On locality in HS theory

based on: 1805.11941 and

D, Gelfond, Korybut, Vasiliev to appear

- Locality problem in HS interactions
- Locality and the conventional homotopy
- Classes of functions
- Vertex $\gamma(\omega, C, C)$
- One-parameter class of local homotopies
- Conclusion

HS equations

$$
\begin{cases}
d\omega + \omega * \omega = \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \dots \\
dC + [C, \omega]_* = \Upsilon(\omega, C, C) + \Upsilon(\omega, C, C, C) + \dots
\end{cases}
$$

$\omega(Y; x)$ is 1-form $\qquad$ $C(Y; x)$ is 0-form

finite dimensional $\qquad$ infinite dimensional

Star-product

$$
f(Y) * g(Y) = f(Y)\, e^{i\epsilon^{\alpha\beta}\overleftarrow{\partial}_\alpha \overrightarrow{\partial}_\beta}\, g(Y)
$$

Generating system

$$d_x W + W * W = 0$$

$$d_x S + [W, S] = 0$$

$$d_x B + [W, B] = 0$$

$$S * S = i\left(\theta^A \theta_A + \eta B * \gamma + \bar{\eta} B * \gamma\right)$$

$$[S, B] = 0$$

$$\gamma = e^{i z_\alpha y^\alpha} \kappa \; \theta^\beta \theta_\beta$$

$$W = \omega(Y; x) + \ldots$$

$$B = C(Y; x) + \ldots$$

Perturbations

$$S^0 = z_A \theta^A \quad , \quad B^0 = 0 \quad , \quad W^0 = \Omega^{AdS}$$

$$[S^0, f] \sim d_z f$$

$$d_z f = J \quad \longrightarrow \quad f = \Delta_0 J$$

conventional homotopy

$$S_1 = -\frac{\eta}{2}\Delta_0\left(C * \gamma\right)$$

$$d_2 B_2 = [S_1, C] \longrightarrow B_2 = \Delta_0\left([S_1, C]\right) = \Delta_0\left([\Delta_0(C*\gamma), C]\right)$$

$$D_\Omega C + [\omega, C] = -[\Omega, B_2]$$

$$\gamma(\Omega, C, C) = [\Omega, B_2] \sim e^{\partial_1 \partial_2} C(y_1)\, C(y_2)\, \Omega$$

<u>Conclusion:</u> Δ_0 results in a highly <u>non-local</u> cubic vertex.

q-homotopies and classes of functions

Locality theorem (Gelfond + Vasiliev)

N- order contribution

$$\int \dots \int dt^n \, C \dots C \, e^{i\left(\tau z_\alpha y^\alpha + A^i \partial_i^\alpha z_\alpha + B^i \partial_i^\alpha y_\alpha + \frac{1}{2} P^{ij} \partial_i^\alpha \partial_{j\alpha}\right)}$$

PLT = z- dominance lemma + Classes of functions

$$\det P_{ij} = 0$$

Gelfond's theorem

Suppose HS master fields B, S, W are solved for using Δ_0 all the way, then

$$i\sum(-)^m A^m = -\tau \qquad (1)$$
$$\sum(-)^m B^m = 0$$
$$i\sum(-)^m P^{mn} = B^n$$

and if $B, S, W \in (1) \Rightarrow B*S,\ S*W,\ B*W \in (1)$

$\Delta_0(\Phi_i * \Phi_j) \in (1)$

$$S * S = i\left(\theta^A \theta_A + \eta\, B * \gamma + \bar{\eta}\, B * \bar{\gamma}\right)$$

Problem: If $B \in (1)$ then $B * \gamma \notin (1)$

B should belong to diff. func. class

$S, W \in (1)$; $B \in (2)$ $\Rightarrow$ $B * \gamma \in (1)$

$$d_z B \sim \sum \left[S^{(n)}, B^{(m)} \right]$$

$O(CC) \rightarrow$

$v_2 - v_1 = 1$

$$z_\alpha \to z_\alpha - i \sum_{n=1}^{N} v^n \partial_\alpha^n \quad ; \quad \sum (-)^n v^n = 1$$

$\boxed{\text{even and odd classes of homotopies}}$

Δ_q-homotopies

$$\Delta_{q+\alpha y}\left(C * \phi(z,y) \right) = C * \Delta_{q+(1-\alpha)p+\alpha y}\, \phi$$

$$d_z f = C(y) * \phi(z,y) \Rightarrow \quad \begin{array}{l} f = \Delta_a (C * \phi) \\ f = C * \Delta_b \phi \end{array}$$

Any vertex $\gamma(\ldots C \ldots C) \sim C * \ldots + C * F(\Delta\gamma, \Delta\Delta\gamma)$

$\gamma(\omega, c, c)$ – vertex

$$S_1 = \Delta_0 (C * \gamma) = C * \Delta_p \gamma$$

$$d_2 B_2 = C * \Delta_{p_1} \gamma * C - C * C * \Delta_{p_2} \gamma = C * C * (\Delta_{p_2} \gamma - \Delta_{p_1 + 2p_2}) \gamma$$

$$B_2 = C * C * \Delta_q (\Delta_{p_2} - \Delta_{p_1 + 2p_2}) \gamma \quad ; \quad q = v_1 p_1 + v_2 p_2$$

$$\Delta B_2 = B_2^{v_1 v_2} - B_2^{v_1' v_2'} = C * C * (h_{v_1 p_1 + v_2 p_2} - h_{v_1' p_1 + v_2' p_2}) \Delta_{p_2} \Delta_{p_1 + 2p_2} \gamma$$

$\Delta B_2 = 0$ iff $v_2 - v_1 = 1$

Typical form of a vertex

$$\gamma(w,C,c) \sim C * C * h_{a_1p_1+a_2p_2} \Delta_{b_1p_1+b_2p_2} \Delta_{c_1p_1+c_2p_2}\, \gamma$$

non-local part $\sim$ $cc\int e^{i(1-(a_2-a_1)\tau_1-(b_2-b_1)\tau_2-(c_2-c_1)\tau_3)\partial_1\partial_2}$

$$\tau_1+\tau_2+\tau_3=1$$

$$a_2-a_1=b_2-b_1=c_2-c_1=1$$

exactly matches with $\boxed{\nu_2-\nu_1=1}$

$$B_2=\frac{\eta}{4i}\, C*C*\Delta_{p_1+2p_2}\Delta_{p_2}\gamma$$

Uniform y-shift freedom

$$S_1 = -\frac{\eta}{2} C * \Delta_{p+\alpha y}\gamma$$

$$W_1 = -\frac{\eta}{4i}\left(C*\omega * \Delta_{p+t+\alpha y}\Delta_{p+2t+\alpha y}\gamma - \omega * C * \Delta_{p+t+\alpha y}\Delta_{p+\alpha y}\gamma\right)$$

correspond to same $\gamma(\omega,\omega,C)$ vertex

2nd order in C

$$B_2 = \frac{\eta}{4i} C*C*\Delta_{p_1+2p_2+\alpha y}\Delta_{p_2+\alpha y}\gamma$$

preserves $\gamma(\omega, C, C)$

Conclusion

- Remarkable star-exchange formulas

$$\omega * \dots * C * \dots * C * F(\Delta\gamma, \Delta\Delta\gamma)$$

- Vertex $\gamma(\omega, C, C)$ is shown to be local
- One-parameter family of homotopies is found to match 1-to-1 with the Gelfond class of functions.