Conformal Higher Spin action from worldline path integrals

Roberto Bonezzi

Physique théorique et mathématique, Université de Mons – UMONS

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Motivations

- Conformal Higher Spin (CHS) theories generalize four dimensional Maxwell and Weyl squared gravity
- In the supersymmetric case, they are HS counterpart of conformal supergravities
- Remarkably, admit a consistent non-linear action around flat space
- Contrary to massless HS in AdS, conformal invariance fixes the derivative structure of vertices $\longrightarrow$ finite derivatives for given spins and number of fields
- Related to boundary values of massless HS in AdS via AdS/CFT
- Despite being non-unitary, there are hints for unitary truncations at tree-level

Outline

- Free theory
- Low spin case: conformal (super)gravity as an induced action
- HS conformal currents from free fields
- CHS action as an induced action
- Worldline model of scalar in CHS background
- Worldline path integral representation for all vertices
- Conclusions

Free theory

- Consider four dimensional Maxwell + Weyl squared gravity

$$S[A,g] = \int d^4x \sqrt{-g}\left[F^{\mu\nu}F_{\mu\nu} + C^{\mu\nu\lambda\sigma}C_{\mu\nu\lambda\sigma}\right]$$

invariant under $U(1)$, diffeos and Weyl local symmetries

- Linearize around flat spacetime $\longrightarrow$

$$S_2[A,h] = \int d^4x \left[A^\mu P_\mu{}^\nu \Box A_\nu + h^{\mu\nu} P_{\mu\nu}{}^{\lambda\sigma} \Box^2 h_{\lambda\sigma}\right]$$

where P_1 and P_2 are transverse-traceless projectors of spin one and two

- Invariant under linearized gauge symmetry

$$\delta A_\mu = \partial_\mu \epsilon \;, \quad \delta h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} + \alpha\, \eta_{\mu\nu}$$

Free theory

- Natural generalization to spin s

  Fradkin, Tseytlin (1989)

$$S[h] = \int d^4x \sum_{s=0}^{\infty} h^{\mu(s)} {P_{\mu(s)}}^{\nu(s)} \Box^s h_{\nu(s)}$$

  where P_s are spin-s transverse-traceless projectors
- Invariant under maximal spin-s gauge symmetry compatible with locality

$$\delta h_{\mu(s)} = \partial_\mu \epsilon_{\mu(s-1)} + \eta_{\mu\mu} \alpha_{\mu(s-2)}$$

- In higher even d, $\Box^s \to \Box^{s+\frac{d-4}{2}}$ with conformal dimension $\Delta_s = 2 - s$
- In general non-unitary for $s \geq 2$
- Although there are conformal scalars with two or higher derivatives, this h_0 is auxiliary in $d = 4$

Induced conformal gravity

- How do we construct the interacting theory? Noether method? Maybe
- Suppose one doesn't know Maxwell and Weyl squared gravity, and take a scalar conformally coupled to the metric + $U(1)$ coupling

$$S[\phi; A, g] = \int d^4x\sqrt{g}\left[g^{\mu\nu}D_\mu\phi^* D_\nu\phi + \tfrac{1}{6} R\phi^*\phi\right] , \quad D_\mu = \partial_\mu - iA_\mu$$

- Compute the scalar effective action

$$\Gamma[A, g] = a_0\,\Lambda^4\mathcal{V} + a_2 \log\Lambda \int d^4x\sqrt{g}\left[F^2_{\mu\nu} + C^2_{\mu\nu\lambda\sigma}\right] + \text{finite}$$

 and "discover" Maxwell+Weyl squared from the log divergent piece
- Notice that around flat space

$$e^{-\Gamma[A,h]} = \left\langle e^{\int J^\mu A_\mu + T^{\mu\nu}_{\text{conf}} h_{\mu\nu} + \mathcal{O}(A,h)^2} \right\rangle_{\text{CFT}}$$

$\mathcal{N}=4$ SYM, AdS/CFT and $\mathcal{N}=4$ C-sugra Liu, Tseytlin (1998)

- Similar construction starting from $\mathcal{N}=4$ SYM: it can be naturally coupled to $\mathcal{N}=4$ conformal supergravity

  Bergshoeff, de Roo, de Wit (1981); de Roo, Wagemans (1985)

- The SYM partition function in C-sugra background

$$e^{-\Gamma[\mathcal{G}]} = \int \mathcal{D}\mathcal{A}\, e^{-S_{\rm SYM}[\mathcal{A};\mathcal{G}]}$$

  yields the C-sugra action as log divergent part
  $\Gamma[\mathcal{G}] = \Gamma_{\rm fin}[\mathcal{G}] + a_2 \log\Lambda S_{\rm CSG}[\mathcal{G}]$

- Again, the part of $S_{\rm SYM}[\mathcal{A};\mathcal{G}]$ linear in CSG fluctuations yields the Noether couplings to SYM operators, so that

$$e^{-\Gamma[\mathcal{G}]} = \left\langle e^{\int O(\mathcal{A})_a \mathcal{G}^a + \mathcal{O}(\mathcal{G}^2)} \right\rangle_{\rm SYM}$$

  that corresponds, at large 't Hooft coupling, to on-shell $\mathcal{N}=8$ gauged sugra in AdS_5

Induced CHS action

- Apply the same idea to the simplest CFT of free scalar field in even d
$$S_0[\phi] = \int d^d x \, \partial^\mu \phi^* \partial_\mu \phi$$
- It possesses infinitely many traceless conserved currents $J_s = \phi^* \partial^s \phi + \ldots$ of conformal dimension $\Delta_{J_s} = d - 2 + s$
- Low spin are scalar operator $J_0 = \phi^* \phi$, $U(1)$ current $J_\mu = \phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*$ and traceless stress energy tensor $J_{\mu\nu} = T^{\rm conf}_{\mu\nu} = \partial_{(\mu} \phi^* \partial_{\nu)} \phi + \ldots$
- Couple to sources $S[\phi; h] = S_0[\phi] + \sum_{s=0}^{\infty} \frac{(i)^s}{s!} \int d^d x \, J^{\mu(s)} h_{\mu(s)}$
- Sources h_s have dimension $\Delta_{h_s} = 2 - s$ and inherit linearized invariance on the scalar mass-shell under $\delta h_s = \partial \epsilon_{s-1} + \eta \, \alpha_{s-2} \longrightarrow$ identified as CHS

Induced CHS action

- Consider the scalar path integral

$$\int \mathcal{D}\phi^* \mathcal{D}\phi \, e^{-S[\phi;h]} := e^{-\Gamma[h]} = \left\langle e^{\int \sum_s J_s \cdot h_s} \right\rangle_{\text{free CFT}}$$

- According to HS AdS/CFT Sezgin, Sundell; Klebanov, Polyakov (2002) conformal spin-s conserved currents are dual to AdS massless spin-s gauge fields $\longrightarrow$
- the above should correspond to the on-shell AdS action of interacting MHS
- From previous discussion, log divergent part of $\Gamma[h]$ defines non-linear CHS action Tseytlin; Segal (2002); Bekaert, Joung, Mourad (2010)
$\Gamma[h] = \sum_{n=1}^{\infty} \Lambda^{2n} \Gamma_n[h] + \log\Lambda \, S_{\text{CHS}}[h] + \Gamma_{\text{fin}}[h]$

Currents, symmetries, effective action

- In terms of the CHS generating function $h(x,u) := \sum_{s=0}^{\infty} \frac{1}{s!} h_{\mu(s)} u^{\mu_1} \dots u^{\mu_s}$ the scalar action can be recast as (basically IBP derivatives on ϕ^*)

$$S_0[\phi] + \int d^d x \, J(x, i\partial_u) h(x,u)|_{u=0} = \langle \phi | \, \hat{P}^2 + \hat{H}(\hat{X}, \hat{P}) \, | \phi \rangle$$

where $\hat{X}^\mu \equiv x^\mu$ and $\hat{P}_\mu \equiv -i\partial_\mu$

- Invariant under the gauge symmetry

$$|\phi\rangle \to \hat{O}^{-1} |\phi\rangle \qquad (\hat{P}^2 + \hat{H}) \to \hat{O}^\dagger (\hat{P}^2 + \hat{H}) \hat{O}$$

- For $\hat{O}(\hat{X}, \hat{P}) = e^{\hat{A} + i\hat{E}}$ at lowest order $\hat{E}$ generates the ϵ differential symmetries, and $\hat{A}$ the HS Weyl symmetries α
- The effective action is thus given by the functional determinant $\Gamma[h] = \mathrm{Tr} \log\left(\hat{P}^2 + \hat{H}\right)$
- Next step: worldline representation of $\Gamma[h]$

Worldline formalism

- The WL formalism is quite effective for such one-loop effective actions. E.G. scalar particle in gravitational and e.m. background

$$S_{g,A}[x,p,e] = \int_0^1 d\tau \left\{ p_\mu \dot{x}^\mu - \frac{e}{2}\left[g^{\mu\nu}(p_\mu - A_\mu)(p_\nu - A_\nu) + \xi\, R(g) \right] \right\}$$

or

$$S_{g,A}[x,e] = \int_0^1 d\tau \left[\frac{1}{2e} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + A_\mu \dot{x}^\mu + \xi \frac{e}{2} R \right]$$

- $e(\tau)$ is the einbein, gauges relativistic hamiltonian and ensures τ-rep invariance
- Path integral on the circle produces the QFT one-loop photon-graviton EA

$$\Gamma[A,g] = \int_{S^1} \frac{DxDe}{\mathrm{VolG}} e^{-S_{g,A}[x,e]}$$

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$$\Gamma[A,g] = \int_{S^1} \frac{DxDe}{\mathrm{VolG}} e^{-S_{g,A}[x,e]}$$

- $e(\tau)$ has a zero mode on S^1 $\to$ gauge fixing $e(\tau) = T$ leaves a modular integration. FP action is locally trivial, but yields T^{-1} factor on S^1 topology

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- Gauge fixed path integral yields heat kernel expansion

$$\Gamma[h] = \int_0^\infty \frac{dT}{T} \int_{S^1} Dx \, e^{-S_{g,A}[x,T]} = \int_0^\infty \frac{dT}{T} K[T; g, A]$$

where $[T] = M^{-2}$ is the proper time and $T \to 0$ is UV region of the QFT (1d version of torus modular parameter $|\tau|$ in String Theory)

Scalar particle in CHS background

- Generalize the hamiltonian action de Wit, Freedman (1980); Segal (2002)

$$S_h[x,p,e] = \int_0^1 d\tau \left[p_\mu \dot{x}^\mu - e\, G(x,p) \right]$$
$$G(x,p) = p^2 + \mathcal{H}(x,p)$$

- The CHS basis is $h(x,p)$ with

$$\mathcal{H}(x,p) = \mathcal{P}_d\, h(x,p) = \sum_{s=0}^{\infty} \sum_{n=0}^{[s/2]} c_n(s,d) \left[\partial^{*2} - \mathrm{Tr}\Box \right]^n h_s(x,p)$$

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- Relativistic invariance under τ-rep with transformations

$$\delta x^\mu = \xi \{x^\mu, G\}_{\text{P.B.}}\ , \quad \delta p_\mu = \xi \{p_\mu, G\}_{\text{P.B.}}\ , \quad \delta e = \dot{\xi}$$

Scalar particle in CHS background

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$$S_h[x,p,e] = \int_0^1 d\tau \Big[p_\mu \dot{x}^\mu - e\, G(x,p) \Big]$$
$$G(x,p) = p^2 + \mathcal{H}(x,p)$$

- CHS background gauge symmetry under

$$\delta\mathcal{H}(x,p) = \{\epsilon(x,p), p^2 + \mathcal{H}(x,p)\}_{\rm P.B.} + \alpha(x,p)\,(p^2 + \mathcal{H}(x,p))$$

with WL variables transforming as

$$\delta x^\mu = \{x^\mu, \epsilon(x,p)\}_{\rm P.B.}\ , \quad \delta p_\mu = \{p_\mu, \epsilon(x,p)\}_{\rm P.B.}\ , \quad \delta e = -\alpha(x,p)\, e$$

- Notice that the α-symmetry is broken by gauge fixing $e(\tau) = T$

Caveat: covariant vs Noether coupling

As in the field theoretic description, the low spin fields do not coincide with the covariant ones (besides the map $\mathcal{H} = \mathcal{P}_d\, h$)

- Take the covariant hamiltonian

$$G_{\rm cov}(x,p) = g^{\mu\nu}(p_\mu - A_\mu)(p_\nu - A_\nu) + \xi\, R(g)\ , \quad g_{\mu\nu} = \eta_{\mu\nu} + h^{\rm w}_{\mu\nu}$$

- Compare with low spin branch of $\mathcal{H}$

$$G_{\rm Noether}(x,p) = p^2 + h_0 - 2\, h^\mu\, p_\mu - h^{\mu\nu}\, p_\mu p_\nu$$

- One has the relation

$$h_{\mu\nu} = h^{\rm w}_{\mu\nu} + r_{\mu\nu}(h_{\rm w}) \quad (g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}_{\rm w} - r^{\mu\nu}(h_{\rm w}))$$

$$h_\mu = A_\mu - h^{\rm w}_{\mu\nu} A^\nu - r_{\mu\nu}(h_{\rm w}) A^\nu$$

$$h_0 = R(\eta + h_{\rm w}) + A^2 - A_\mu A_\nu (h^{\mu\nu}_{\rm w} + r^{\mu\nu}(h_{\rm w}))$$

- Compositeness of h_0 can be deduced, for instance, by its $U(1)$ transformation $\delta_{U(1)} h_0 = 2 g^{\mu\nu} A_\mu \partial_\nu \epsilon$

CHS action from WL path integral

- CHS effective action from WL path integral on the circle

$$\Gamma[h] = \int_{S^1} \frac{DxDpDe}{\mathrm{Vol\,G}} e^{-S_h[x,p,e]}$$

- Gauge fixed path integral $e(\tau) = T \rightarrow$ Schwinger proper time expansion

$$\Gamma_\Lambda[h] = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{dT}{T} K[T; h]$$

- Log divergent piece is the $\mathcal{O}(T^0)$ part of $K[T; h]$

CHS action from WL path integral

- Gauge fixed path integral $\Gamma_\Lambda[h] = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{dT}{T} K[T; h]$
- Trace of heat kernel $K[T; h] = \int_{S^1} DxDp\, e^{-S_h[x,p;T]}$
- Extract x zero mode: $x(\tau) = x + q(\tau)$ and average w.r.t. free action

$$K[T; h] = \int \frac{d^d x}{(4\pi T)^{d/2}} \left\langle e^{-T \int_0^1 d\tau\, \mathcal{H}(x+q,p)} \right\rangle = \int \frac{d^d x}{(4\pi T)^{d/2}} \sum_{n=0}^{\infty} T^n \mathcal{V}_n[T; h]$$

- $(4\pi T)^{-d/2}$ is the free PI on the circle; $\mathcal{V}_n[T; h]$ yields the n-point effective vertex. $\mathcal{V}_n$ is analytic in T for $s \leq 2$, but *not* for HS

CHS action from WL path integral

- Gauge fixed path integral $\Gamma_\Lambda[h] = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{dT}{T} K[T;h]$

$$K[T;h] = \int \frac{d^d x}{(4\pi T)^{d/2}} \sum_{n=0}^{\infty} T^n \mathcal{V}_n[T;h]$$

- Effective vertex is given by a differential operator acting on a string of $\mathcal{H}$'s

$$\mathcal{V}_n[T;h] = \hat{V}_n(T;\partial_{x_i},\partial_{u_i})\,\mathcal{H}(x_1,u_1)\cdots\mathcal{H}(x_n,u_n)|_{\substack{x_i=x\\u_i=0}}$$

(expand $\mathcal{H}(x+q_i,p_i)$ around $(x,0)$ and compute exact PI)

- Freedom in IBP expressed as $\sum_{i=1}^{n} \partial_{x_i} \sim 0$

CHS action from WL path integral

- Gauge fixed path integral $\Gamma_{\Lambda}[h] = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{dT}{T} K[T;h]$

$$K[T;h] = \int \frac{d^d x}{(4\pi T)^{d/2}} \sum_{n=0}^{\infty} T^n \mathcal{V}_n[T;h]$$

- $\mathcal{V}_n[T;h] = \hat{V}_n(T;\partial_{x_i},\partial_{u_i})\,\mathcal{H}(x_1,u_1)\cdots\mathcal{H}(x_n,u_n)|_{\substack{x_i=x \\ u_i=0}}$
- WL path integral yields the result in terms of parametric proper time integrals

$$\hat{V}_n(T;\partial_{x_i},\partial_{u_i}) = \frac{(-1)^n}{n} e^{\frac{1}{4T}\partial_U^2} \int_0^1 d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{n-2}} d\tau_{n-1}$$

$$\times \exp \sum_{i<j}^{(\tau_n=0)} \left\{ -i(\tau_{ij} - \tfrac{1}{2})(\partial_{x_i}\cdot\partial_{u_j} - \partial_{x_j}\cdot\partial_{u_i}) + T\,\tau_{ij}(\tau_{ij}-1)\partial_{x_i}\cdot\partial_{x_j} \right\}$$

CHS action from WL path integral

- $\mathcal{V}_n[T;h] = \hat{V}_n(T;\partial_{x_i},\partial_{u_i})\,\mathcal{H}(x_1,u_1)\cdots\mathcal{H}(x_n,u_n)|_{\substack{x_i=x\\u_i=0}}$

$$\hat{V}_n(T;\partial_{x_i},\partial_{u_i}) = \frac{(-1)^n}{n}e^{\frac{1}{4T}\partial_U^2}\int_0^1 d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-2}}d\tau_{n-1}$$
$$\times\ \exp\sum_{i<j}^{(\tau_n=0)}\left\{-i(\tau_{ij}-\tfrac{1}{2})(\partial_{x_i}\cdot\partial_{u_j}-\partial_{x_j}\cdot\partial_{u_i}) + T\,\tau_{ij}(\tau_{ij}-1)\partial_{x_i}\cdot\partial_{x_j}\right\}$$

- When acting on a definite spin basis $\{s_i\}$ of $h(x_i,u_i)$ there are at most $s_{\rm tot}=\sum_{i=1}^n s_i$ u-derivatives $\rightarrow$ only polynomials from the inverse T and T-independent exp

CHS action from WL path integral

- $\mathcal{V}_n[T;h] = \hat{V}_n(T;\partial_{x_i},\partial_{u_i})\,\mathcal{H}(x_1,u_1)\cdots\mathcal{H}(x_n,u_n)|_{\substack{x_i=x\\u_i=0}}$

$$\hat{V}_n(T;\partial_{x_i},\partial_{u_i}) = \frac{(-1)^n}{n}e^{\frac{1}{4T}\partial_U^2}\int_0^1 d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-2}}d\tau_{n-1}$$
$$\times\exp\sum_{i<j}^{(\tau_n=0)}\left\{-i(\tau_{ij}-\tfrac{1}{2})(\partial_{x_i}\cdot\partial_{u_j}-\partial_{x_j}\cdot\partial_{u_i})+T\,\tau_{ij}(\tau_{ij}-1)\partial_{x_i}\cdot\partial_{x_j}\right\}$$

- For every s_{tot}, $\hat{V}_n$ has a well defined Laurent expansion in T

$$\mathcal{V}_n[T;h] = \sum_{k=-\infty}^{\infty}T^k\,\mathcal{V}_n^{(k)}[h]$$

and the component giving rise to the log is $k=\frac{d}{2}-n$

CHS action from WL path integral

Finally, the classical CHS action is given by

$$S_{\mathrm{CHS}}[h] = \int d^d x \sum_{n=2}^{\infty} \mathcal{V}_n^{(d/2-n)}[h]$$

- For a given set of spins $\{s_i\}$ with $\sum_{i=1}^n s_i = s_{\mathrm{tot}}$ locality is manifest at all orders n. In fact, the number of derivatives p is fixed to

$$p = d + s_{\mathrm{tot}} - 2n$$

- E.g. the diagonal quadratic part has $p = d - 4 + 2s$
- Only spin one in 4d: $p = 4 - n$ accounts for YM cubic and quartic vertices
- Only spin two in 4d: $p = 4$

CHS action from WL path integral

Finally, the classical CHS action is given by

$$S_{\text{CHS}}[h] = \int d^d x \sum_{n=2}^{\infty} \mathcal{V}_n^{(d/2-n)}[h]$$

- The quadratic part has been rederived and indeed reproduces Fradkin-Tseytlin

$$S_2[h] = \sum_{s=0}^{\infty} c_s \int d^d x \left[h_s \, P_s \Box^{s+\frac{d-4}{2}} h_s\right]$$

- Mind that the differential operator $\hat{V}_n^{(d/2-n)}$ has to undo the map $\mathcal{H} = \mathcal{P}_d \, h$
- The constants c_s are irrelevant at the free level, but they set the relative normalization of all couplings

Conclusions and outlook

- WL representation of CHS vertices, similar to string 1st quantized approach
- Technically annoying fact: the map $\mathcal{H} = \mathcal{P}_d\, h \rightarrow$ In TT gauge reduces to identity and, in 4d, one could use spinors basis to simplify
- Zero total dof, anomaly cancellations and hints for S-matrix triviality Beccaria, Bekaert, Joung, Mourad, Tseytlin; McLoughlin, Hähnel (2013–> now) rely on regulated spin sums $\rightarrow$ develop a 1st quantized WL or worldsheet model for CHS, would help understanding
- WL approach can simplify manifest covariant coupling to spin 2 Beccaria, Grigoriev, Tseytlin (2016-2017)
- Possible unitary truncations of amplitudes?

  McLoughlin, Hähnel (2016-2017)

THANKS FOR YOU ATTENTION!