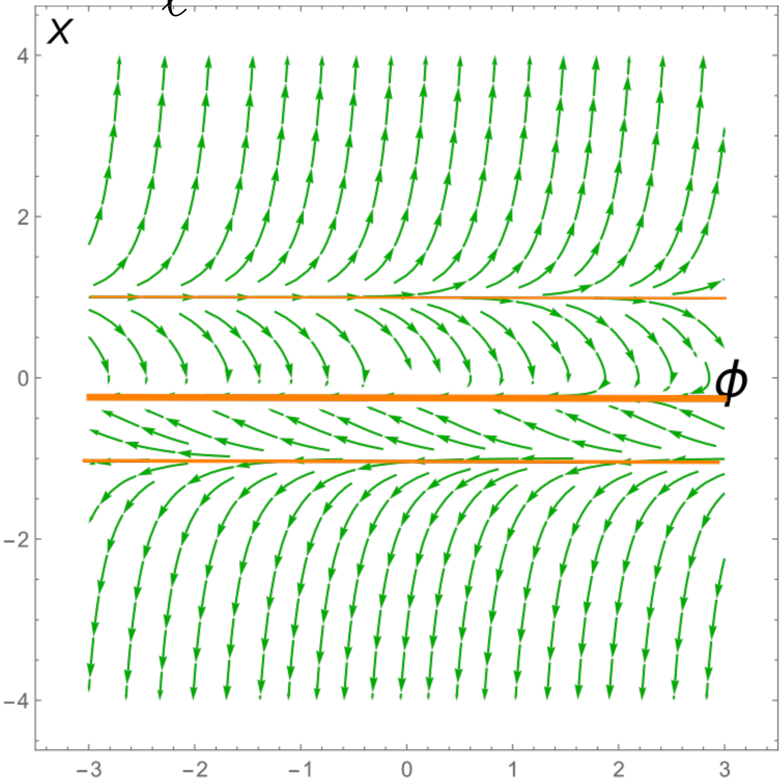

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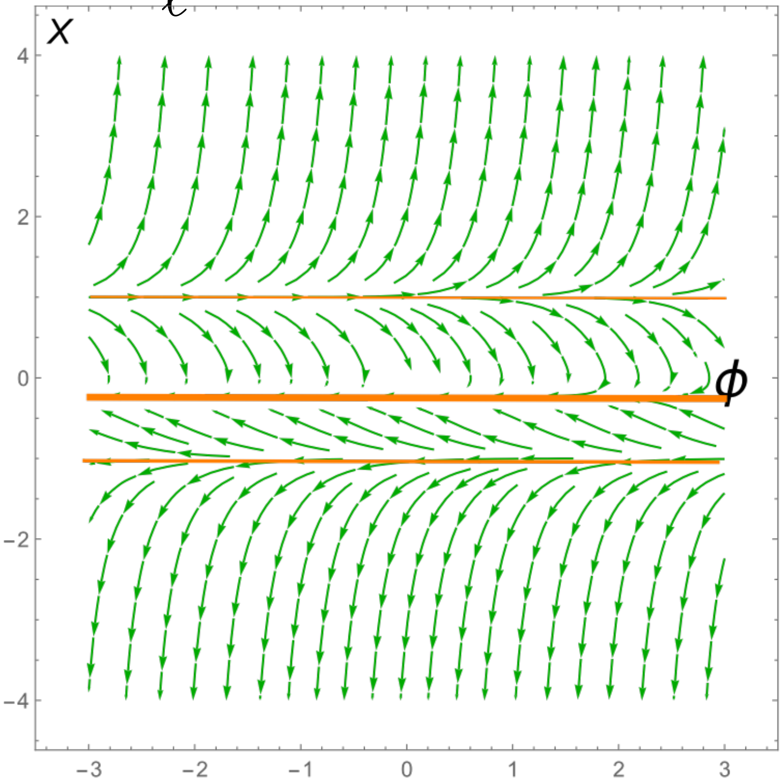

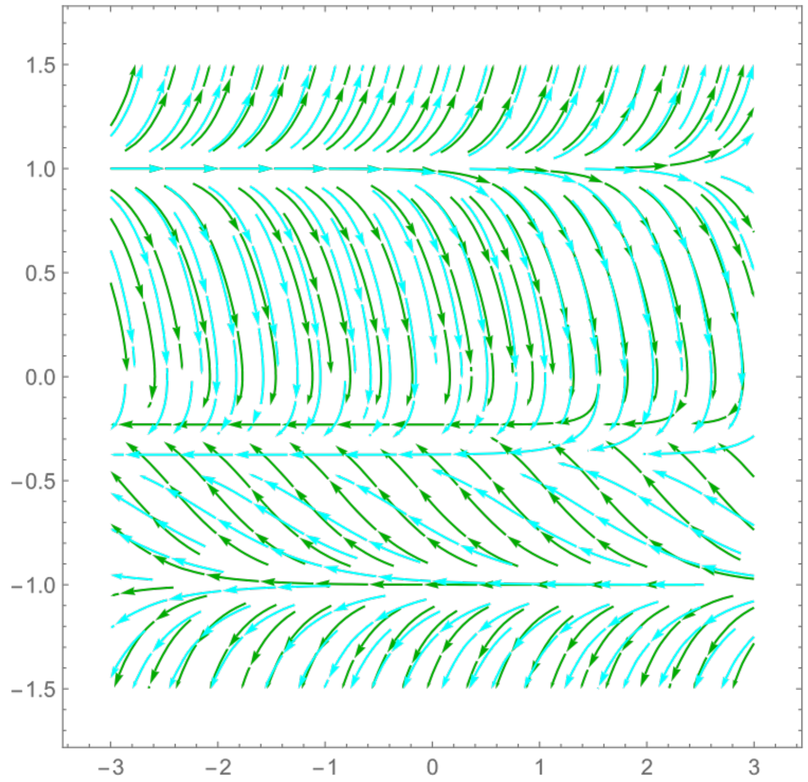

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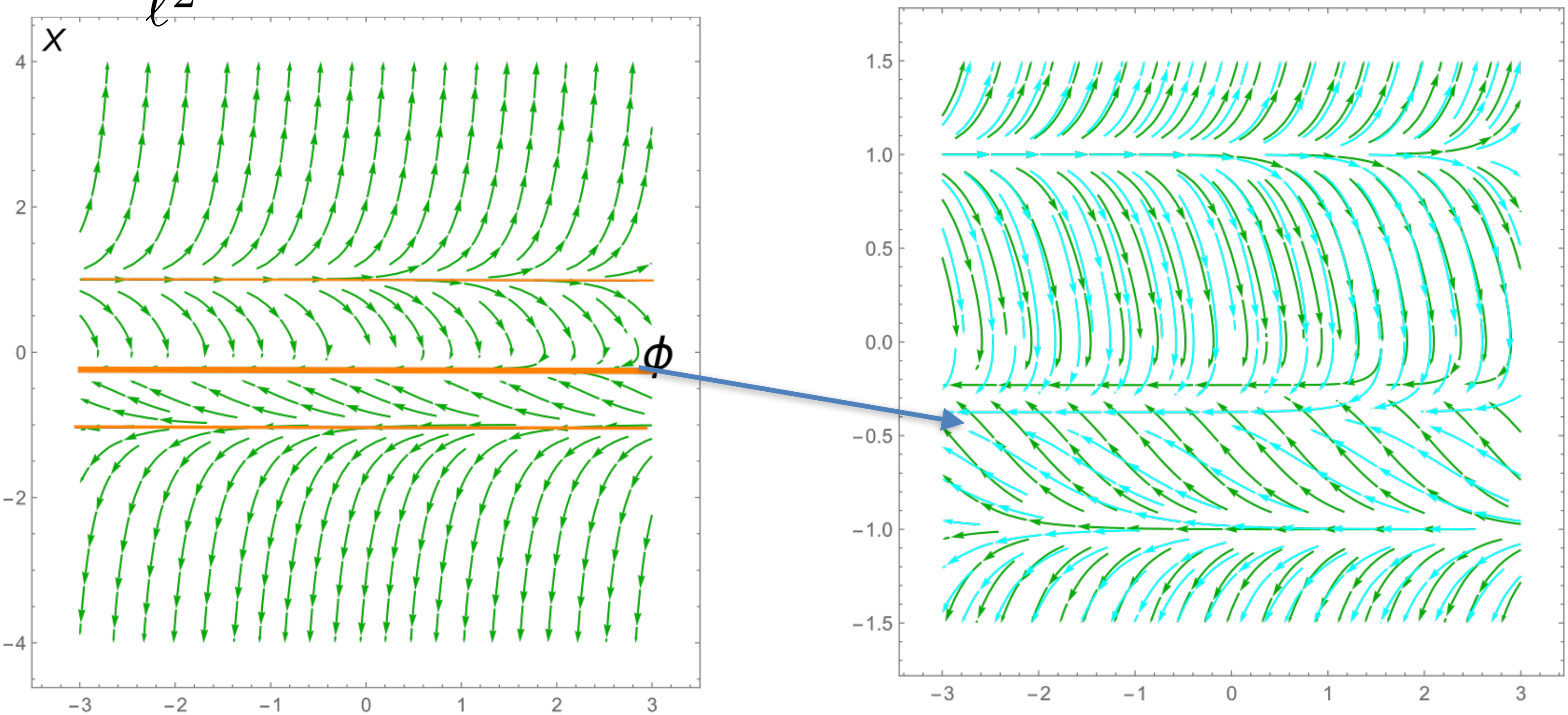

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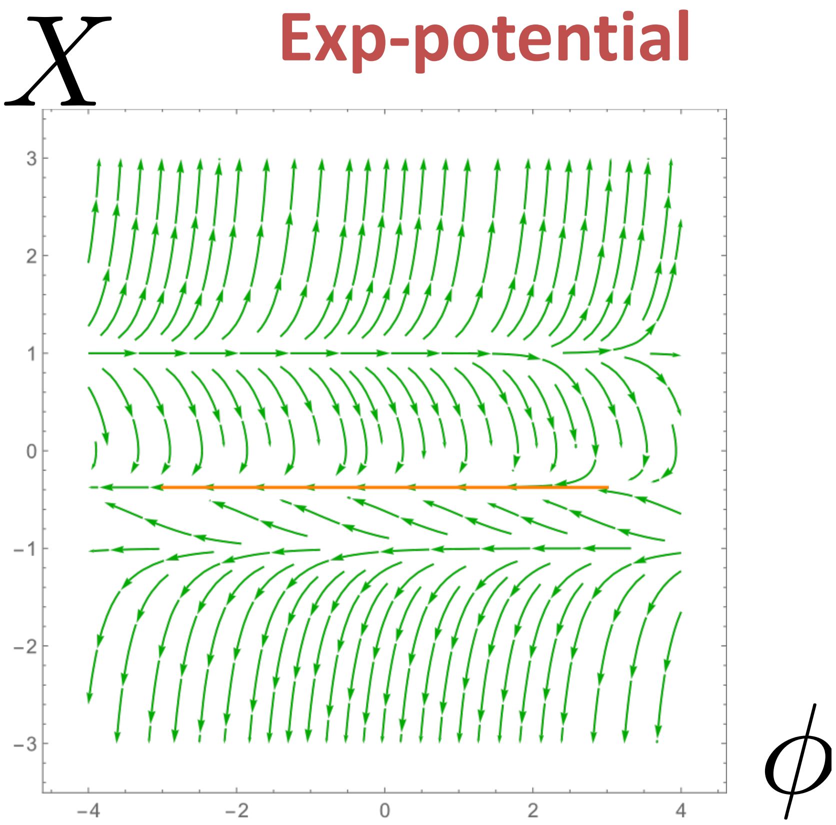
Chamblin-Reall-model, 99



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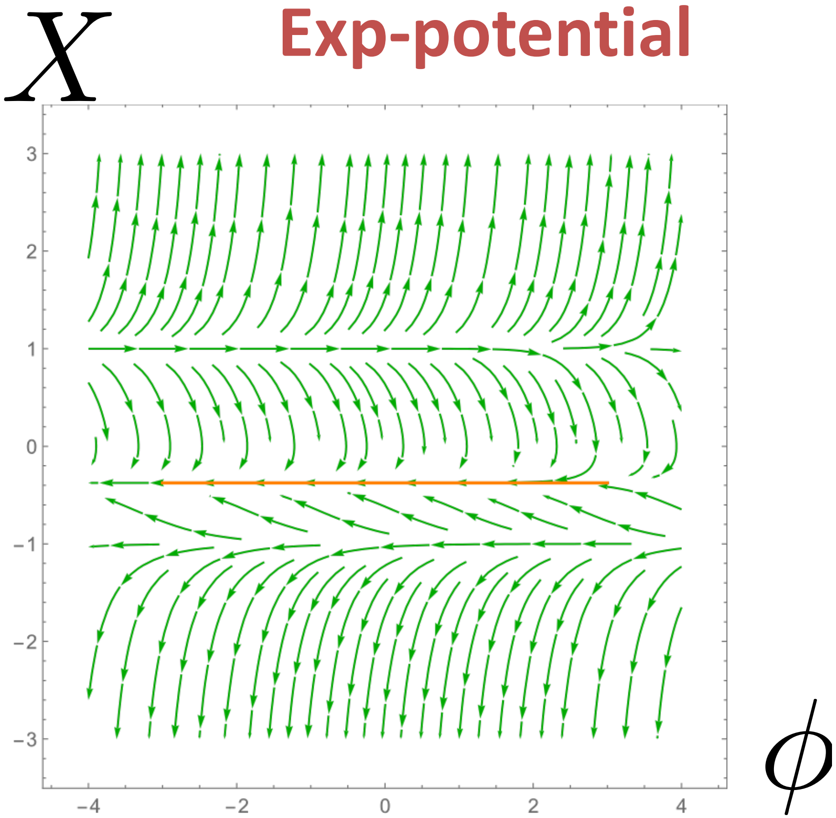
Exp-potential



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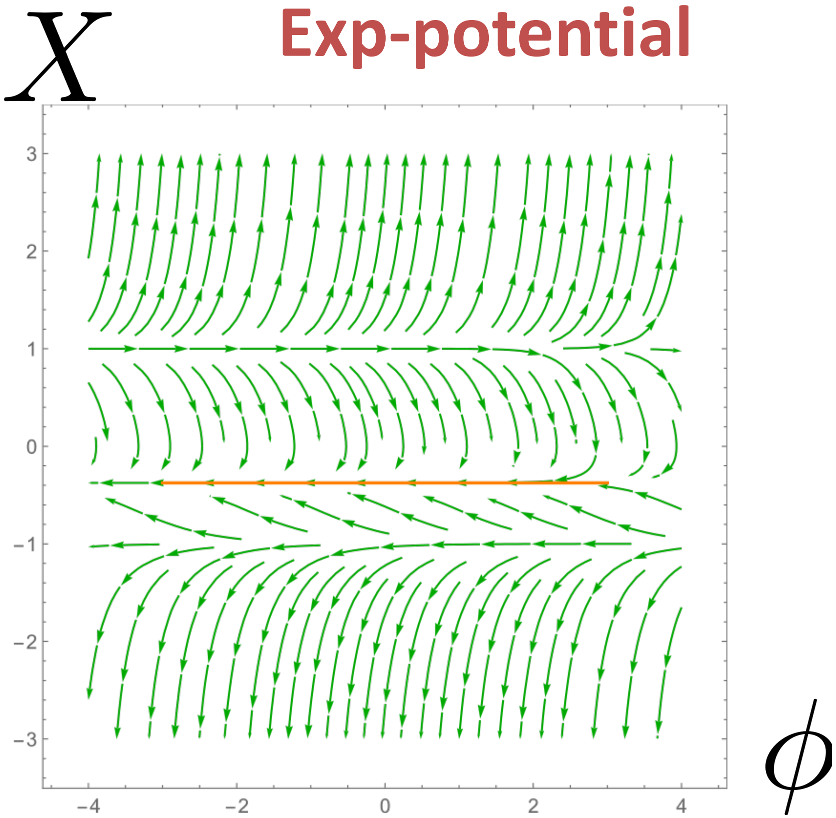

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$$W(\phi) = 4e^{-\frac{4x\phi}{3}}$$

$$\frac{1}{16}W^2 - \frac{9}{256}W'^2 = V$$

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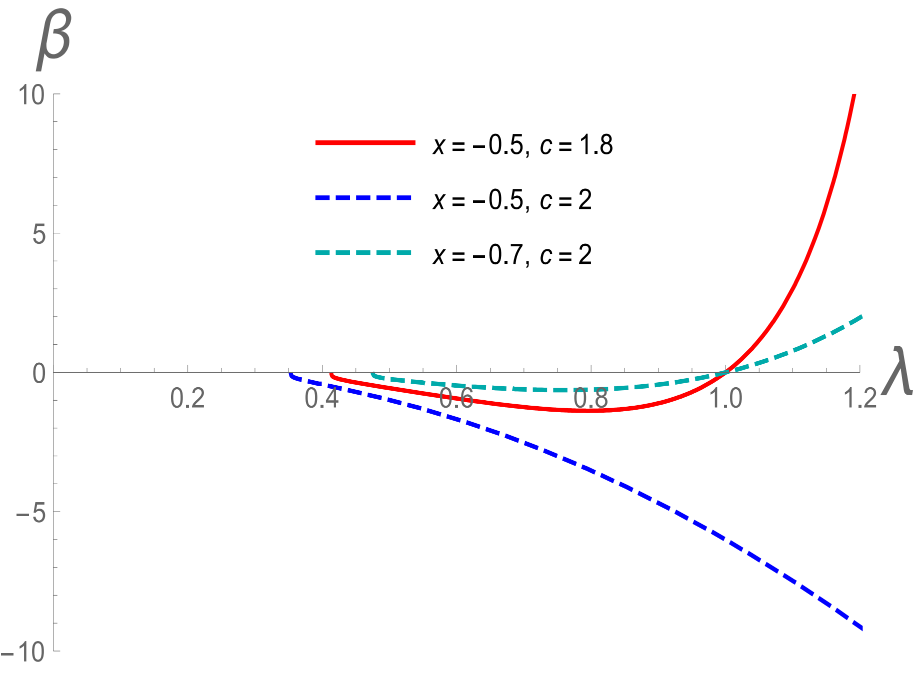

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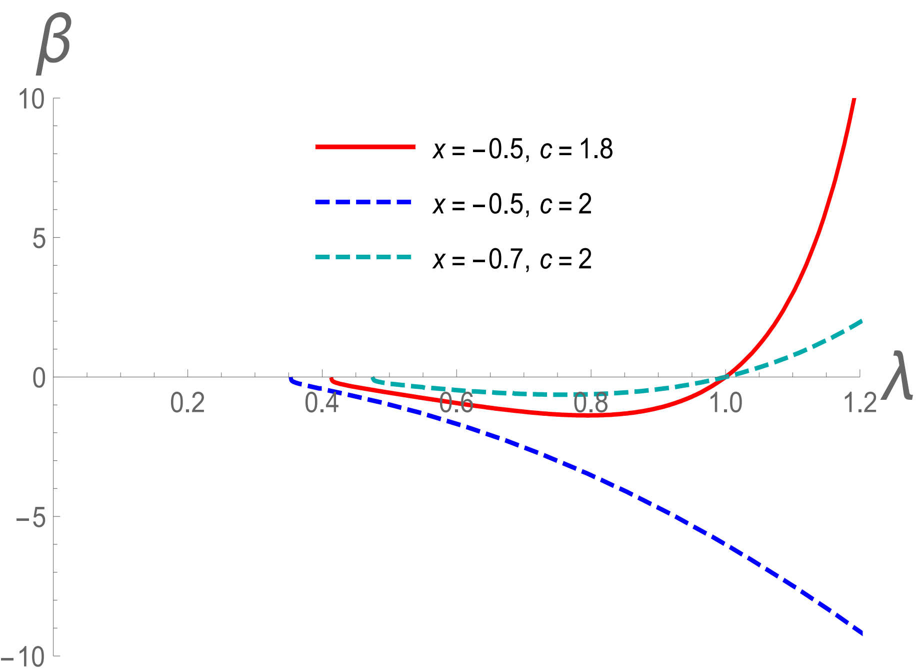

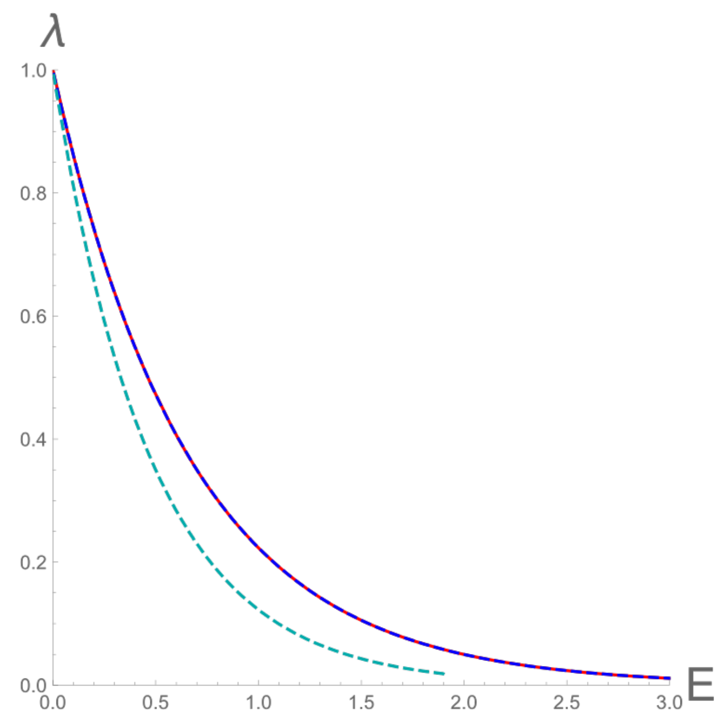

Two Exp-potential provides an essentially more rich structure

I.A., A.Golubtsova and G. Policastro,

«Exact holographic RG flows and the A_1x A_1 Toda chain», arXiv: 1803.06764

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Two motivations:

1) Relation with realistic model

2) **Explicit solution, relation with group theory and possible generalizations**

Holographic RenormGroup Flow,T=0

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$$V_{AR}(\phi) = V_0 - C_1 e^{K_1 \phi} + C_2 e^{K_2 \phi}$$

$$V_0 = -0.6, \;\; K_1 = 0.8, \quad K_2(4.5) = 2.1$$
$$C_1 = 23, \;\; C_2 = 0.06$$

I.A., Rannu, arXiv: 1802.05652

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$$\varphi = 0.47\phi \qquad k = 0.85$$

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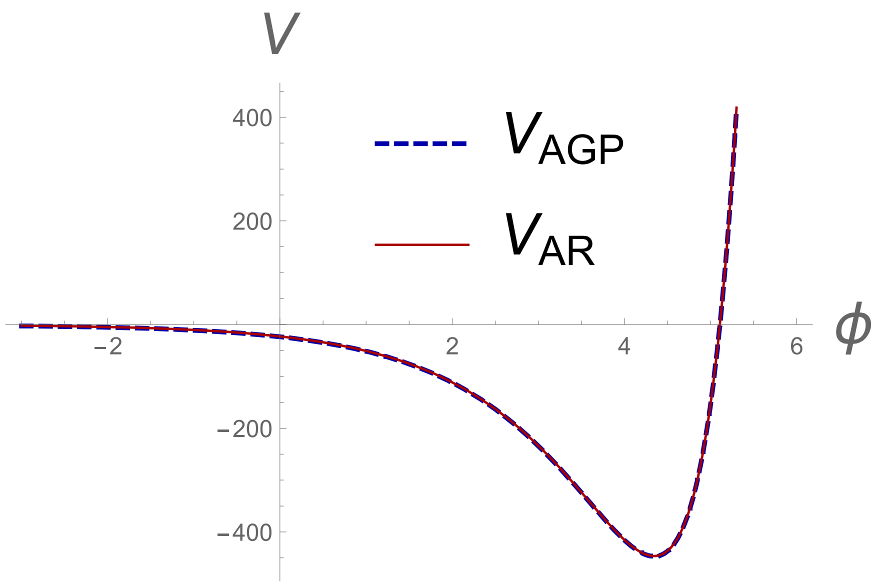

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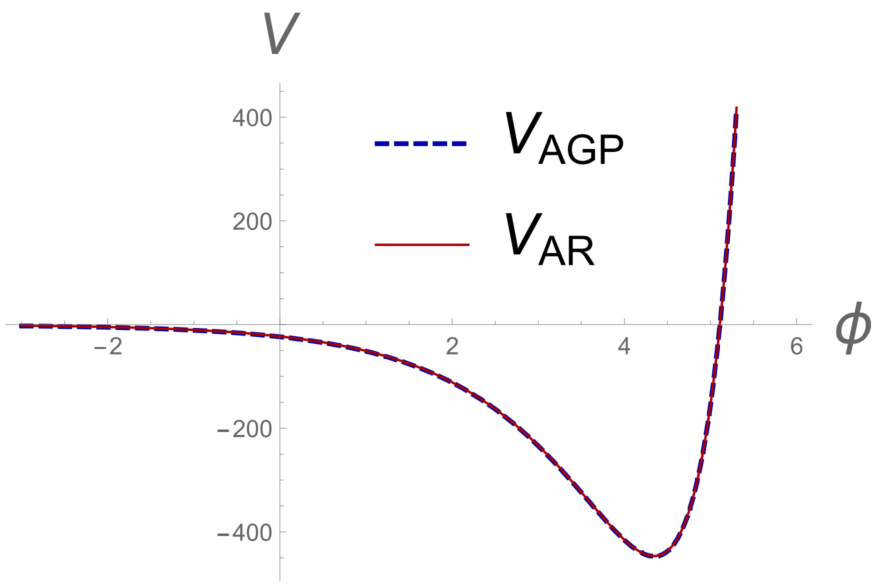

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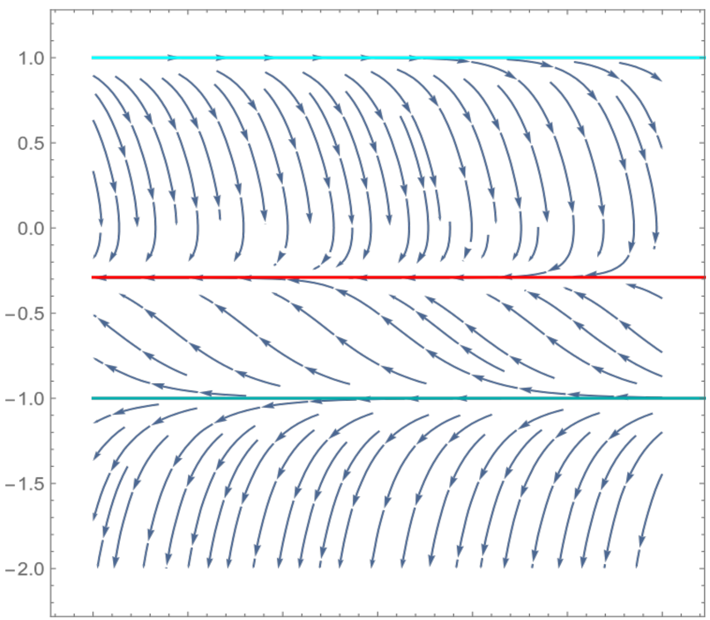
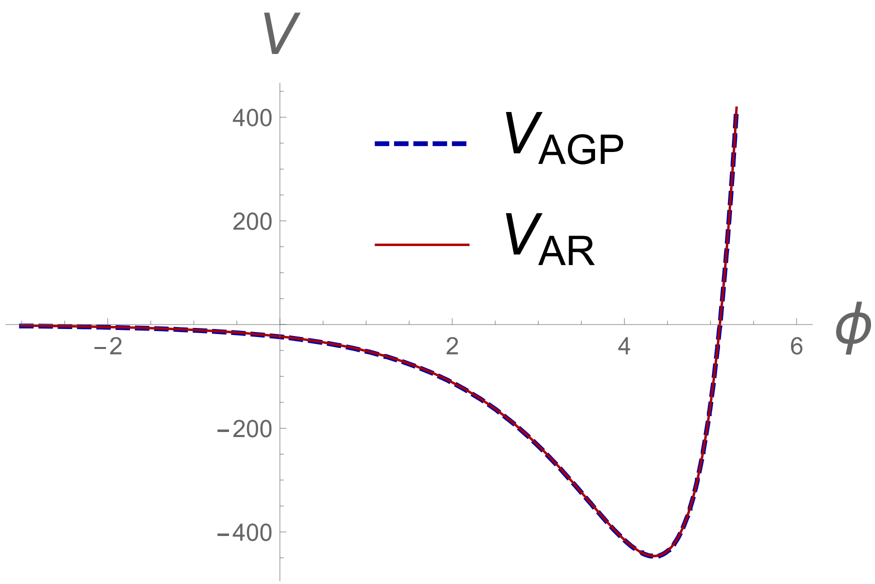


Holographic RenormGroup Flow,T=0



$$V_{AR}(\phi) = V_0 - C_1 e^{K_1 \phi} + C_2 e^{K_2 \phi}$$

$$V_0 = -0.6, \ K_1 = 0.8, \quad K_2(4.5) = 2.1$$
$$C_1 = 23, \ C_2 = 0.06$$

I.A., Rannu, arXiv: 1802.05652

I.A.,Golubtsova, Policastro,arXiv: 1803.06764

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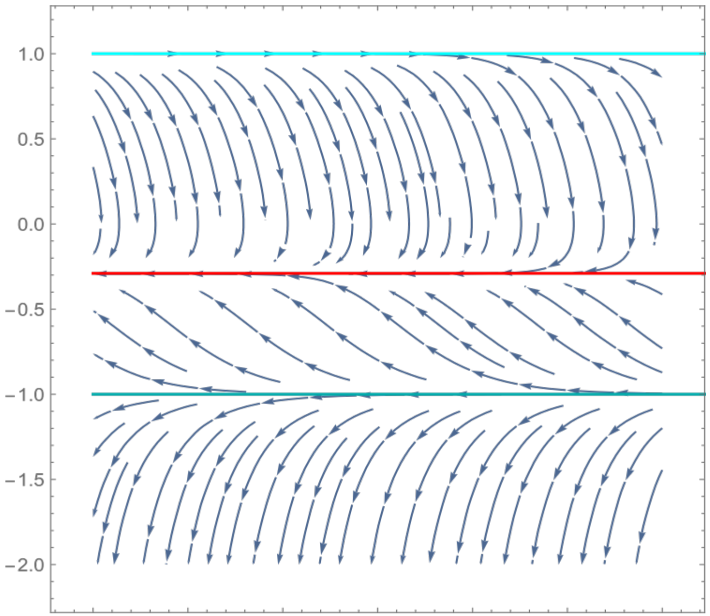

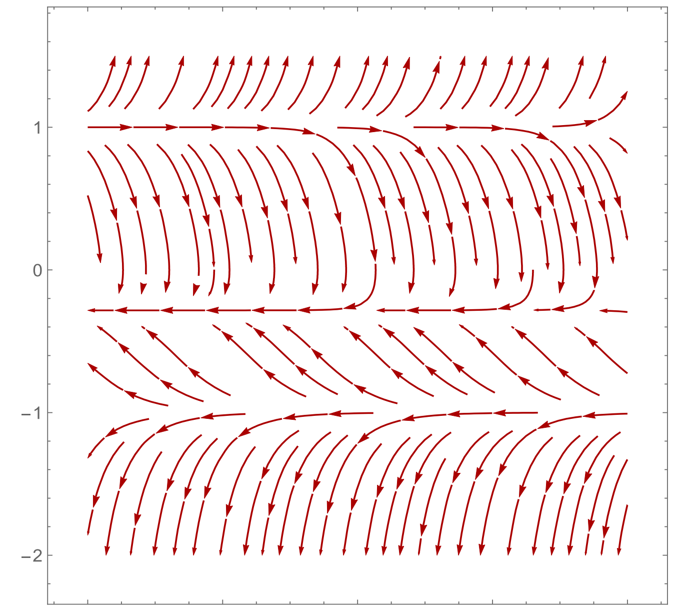

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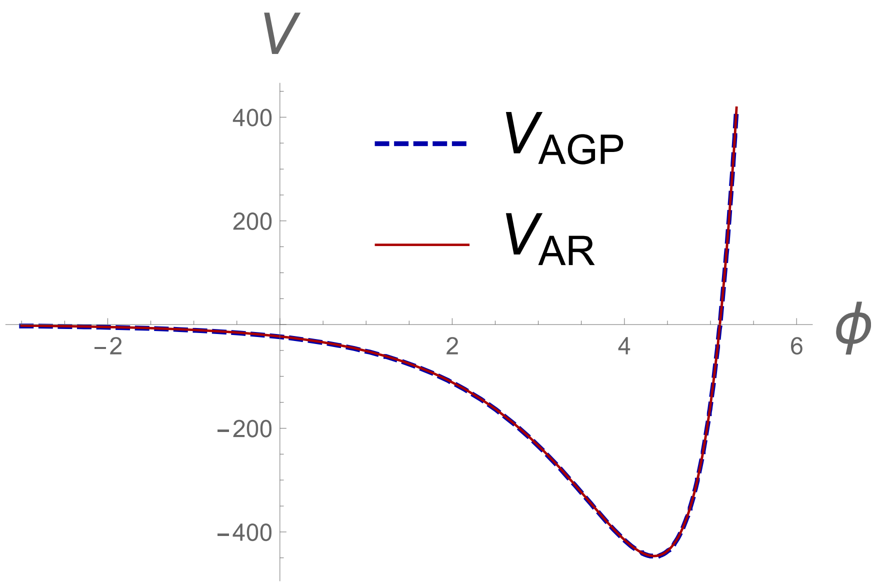

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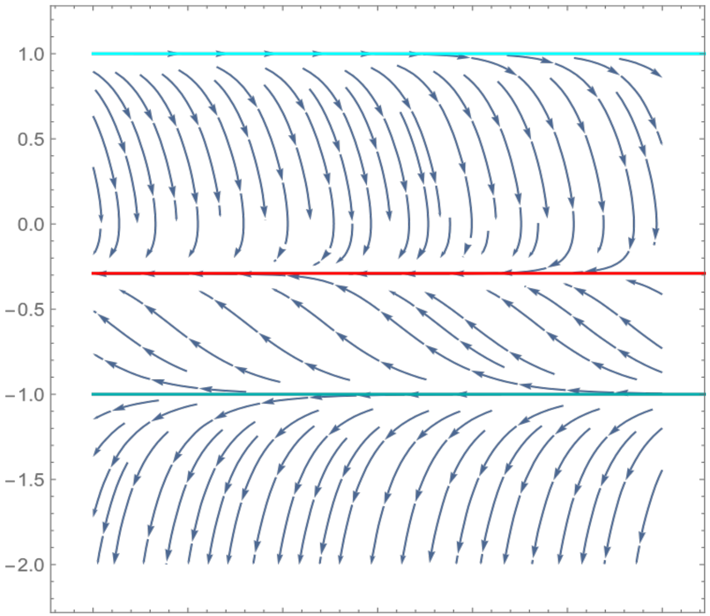
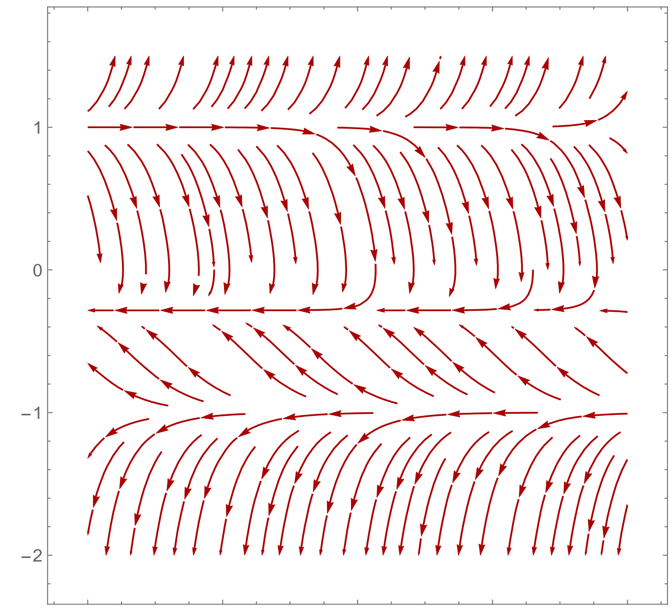


V_{AR}



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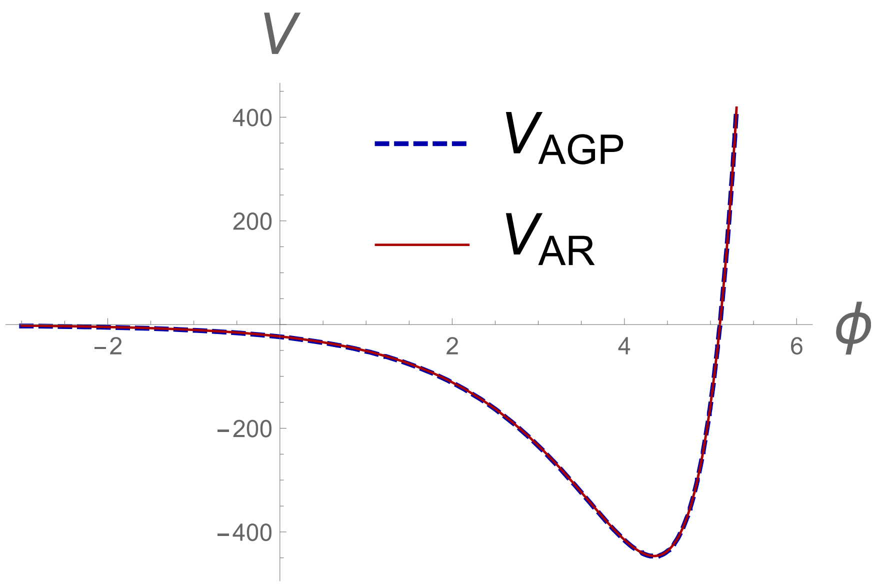

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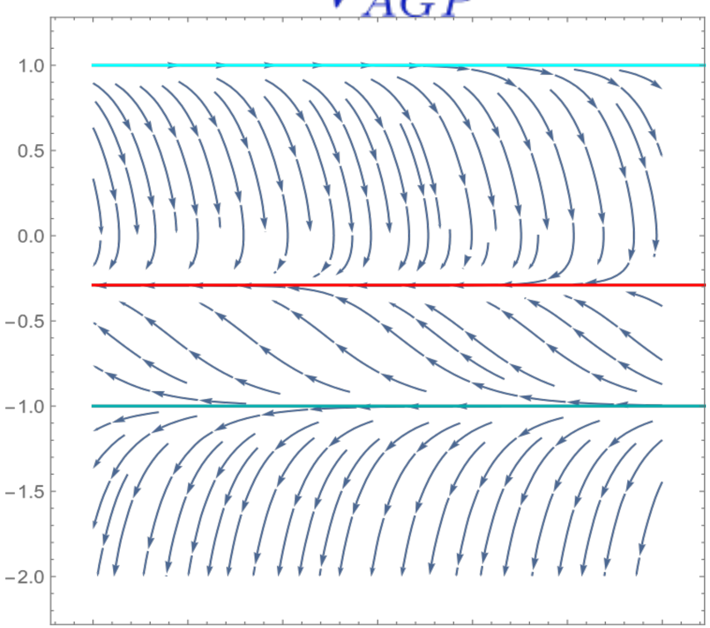
V_{AGP}



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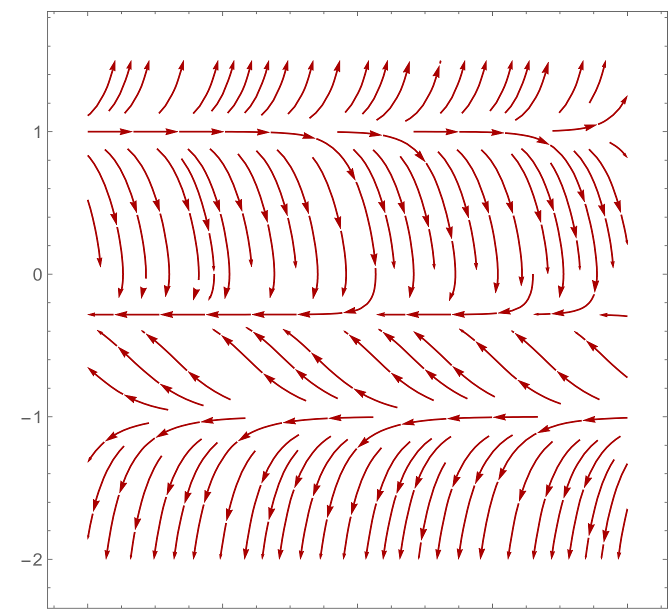

Holographic RG Flow

$$V_{AGP}(\varphi) = C_1 e^{2k\varphi} + C_2 e^{\frac{32}{9k}\varphi}$$

Generalization of Chamblin&Reall model, hep/th9903225

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2\right)\left(1 + \frac{3}{8}\frac{1}{X}\frac{V'}{V}\right)$$

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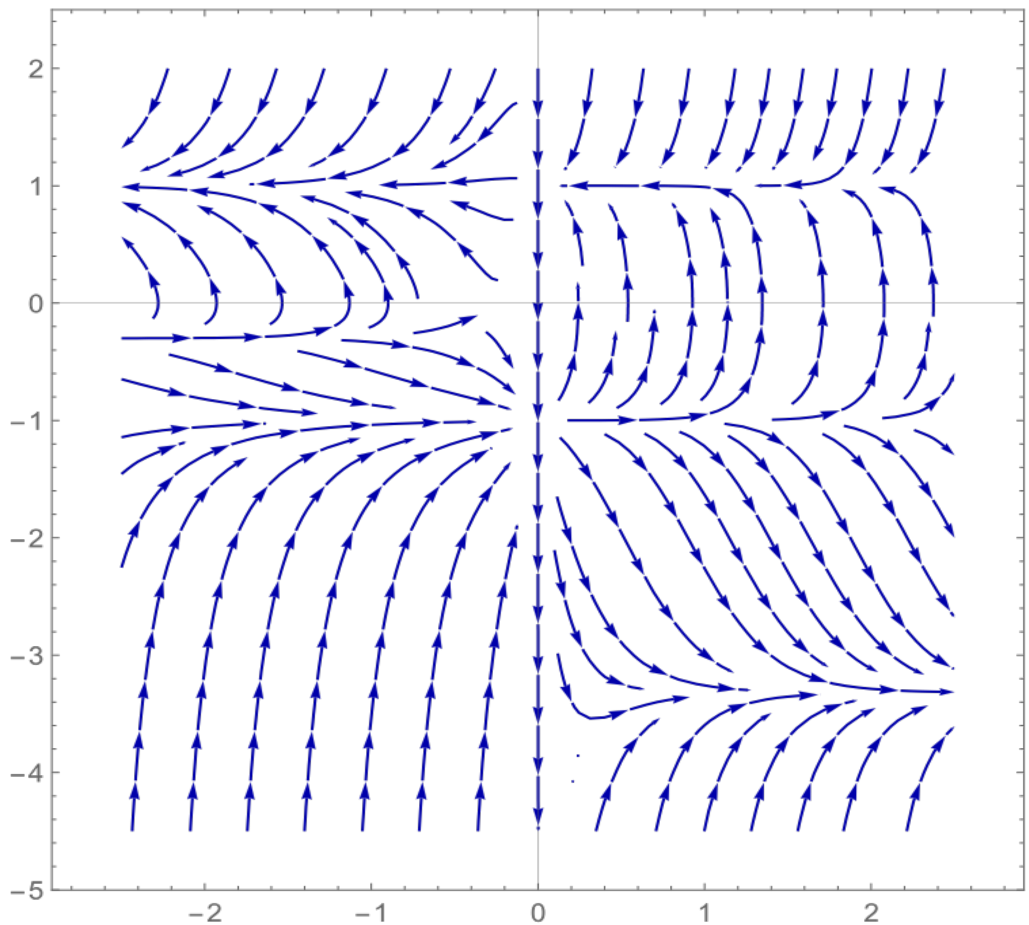

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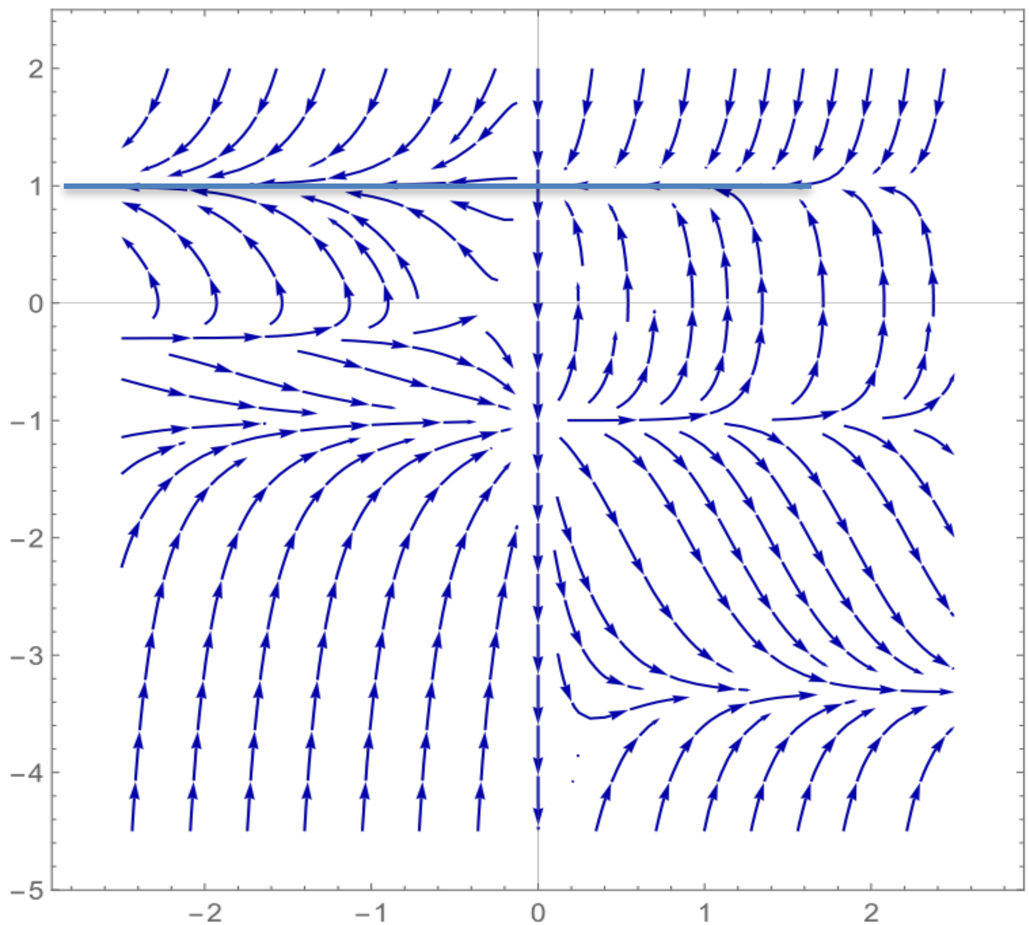

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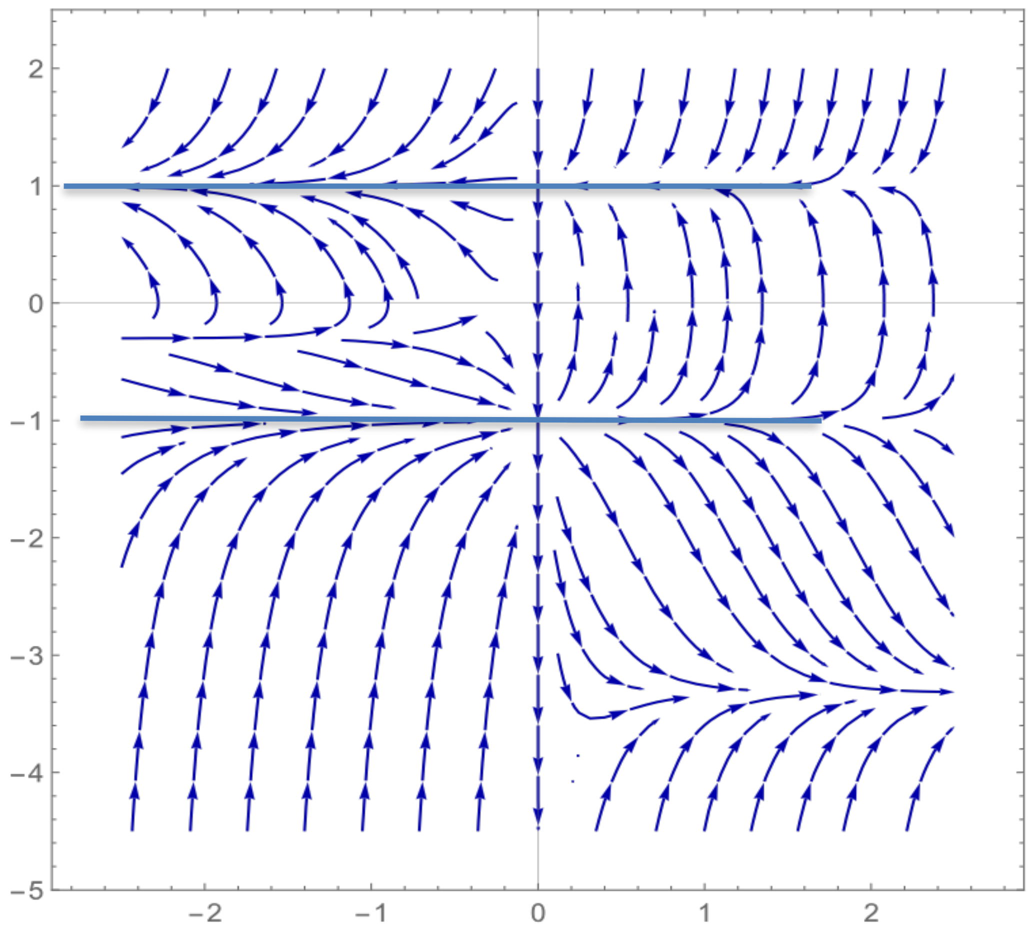

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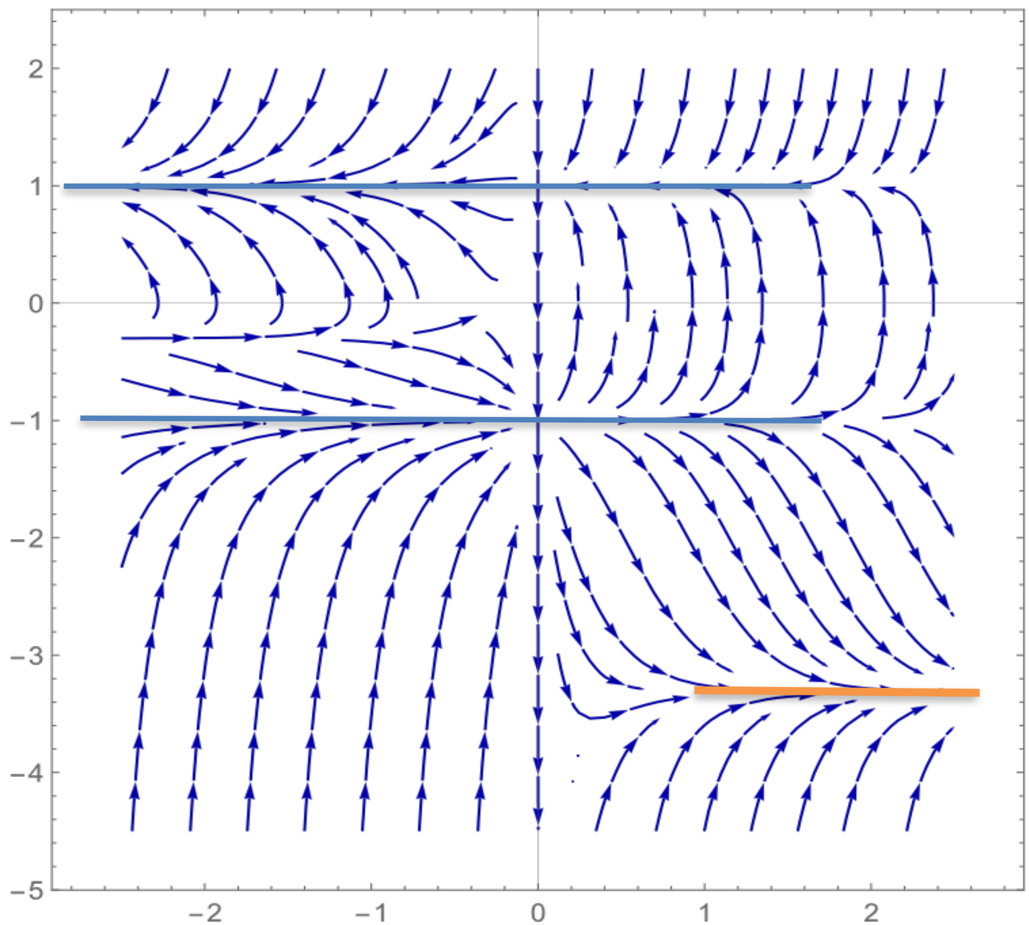

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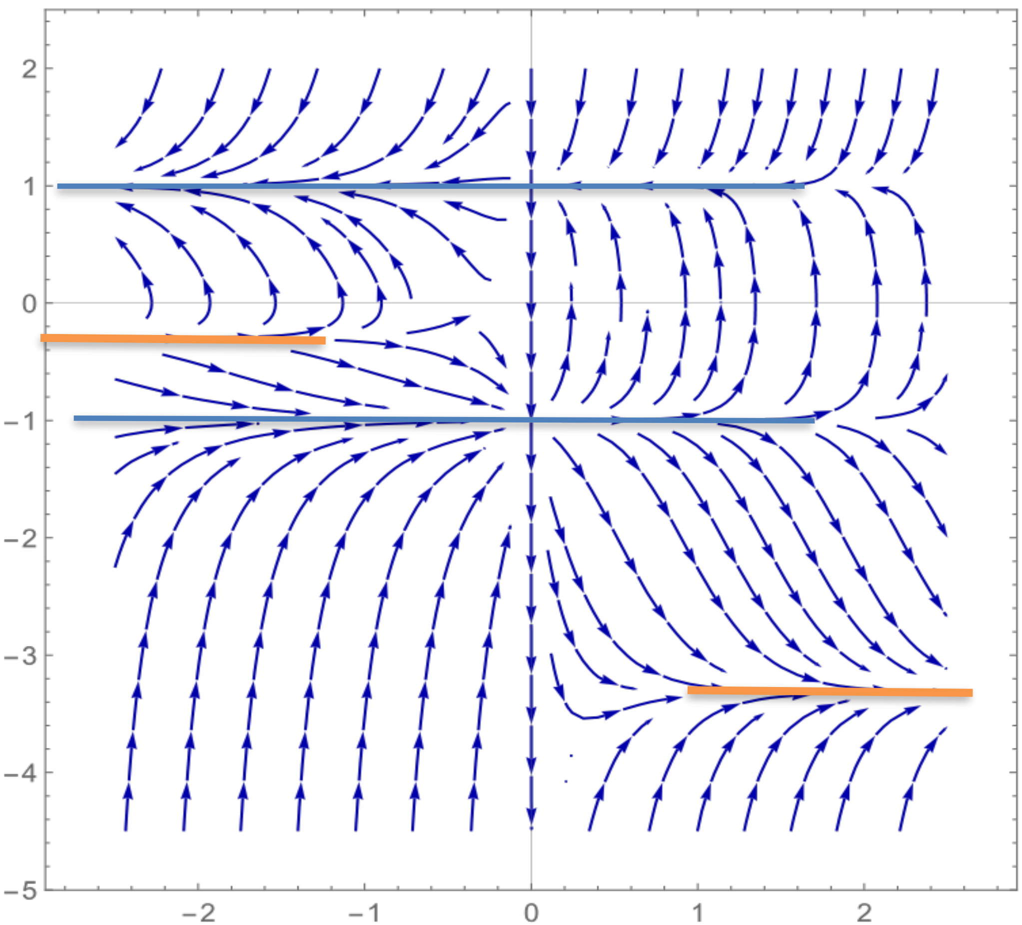

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The solution for the metric and the dilaton

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2$$

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Explicit answer due to integration of the mechanical model (A_1xA_1 Toda model)

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$$s = 1,2, \quad \mu_1 = \sqrt{|\frac{3E_1}{2}(k^2 - \frac{16}{9})|}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\left(\left(\frac{16}{9}\right)^2 \frac{1}{k^2} - \frac{16}{9}\right)\right|}$$

The solution for the dilaton - 3 regions!

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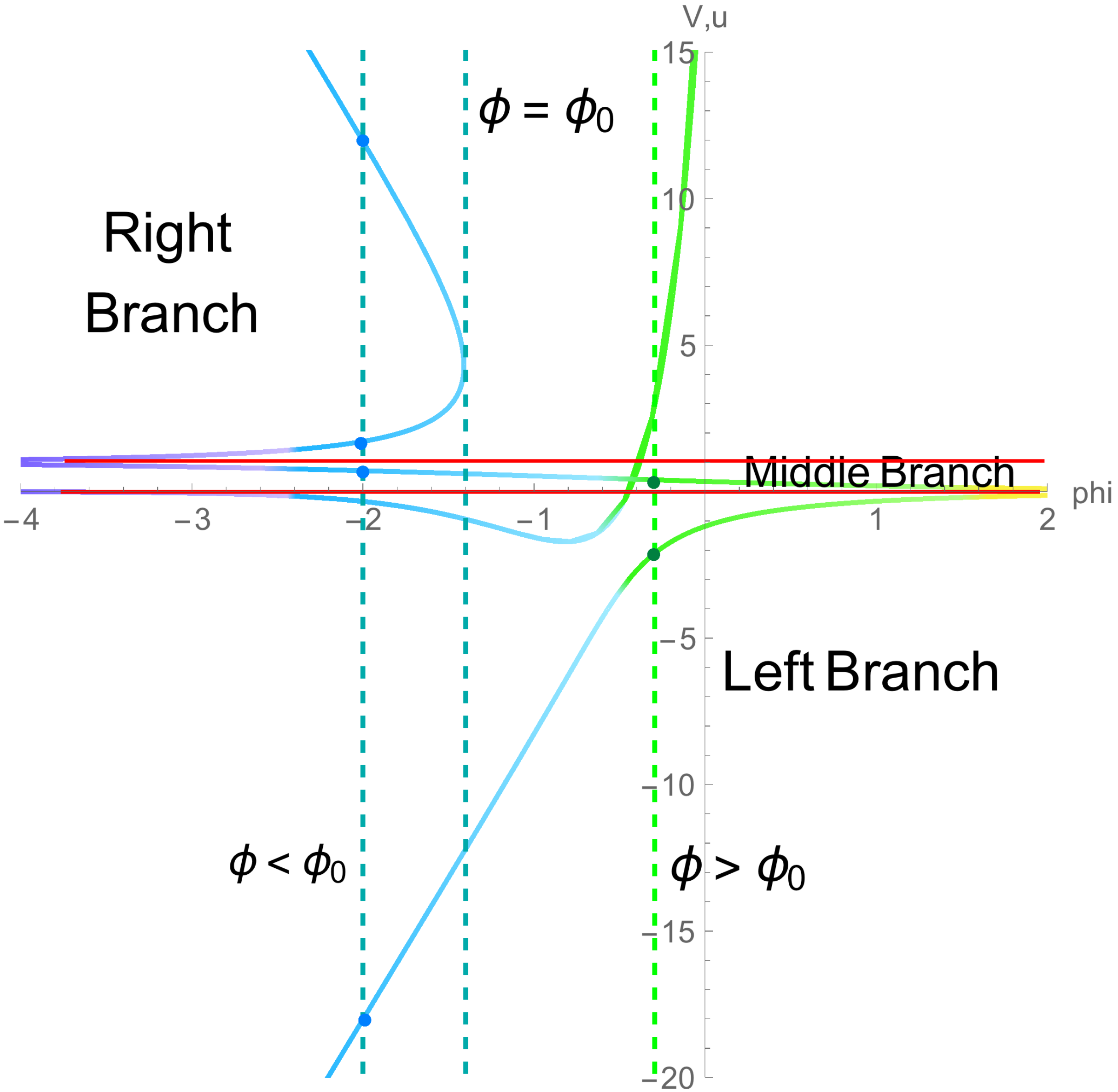

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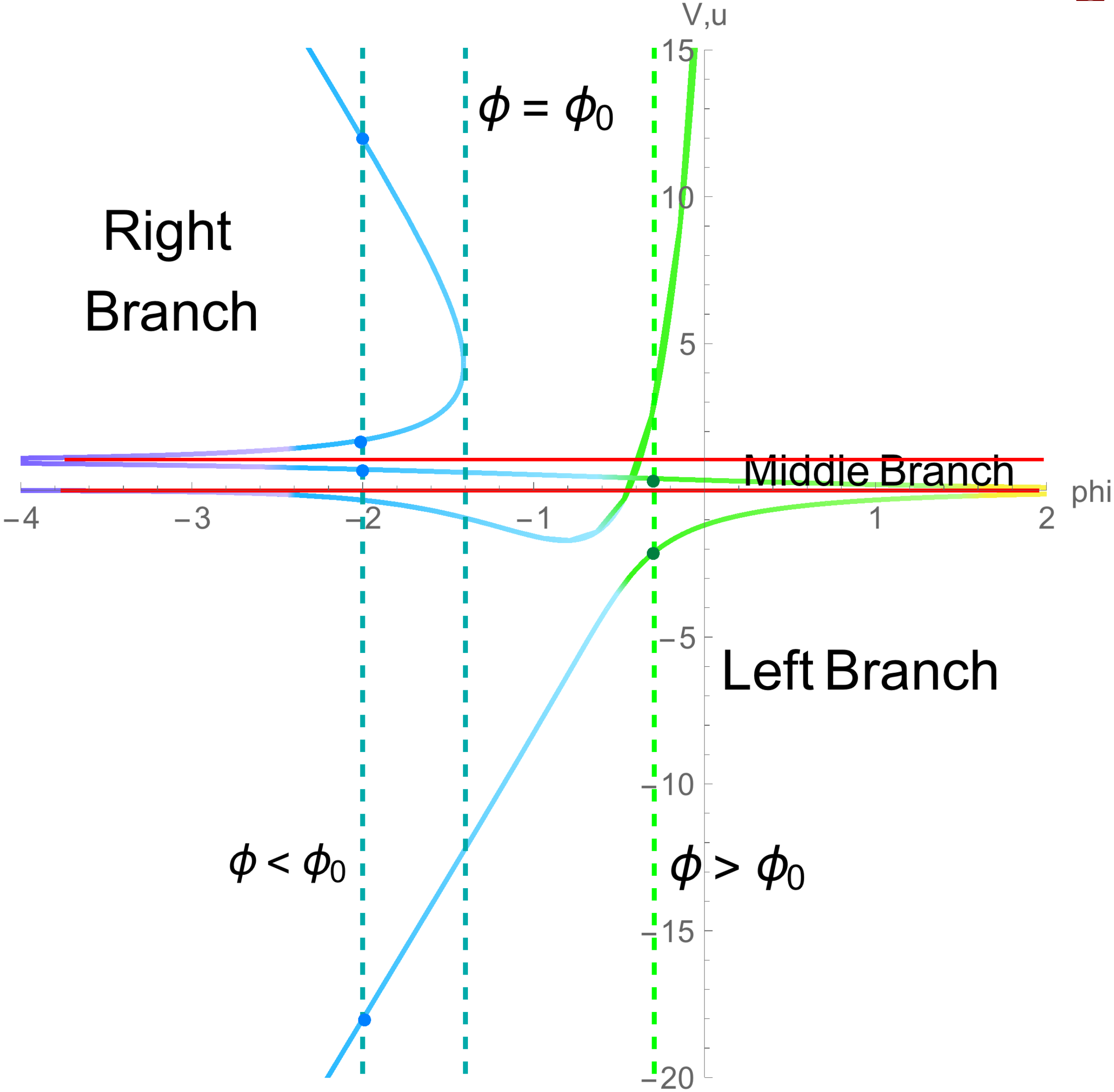

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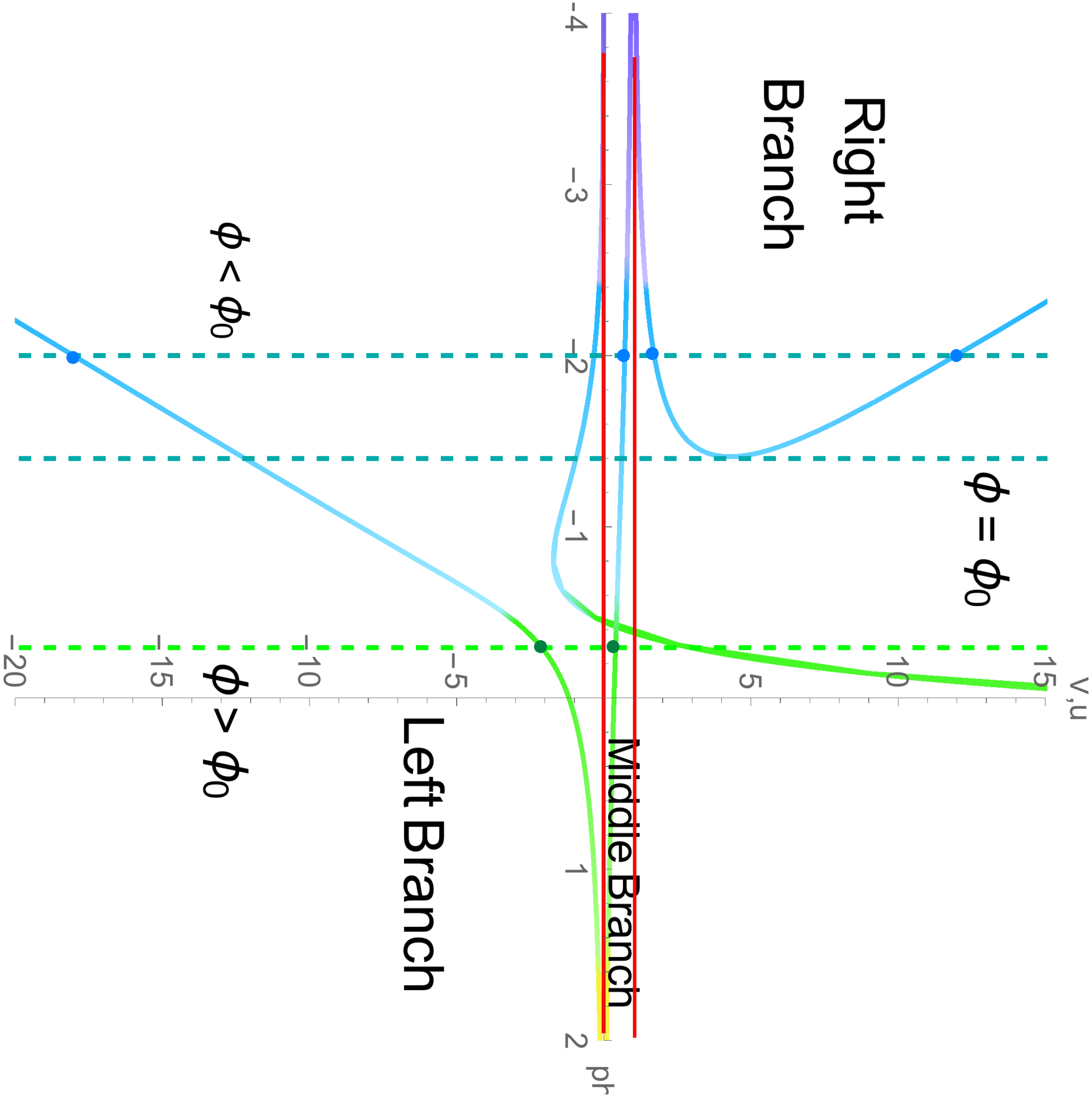

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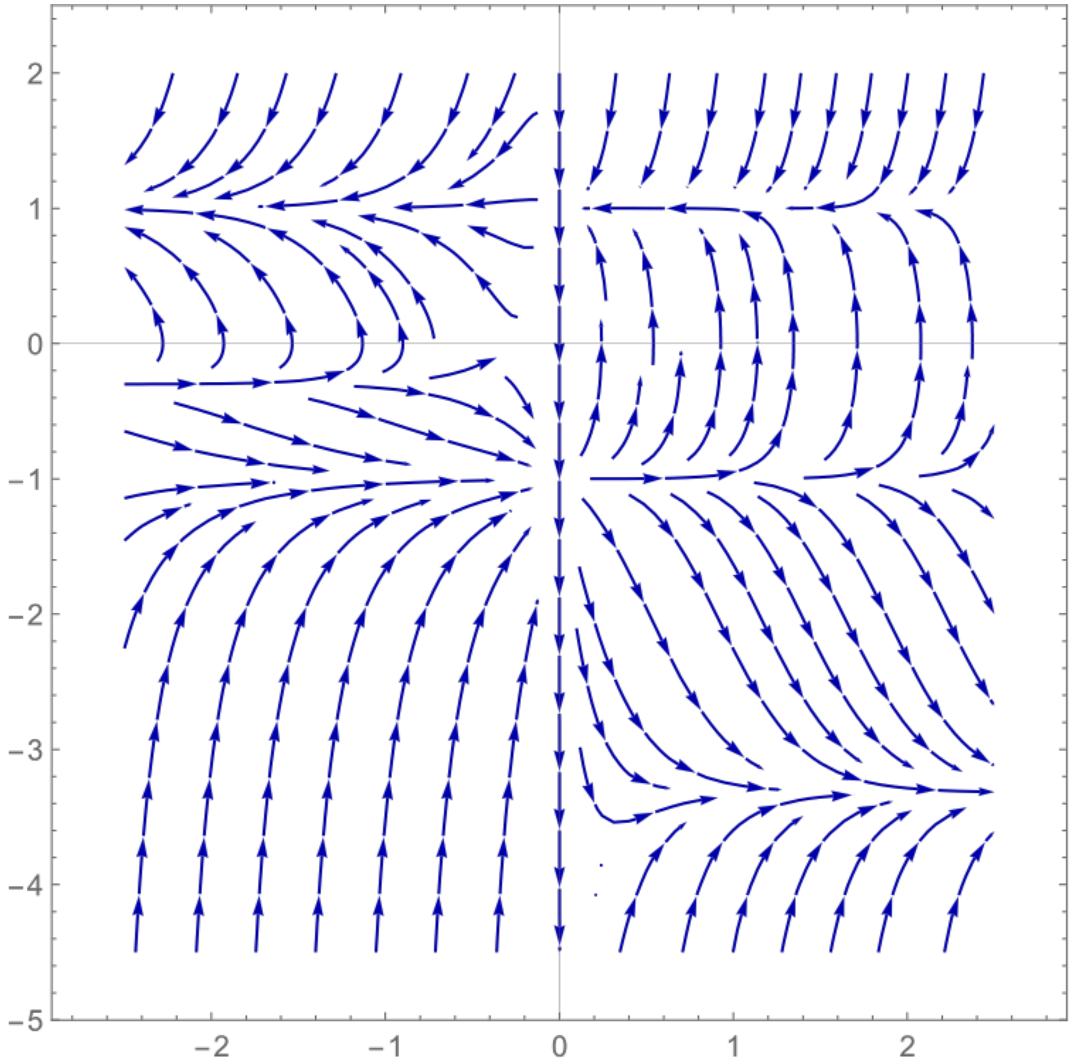

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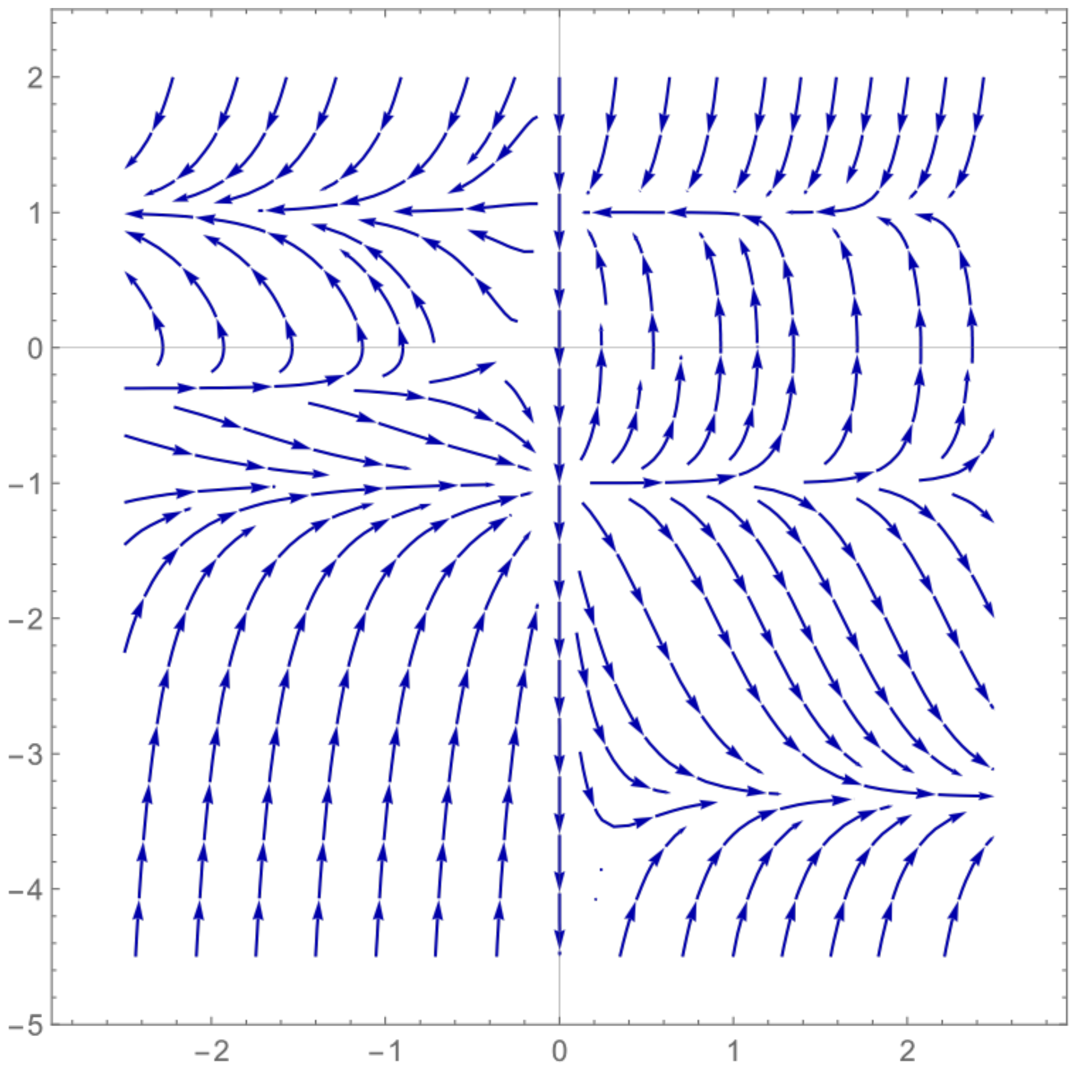

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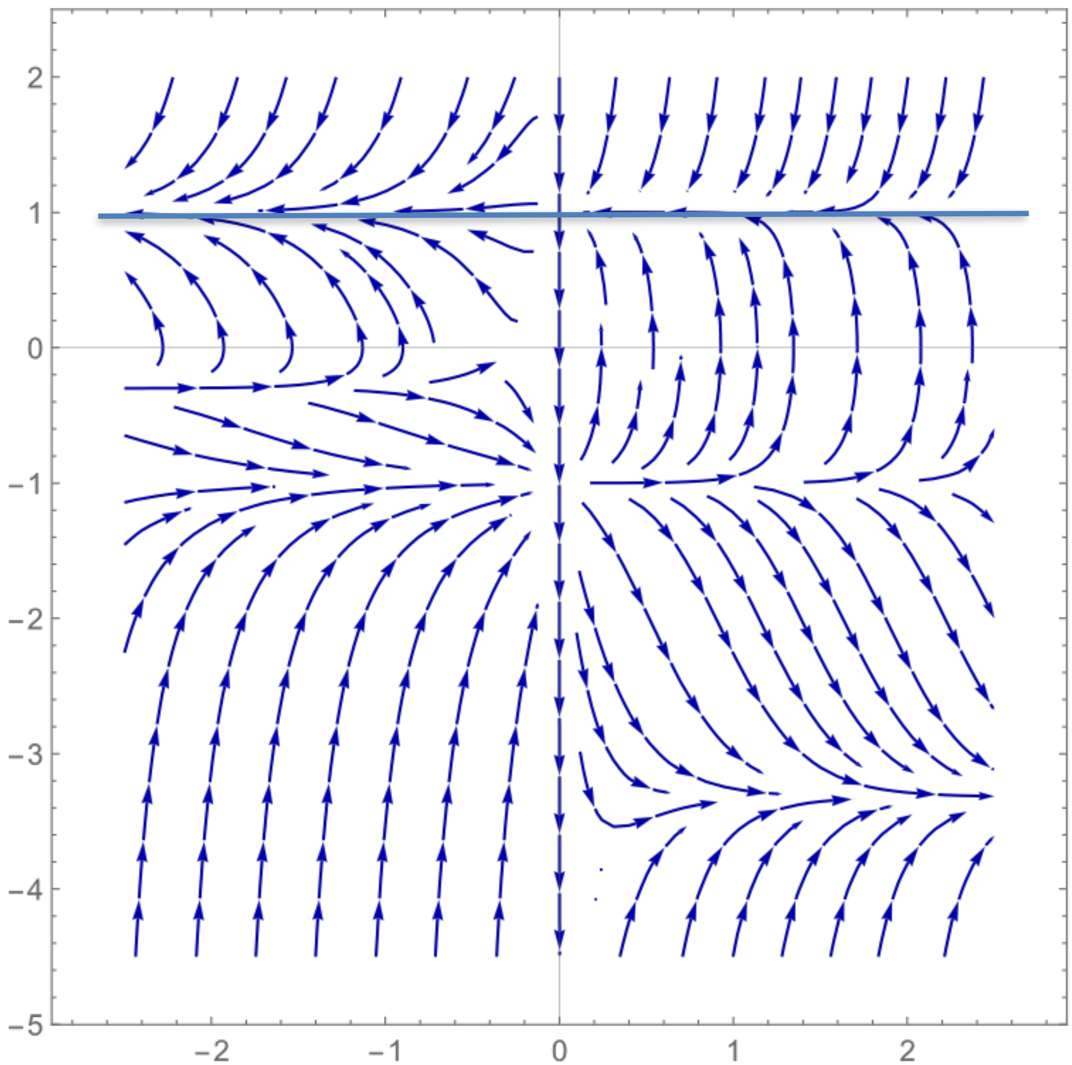

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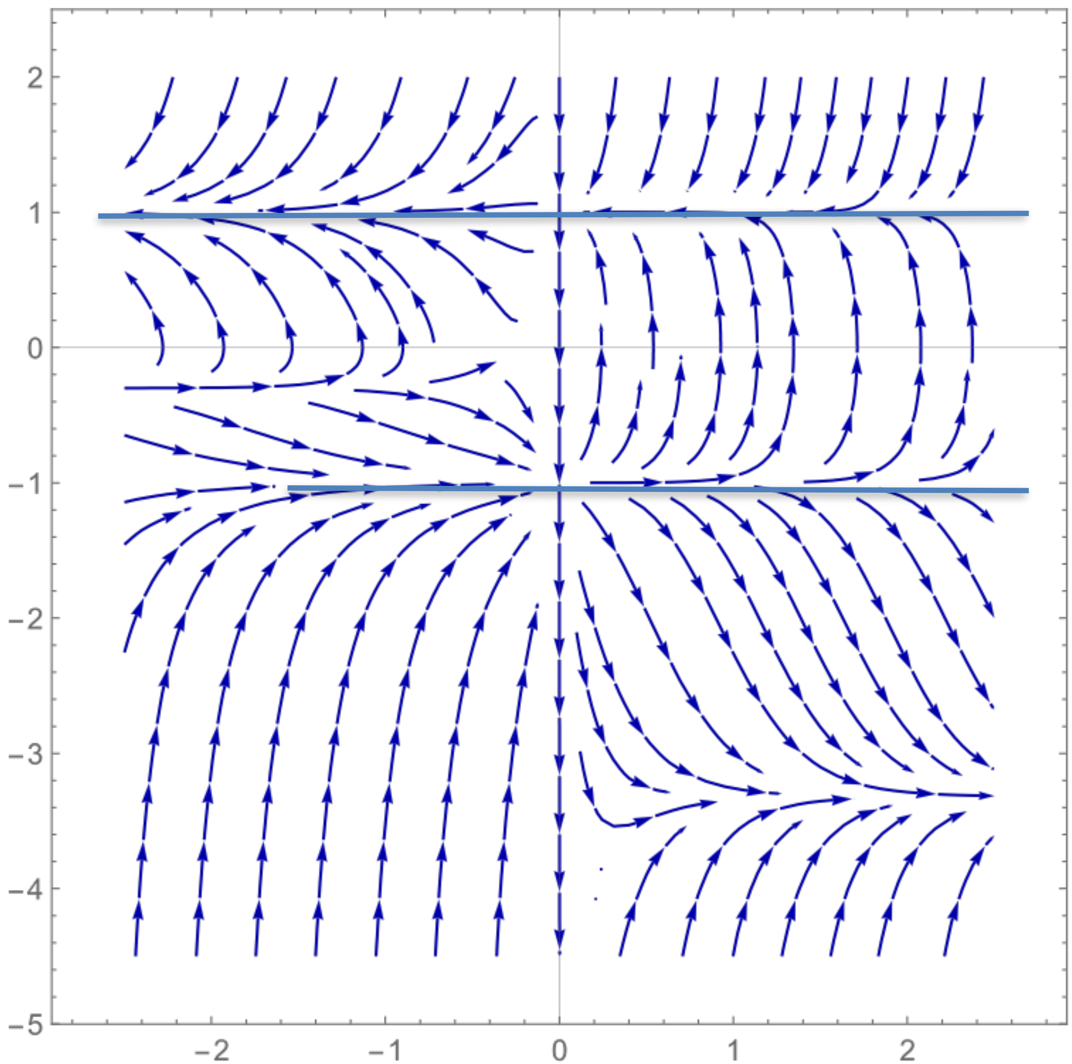

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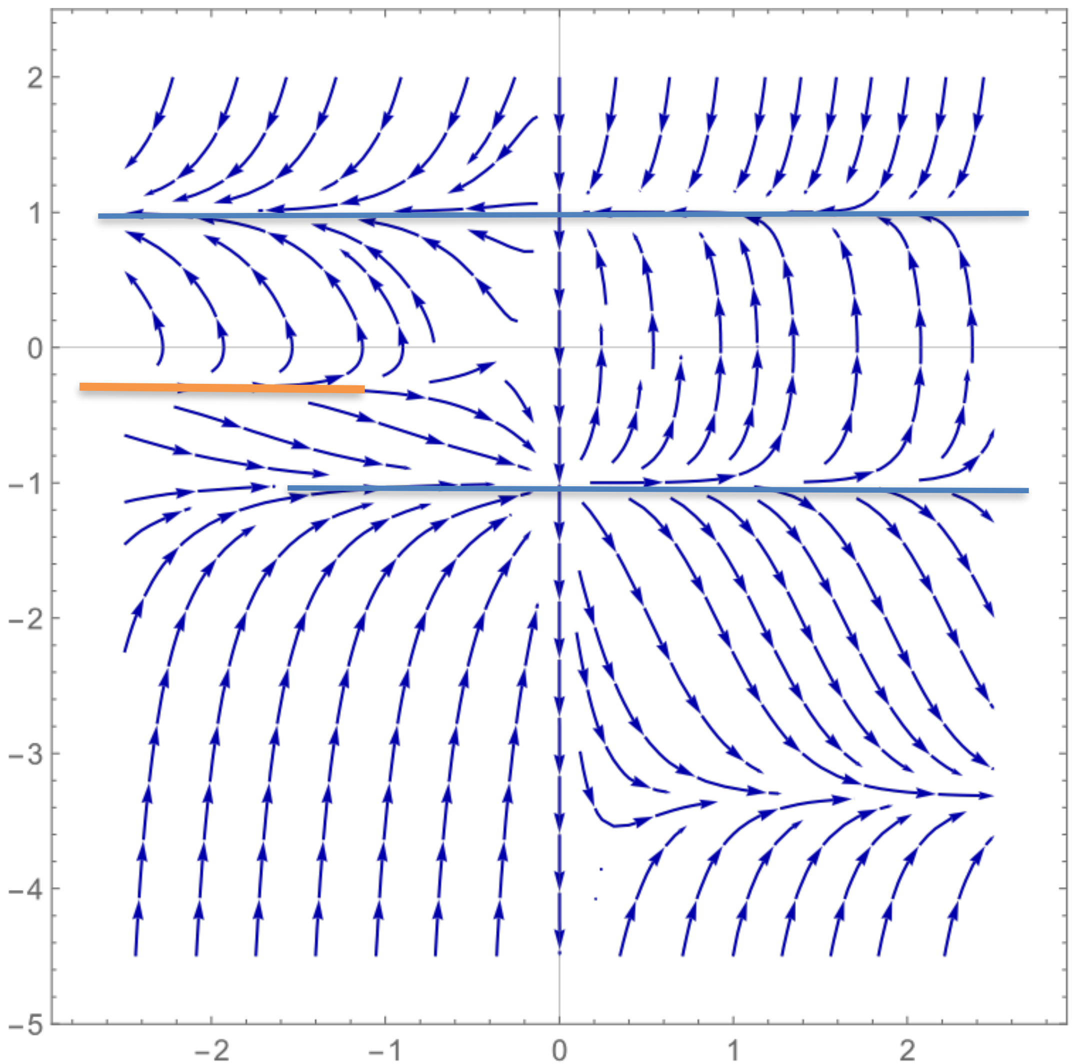

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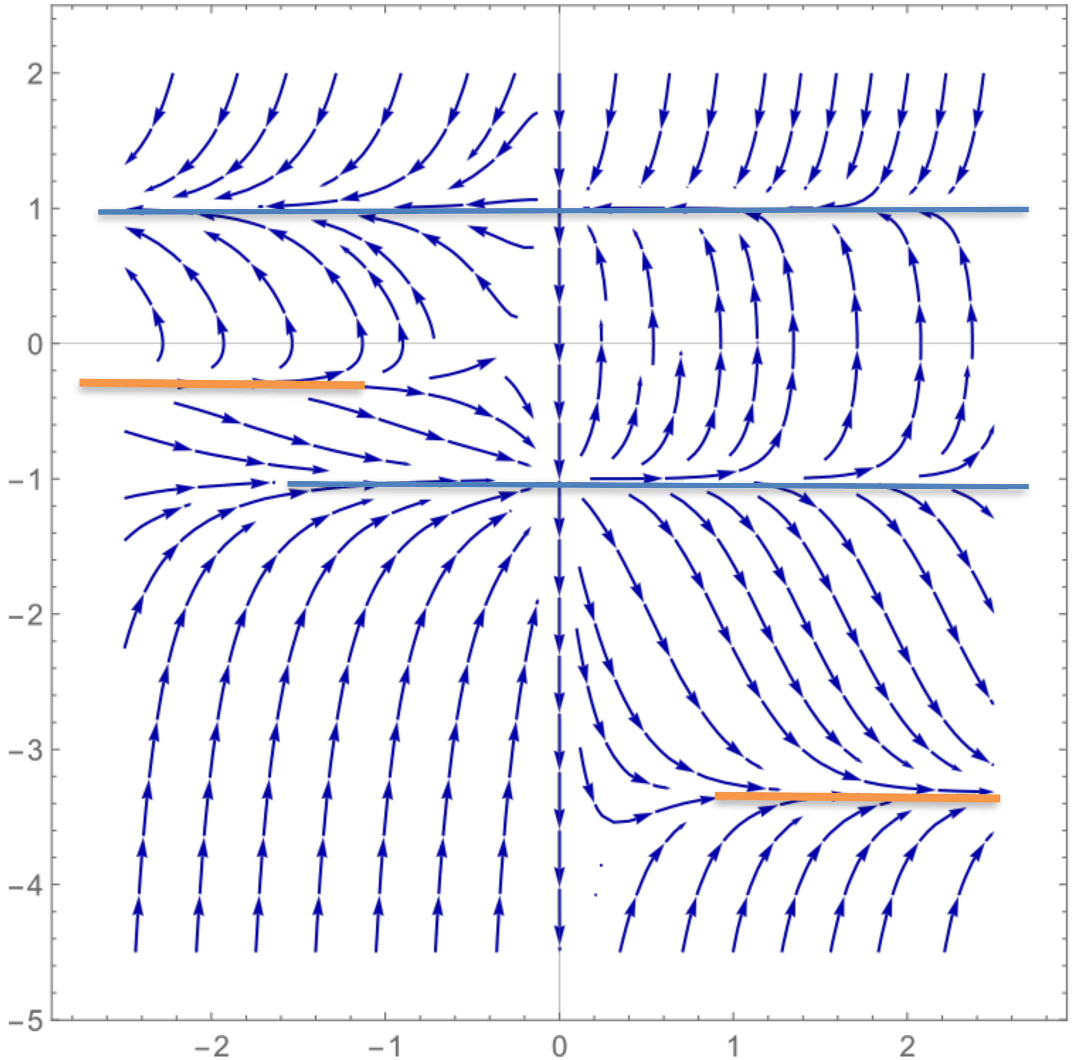
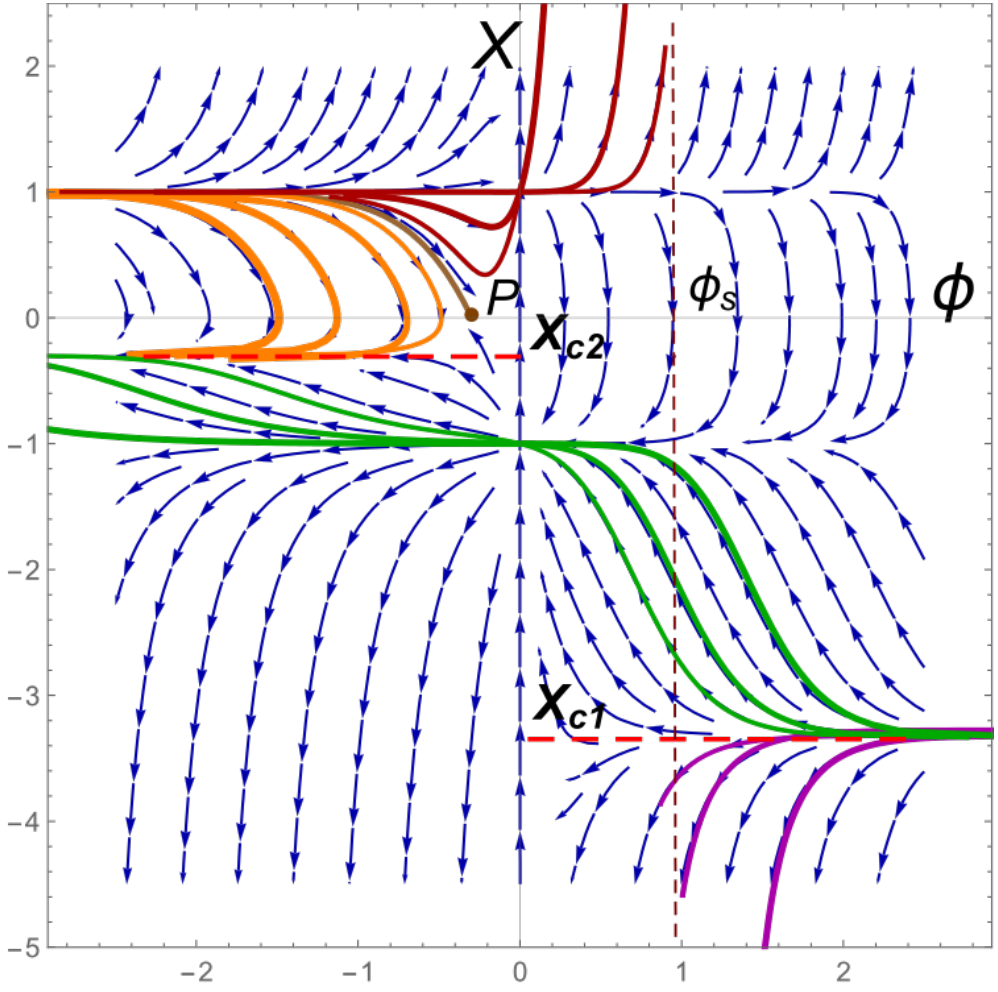


$\sinh - solutions$



XPhivac.pdf

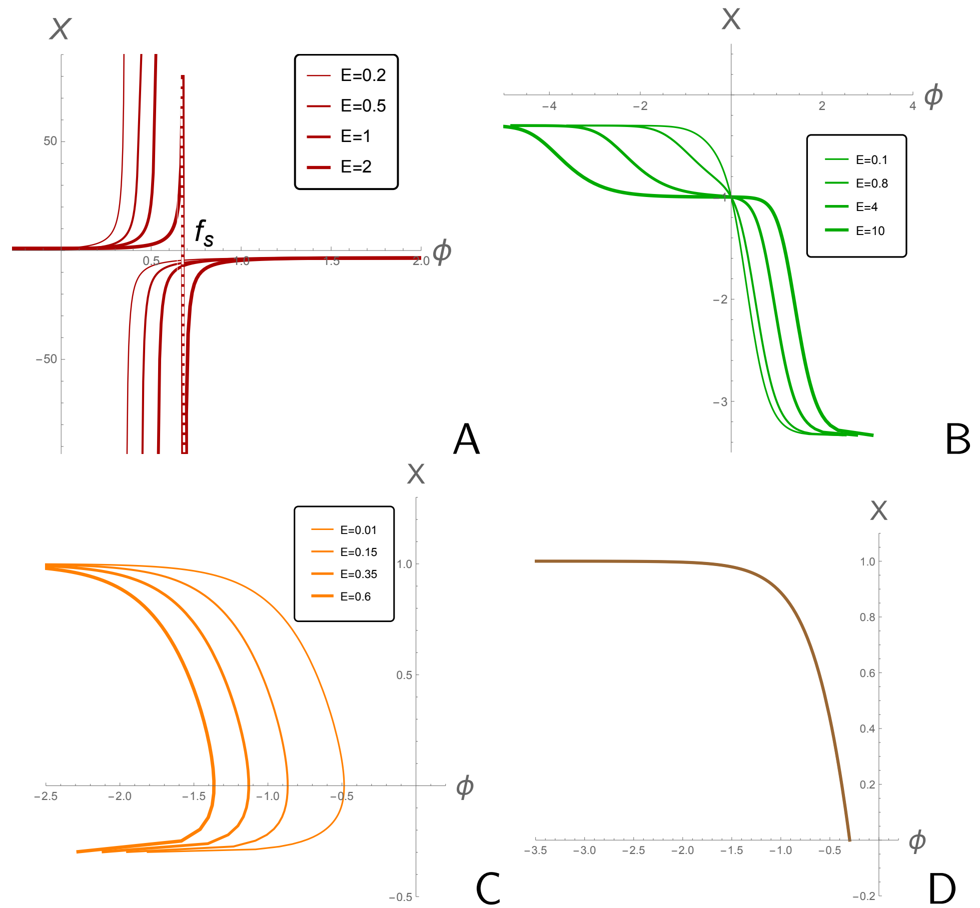
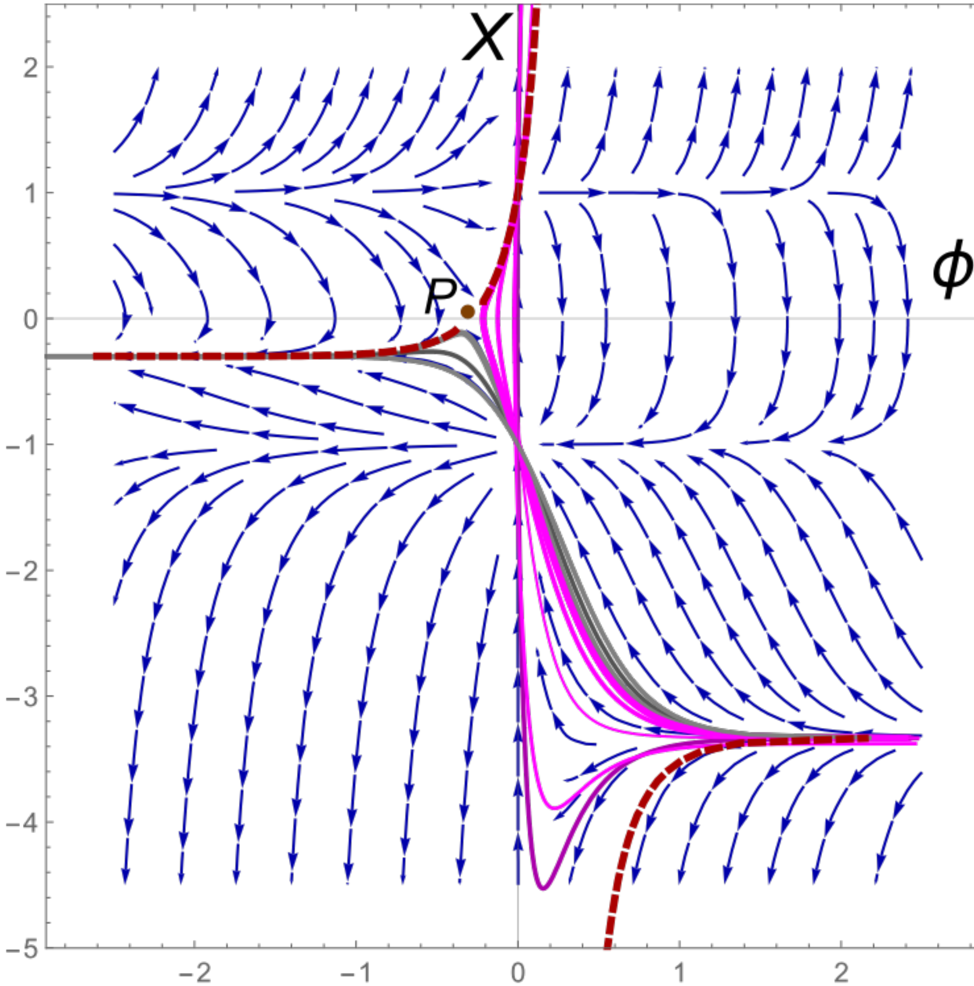
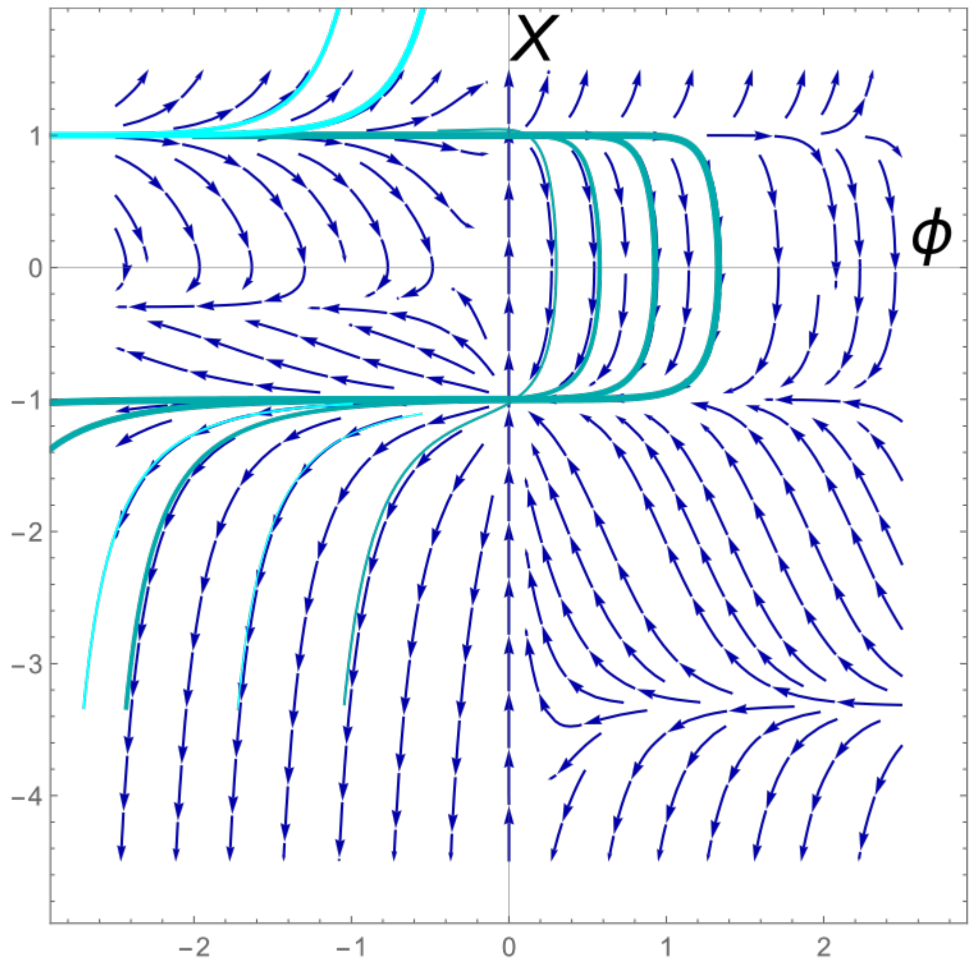


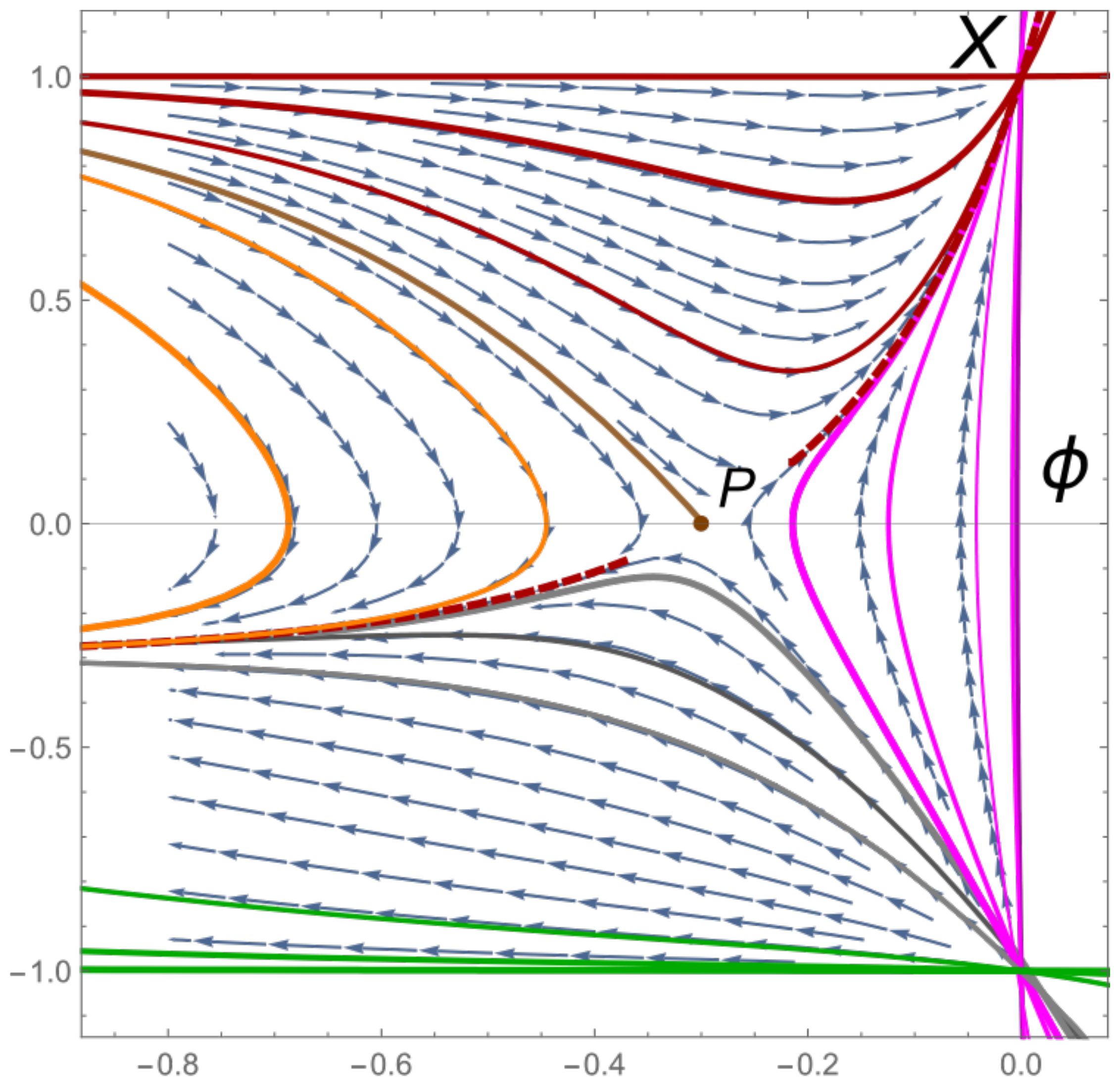
Figure: The behaviour of the X-function with the dependence on the dilaton plotted using the solutions for $\mathcal{A}$. A)left B) middle C)right D) $u_{01} = u_{02}$

$\sin - solutions$



$$\cosh - solutions$$





X
ϕ
P
1.0
0.5
0.0
−0.5
−1.0
−0.8
−0.6
−0.4
−0.2
0.0

Near the boundaries

Near the boundaries

- The left solution $u < u_{02}$ (the conformally flat spacetimes)
  - $u \to -\infty$ $ds^2 \sim z^{2/3}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$, $z \sim e^{-\frac{3\mu_1 u}{4+3k}}$
    $\phi \sim \frac{9k}{16-9k^2}(\mu_2 - \mu_1)u \sim \log z \to -\infty$
  - $u \to u_{02} - \epsilon$ $ds^2 \sim z^{\frac{18k^2}{64-9k^2}}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
    $z \sim \frac{64-9k^2}{4(16-9k^2)}(u - u_{02})^{\frac{64-9k^2}{4(16-9k^2)}}$,
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- The middle solution $u_{02} < u < u_{01}$ (the conformally flat spacetimes)
  - $u \to u_{02} + \epsilon$ the same as for the left solution at $u \to u_{02} - \epsilon$
  - $u \to u_{01} - \epsilon$ $ds^2 \sim z^{\frac{8}{9k^2-4}}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
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- The right solution $u > u_{01}$ (the conformally flat spacetimes)
  - $u \to u_{01} + \epsilon$ the same as for the middle solution at $u \to u_{01} + \epsilon$
  - $u \to +\infty$ $ds^2 \sim z^{2/3}\left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
    $\phi \sim \log z \to -\infty$

The behaviour of the running coupling λ on the energy scale

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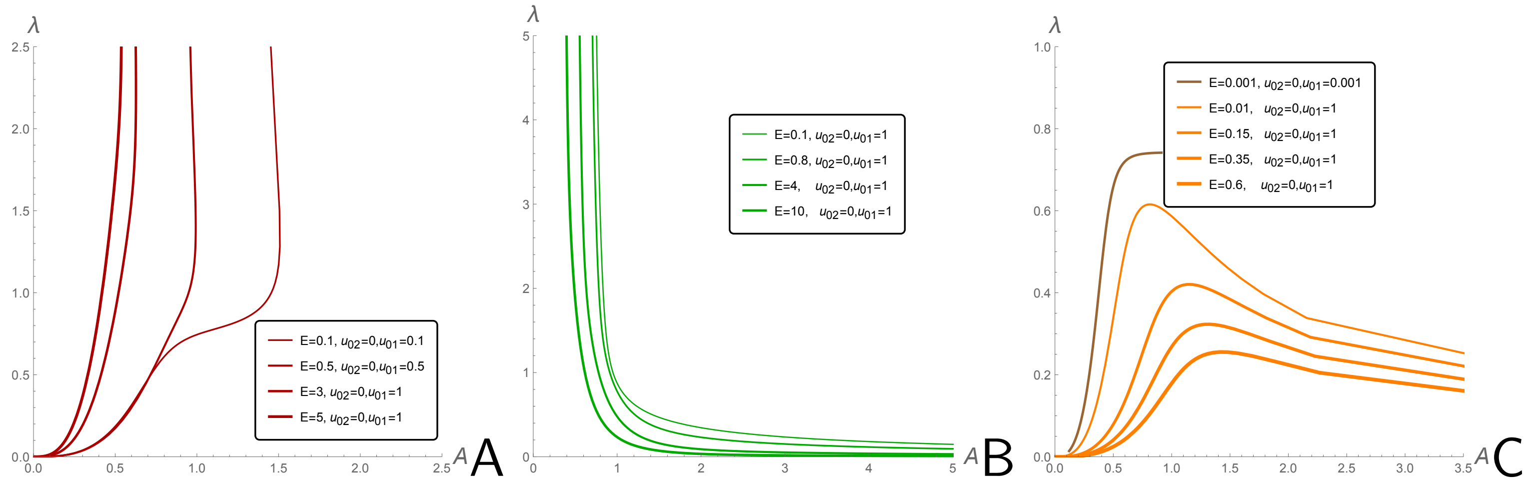

Figure: λ on the energy A on the dilaton plotted using the solutions for $\mathcal{A}$ and ϕ.A) the left branch with $u_{02} > u$, B) the middle branch $u_{02} < u < u_{01}$; C) the right branch $u > u_{01}$. For all plots $k = 0.4$, $C_1 = -2$, $C_2 = 2$, different curves on the same plot corresponds to the different values of $|E_1| = |E_2|$, labeled as E on the legends and different u_{01} and u_{01} also indicated on the legends.

Black Brane solutions

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$$ds^2 = e^{2A - \frac{2}{3}\alpha^1 u}\left(-e^{\frac{8}{3}\alpha^1 u}dt^2 + d\vec{y}^2 + e^{6A + \frac{2}{3}\alpha^1 u}du^2\right)$$

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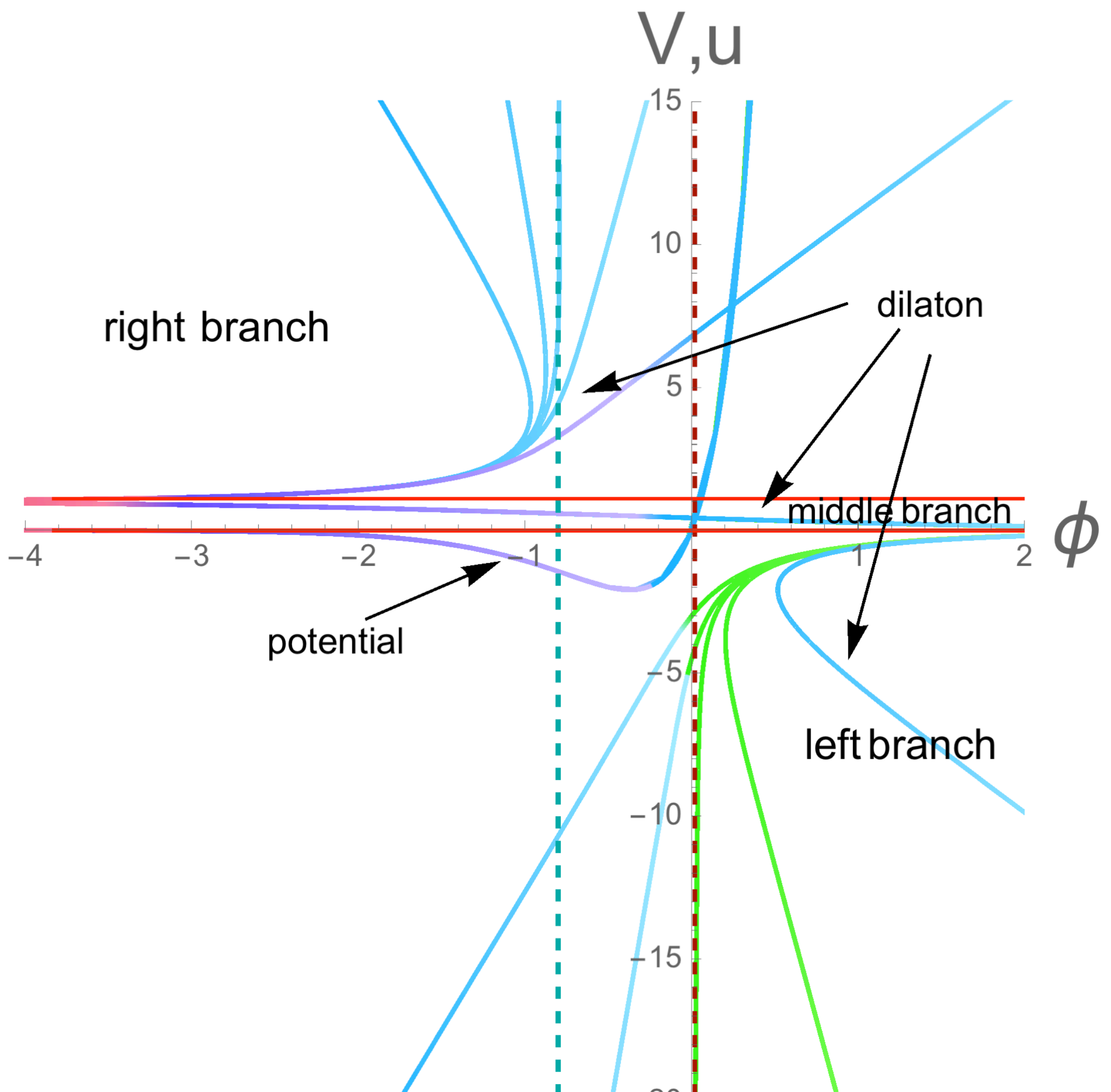

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Removing conical singularities

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$$\frac{1}{\beta} = T = \frac{2}{3\pi}\frac{\alpha^1}{\mathcal{C}^{3/2}}$$

Holographic Isotropic zero μ RG Flow

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$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right),$$

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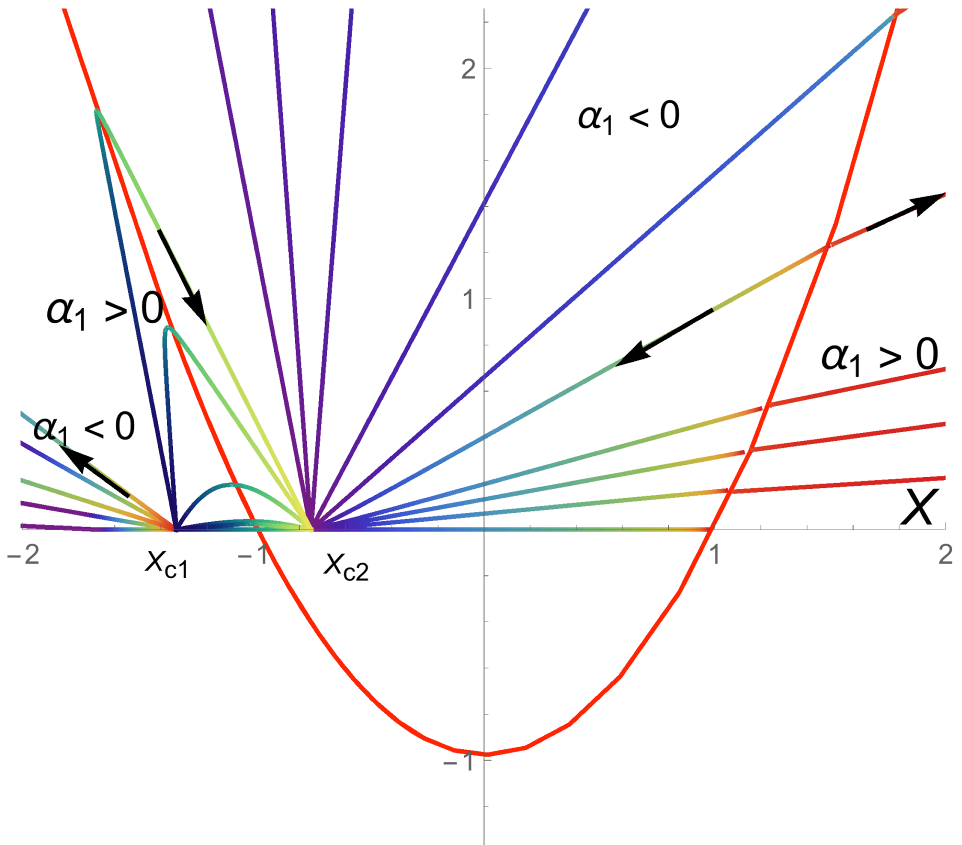

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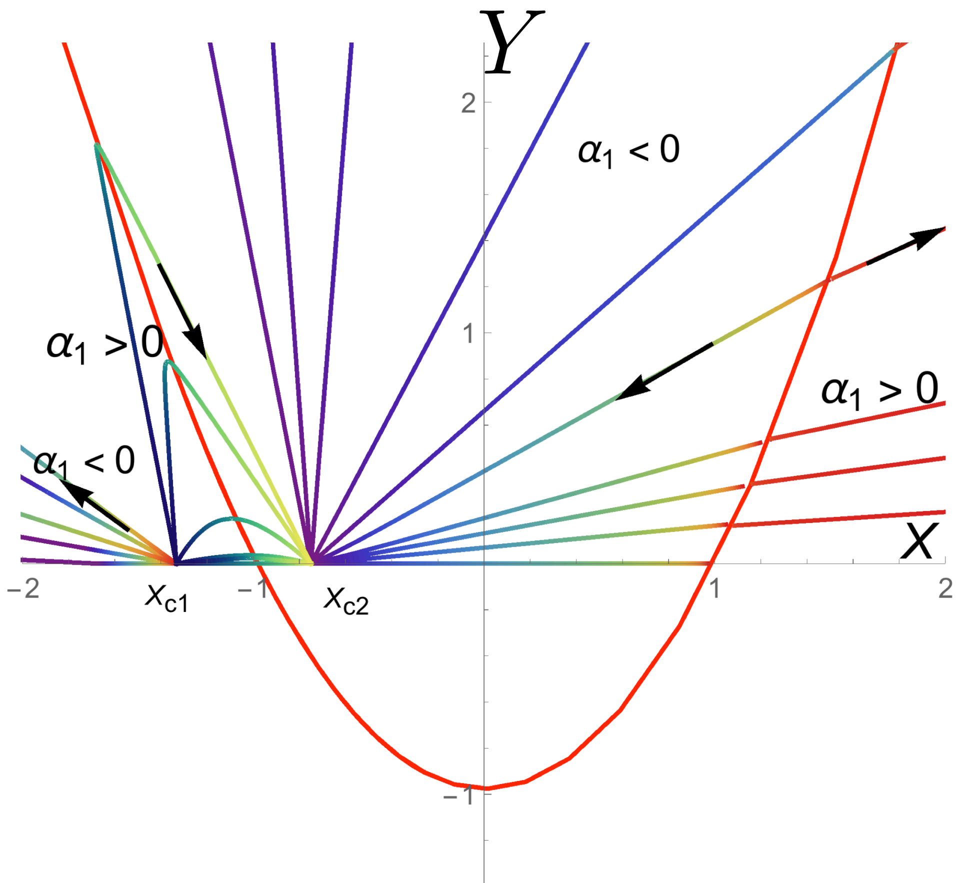

Holographic Isotropic Zero $\mu \neq 0$ Flow

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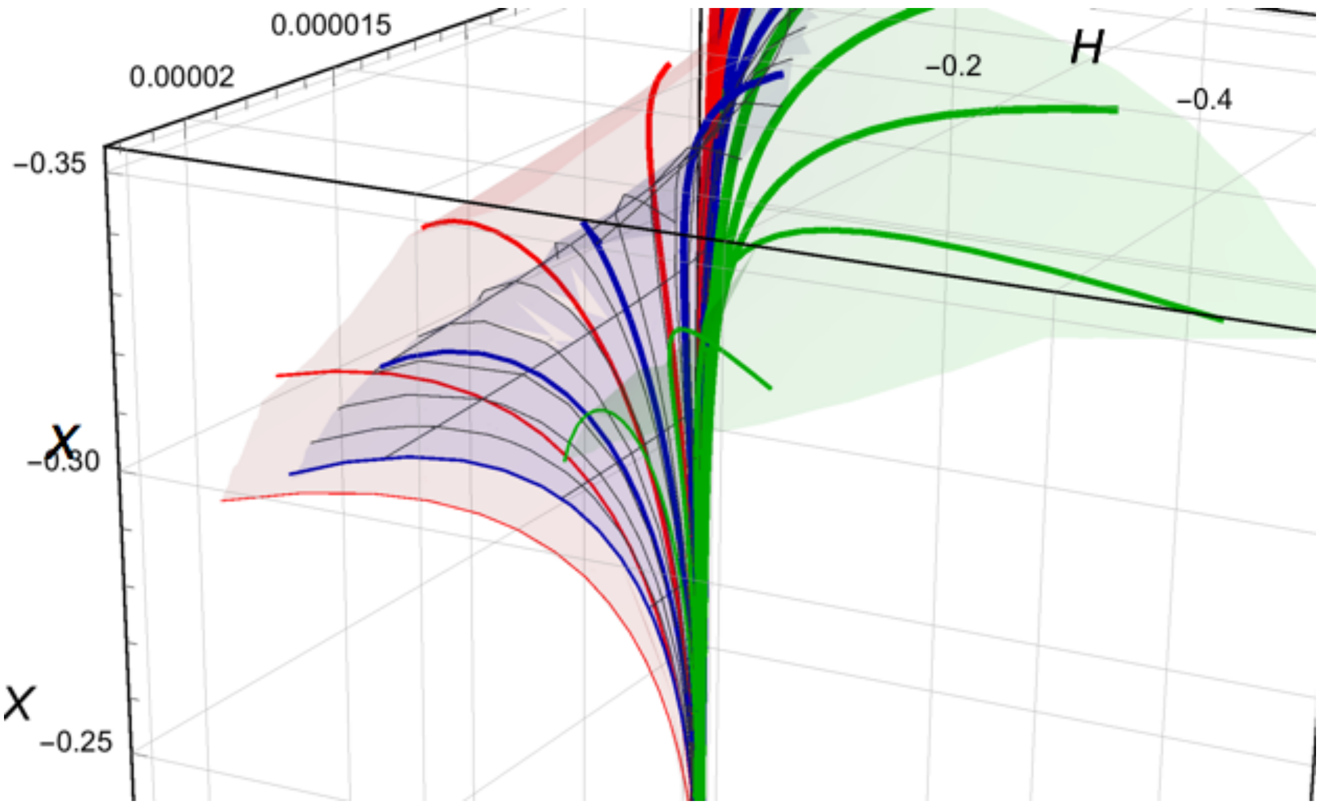
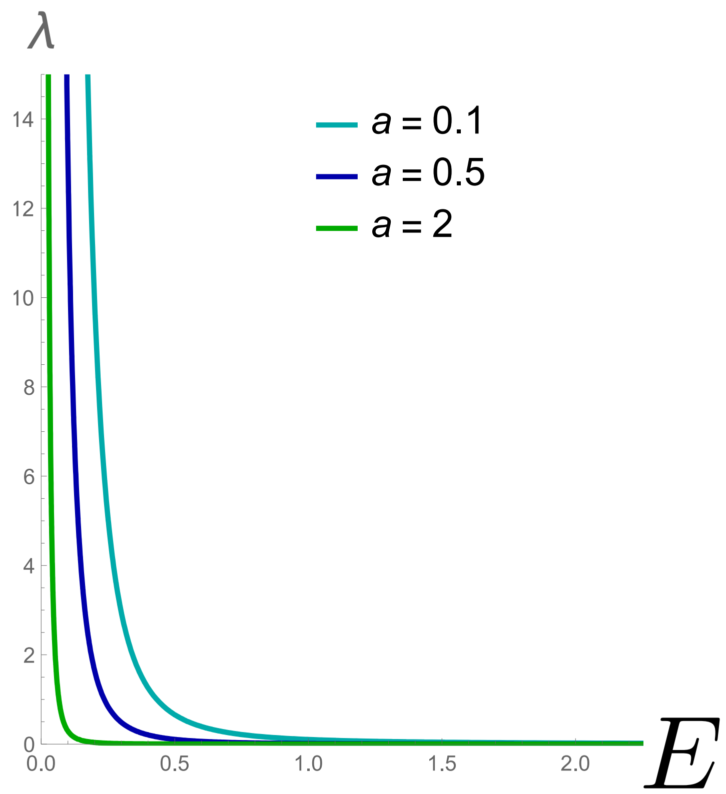


Running coupling constant



Relation with HJ method and refs. Backup

Based on Hamiltonian formalism of gravity

Boer, E.Verlinde, H.Verlinde,'99
Heemskerk, Polchinski, '11

$$-\frac{1}{2\kappa^2}\int_M d^{d+1}x\,\sqrt{g}\left(\mathcal{R}[g]-g^{\mu\nu}G_{IJ}\partial_\mu\Phi^I\partial_\nu\Phi^J-V(\Phi)\right)-\frac{1}{\kappa^2}\int_{\partial M}d^dx\,\sqrt{\gamma}K$$

$$ds^2=dr^2+\gamma_{ij}(r)dx^idx^j$$

$$S=-\frac{1}{2\kappa^2}\int_M d^dx\,dr\sqrt{\gamma}\Big(R[\gamma]+K^2-K_{ij}K^{ij}-G_{IJ}\dot\Phi^I\dot\Phi^J-\gamma^{ij}G_{IJ}\partial_i\Phi^I\partial_j\Phi^J-V(\Phi)\Big)$$

$$K^i_j=\frac{1}{2}\gamma^{ik}\dot\gamma_{kj}$$

$$\pi^{ij}=\frac{\delta S}{\delta\dot\gamma_{ij}}=\frac{1}{2\kappa^2}\sqrt{\gamma}\left(K^{ij}-K\gamma^{ij}\right)\quad\text{and}\quad\pi_I=\frac{\delta S}{\delta\dot\Phi^I}=\frac{1}{\kappa^2}\sqrt{\gamma}G_{IJ}\dot\Phi^J$$

$$H=\int_{\partial M}d^dx\left(\pi^{ij}\dot\gamma_{ij}+\pi_I\dot\Phi^I-\mathcal{L}\right)$$

$$H=\frac{1}{2\kappa^2}\int_{\partial M}d^dx\sqrt{\gamma}\left(R[\gamma]-K^2+K_{ij}K^{ij}+G^{IJ}p_Ip_J-\gamma^{ij}G_{IJ}\partial_i\Phi^I\partial_j\Phi^J-V(\Phi)\right)$$

$$H+\frac{\partial S_{\text{on-shell}}}{\partial r}=0\qquad S_{\text{on-shell}}=\frac{1}{\kappa^2}\int_{\partial M_\epsilon}d^dx\sqrt{\gamma}\,U(\gamma,\Phi,r)$$

$$R[\gamma]+K_{ij}K^{ij}-K^2+G^{IJ}p_Ip_J-\gamma^{ij}G_{IJ}\partial_i\Phi^I\partial_j\Phi^J-V(\Phi)+2\frac{\partial U}{\partial r}=0$$

Relation with HJ method and refs. Backup

$$S = \int d^4x dr \sqrt{g} \left(-\frac{1}{4} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

$$ds^2 = e^{2A(r)} (\eta_{ij} dx^i dx^j) + dr^2$$

GKMN'08—HRG EQ. $X' = -\frac{4}{3}(1 - \frac{3}{2}X^2)(1 - \frac{1}{4}\frac{1}{X}\frac{V'}{V})$

Relation with HJ method and refs. Backup

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EXRA DIM: DeWolfet al, 99
Cosmology: IA.,Koshelev, VERNOV, 05
Supersymmetry transformatio
Skenderis,Townsend '06

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$$=\frac{1}{8}W'(\phi)^2-\frac{1}{3}W(\phi)^2. \qquad \dot{A}+\frac{1}{3}W=0\Rightarrow\dot{\phi}-\frac{1}{2}W'=0$$

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GKMN'08—HRG EQ. $X'=-\frac{4}{3}(1-\frac{3}{2}X^2)(1-\frac{1}{4}\frac{1}{X}\frac{V'}{V})$

$$X=\frac{1}{3}\frac{\dot{\phi}}{\dot{A}}=-\frac{1}{2}\frac{W'}{W} \qquad X'=-\frac{1}{2}[\frac{W''}{W}-\frac{W'^2}{W^2}]$$

$$1-\frac{3}{2}X^2=1-\frac{3}{8}\frac{W'^2}{W^2} \qquad 1+\frac{1}{4}\frac{1}{X}\frac{V'}{V}=1-\frac{W''W-\frac{8}{3}W^2}{W'^2-\frac{8}{3}W^2}$$

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