

# Failure of mean field approximation in weakly coupled dilaton gravity\*

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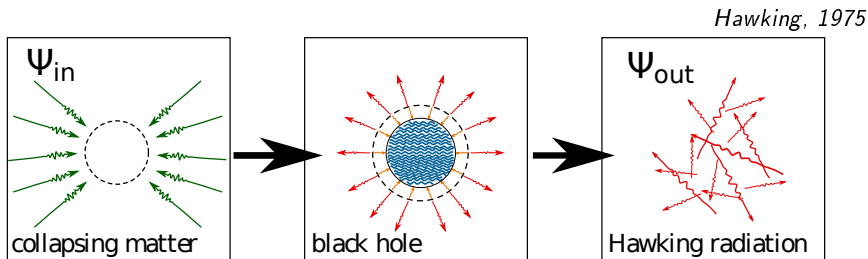
*Higher Spin Theory and Holography-7*

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\* - *in collaboration with D.G. Levkov, S.M. Sibiryakov, and Y.A. Zenkevich*

# Motivation

- Information paradox: apparent loss of quantum coherence.



$$\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \mapsto \hat{\rho}_{out} = \text{Tr}_{BH}(|\Psi_{ext}\rangle\langle\Psi_{BH}|\langle\Psi_{BH}|\langle\Psi_{ext}|), \quad \text{Tr}(\hat{\rho}_{out}^2) < 1$$

- ▶ Responses: AdS/CFT correspondence, “complementarity”, etc.
- ▶ **AMPS-firewall**. Unitarity versus Equivalence principle.

Almheiri et al, 2012

We need useful solvable models!

- Toy models: 2D dilaton gravity. Period of activity:  $t \in (1991, 1996)$ .  
Problem: apparent non-unitarity persists.
- Idea: revive 2d gravity with new semi-classical methods!
  - ▶ S-matrix as path integral: complex classical solutions.

# Outline

- 1 Weakly coupled dilaton gravity
- 2 Failure of mean field theory
- 3 Semiclassical S-matrix elements

# Weakly coupled dilaton gravity

$$S = \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2}(\nabla f)^2 \right]$$

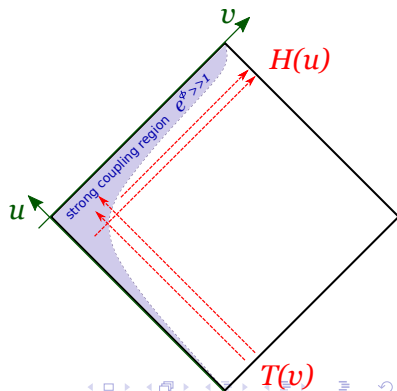
ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

In the bulk:

$$ds^2 = -e^{2\phi} dvdu, \quad f(v, u) = f_{out}(u) + f_{in}(v)$$

$$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$$

$$\partial_v^2 \mathcal{T} = (\partial_v f_{in})^2/2, \quad \partial_u^2 \mathcal{H} = (\partial_u f_{out})^2/2$$



# Weakly coupled dilaton gravity

$$S = \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2}(\nabla f)^2 \right] + 2 \int_{\partial\mathcal{M}} d\tau e^{-2\phi} (K + 2\lambda)$$

ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

ArXiv:1702.02576 [hep-th] M.F., D. Levkov, Y. Zenkevich, 2017

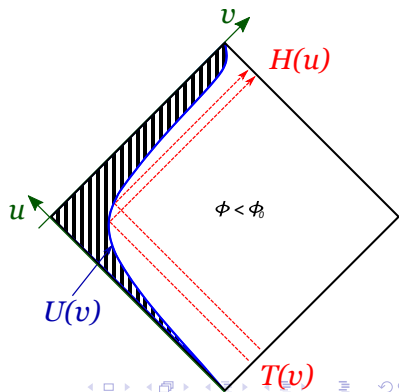
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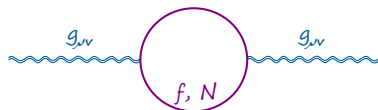
$$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$$

$$\partial_v^2 \mathcal{T} = (\partial_v f_{in})^2 / 2, \quad \partial_u^2 \mathcal{H} = (\partial_u f_{out})^2 / 2$$

- Weak coupling:  $g_{gr} = e^\phi \leq e^{\phi_0} \ll 1$
- Minimal black hole mass  
 $M_{cr} = 2\lambda e^{-2\phi_0}$
- Reflecting condition  
 $f_{out}(U(v)) = f_{in}(v)$ .



# One-loop effective action

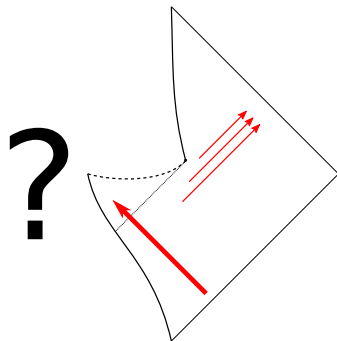


Includes Hawking radiation and backreaction on metric.

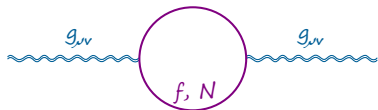
One loop from  $N = 24Q^2$  scalars  $f_i \Rightarrow$   
Liouville-Polyakov action

$$S_{PL} = -\frac{Q^2}{2} \int dx \sqrt{-g} \int dy \sqrt{-g} R \frac{1}{\square} R$$

$$S_{PL} \underset{\text{on-shell}}{=} \int d^2x \sqrt{-g} \left[ -\frac{1}{2} (\nabla \chi)^2 + Q \chi R \right]$$



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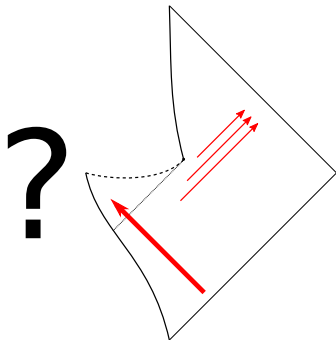
To restore solvability: *ArXiv:9206070 [hep-th]*

*J. Russo, L. Susskind, L. Thorlacius, 1992*

$$\Delta S_{RST} = -Q^2 \int d^2x \sqrt{-g} \phi R$$

Boundary terms: fixed by **Wess-Zumino**

$$\Delta S_{\text{boundary}} = 2 \int d\tau \left[ (-Q^2 \phi + Q \chi) K + \lambda Q^2 \right]$$



*M.F., D. Levkov, Y. Zenkevich, in preparation*

## Correspondence $\mathcal{Q}RST \simeq \mathcal{Q}JT$

Conjecture: correct quantization via **Weyl rescaling**:  $g_{\mu\nu} = e^{2\phi} \hat{g}_{\mu\nu}$ .

No contribution from ghosts.

A. Strominger, 1992

It yields  $S[g_{\mu\nu}, \phi, f_j] \mapsto \hat{S}[\hat{g}_{\mu\nu}, \phi, f_j] + S_A$ ,  $S_A$  - quantum anomaly term:

$$\boxed{\int [Df_j]_g e^{iS_f[f_j, g_{\mu\nu}]} = \int [Df_j]_{\hat{g}} e^{iS_A + iS_f[f_j, \hat{g}_{\mu\nu}]}, \quad S_A = \int \hat{R} \square^{-1} \hat{R} - \int R \square^{-1} R.}$$



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With field redefinition  $\Omega = e^{-2\phi} + Q^2 \phi$  we obtain **Jackiw-Teitelboim** gravity with boundary

$$\hat{S}[\hat{g}_{\mu\nu}, \Omega, f_j] = \int d^2x \sqrt{-\hat{g}} \left[ \Omega \hat{R} + 4\lambda^2 - \frac{1}{2} \sum_{j=1}^N \hat{g}^{\mu\nu} \partial_\mu f_j \partial_\nu f_j \right] + \text{boundary terms}.$$

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Statement: **JT model** minimally coupled to arbitrary matter

$$S_{JT} = \int d^2x \sqrt{-\hat{g}} \left( \hat{\phi} \hat{R} - \Lambda \right) + S_{matter}[\hat{g}_{\mu\nu}, \hat{f}] \quad \text{has unitary S-matrix.}$$

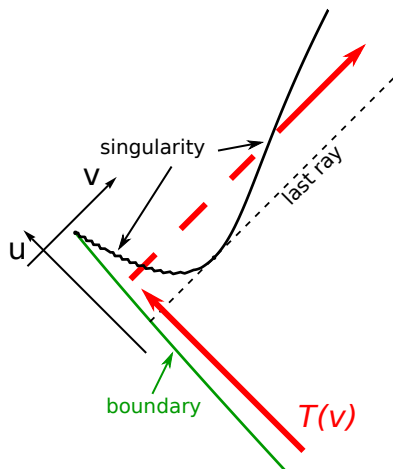
ArXiv:1706.06604 [hep-th] S. Dubovsky, V. Gorbenko, M. Mirbabayi, 2018

## Solution for black hole

$U(v)$  - boundary in light-cone coordinates  $u, v$ ;  $T(v)$  - incoming matter  $f_{in}$ .

$$\text{Boundary condition: } \partial_v U = \text{const} \left( \partial_v T + \lambda^2 U + \frac{Q^2 \partial_v^2 U}{2\partial_v U} \right)^2$$

$E > E_{cr} \Rightarrow$  2nd branch  $\bar{U}(v) \neq U(v)$  of  $\phi(v, u) = \phi_0$  is the singularity.



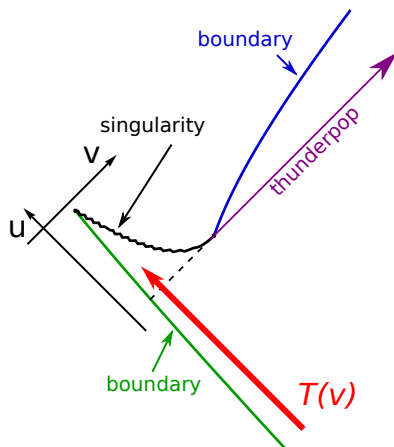
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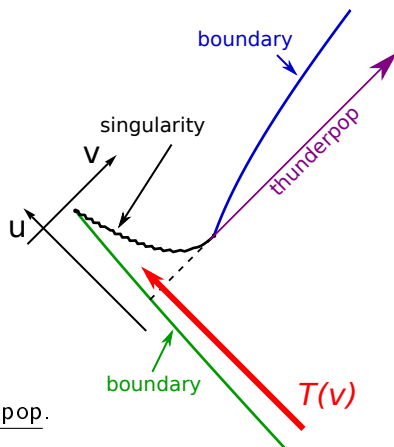
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## Problems:

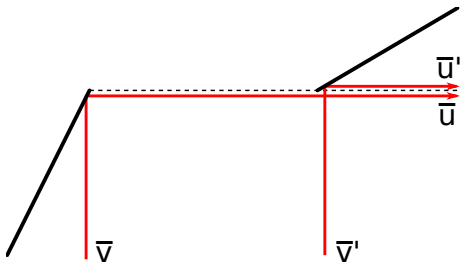
- Non-analyticity.
- Ambiguity:  $\partial_v U \Big|_{\text{end point}} - ?$
- Thunderpop:  $E_{th} \sim -\lambda Q^2$ .

$\Rightarrow$  has to introduce smearing around thunderpop.



# Failure of mean field theory

Vacuum correlation function  $G_{vac}(\bar{v}, \bar{v}') \equiv \langle f(\bar{v})f(\bar{v}') \rangle \propto \ln |\bar{v} - \bar{v}'|$



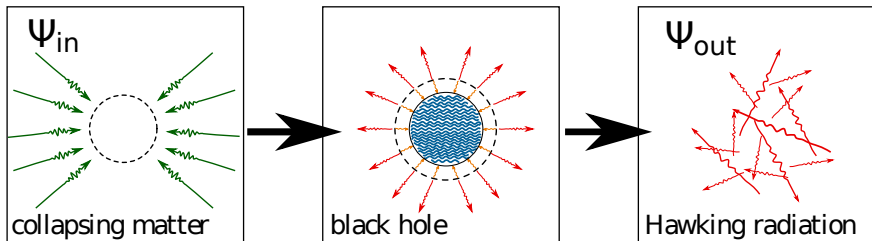
Near the thunderpop  $\bar{u}, \bar{u}' \simeq \bar{u}_{end}$ ,  
 $G(\bar{u}, \bar{u}') = \langle f_{out}(\bar{u})f_{in}(\bar{u}') \rangle = \langle f_{in}(\bar{v}(\bar{u}))f_{in}(\bar{v}(\bar{u}')) \rangle \neq G_{vac}(\bar{u}, \bar{u}')$

$\Rightarrow$  thunderbolt: particles with arbitrary large momenta  $k \sim \Delta\bar{u}^{-1}$ .

Strominger, 1994

Energy conservation  $\Rightarrow \Delta\bar{u} \simeq Q/M_{cr}$  - characteristic size of quantum area where semiclassics always fails.

# Failure of mean field theory



$$\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle = \int \mathcal{D}\Phi \Psi_{out}^* \Psi_{in} \exp\left\{\frac{i}{\hbar} S[\Phi]\right\}, \quad \Phi = \{g_{\mu\nu}, \phi, f\}$$

Semiclassics  $\Rightarrow \frac{\delta}{\delta\Phi} S = 0 \Rightarrow$  saddle point  $\Phi_s$  - can not be singular.

- MFA fails at 0th order: glued solutions are **incorrect saddle points!**
- Another possible answer: stiff boundary condition is inconsistent.
  - ▶ Analogy: Klein paradox in QM. Second quantization of the boundary?

# Semiclassical S-matrix elements

- But we want to consider the whole solution  $\Phi_s$  corresponding to  $\Psi_{in} \mapsto \Psi_{out}$  (asymptotically flat to asymptotically flat).
  - ▶ At  $E < E_{cr} \Rightarrow$  semiclassical S-matrix exists:  $\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle \approx \exp\left\{\frac{i}{\hbar} S[\Phi_c]\right\}$
  - ▶ At  $E > E_{cr} \Rightarrow$  no flat asymptotics of classical solutions at  $t \rightarrow +\infty \Rightarrow$  ill-defined S-matrix.
- Idea: obtain physical solutions at  $E > E_{cr}$  via analytic continuation.
  - ▶ Problem: many possibilities for deformation: suppressed, unphysical solutions. We need leading contribution!

We need a criterion to chose physical branch at  $E > E_{cr}$ ,  $\Im m E \rightarrow 0$ .



# Criterion: start from the “shell” model

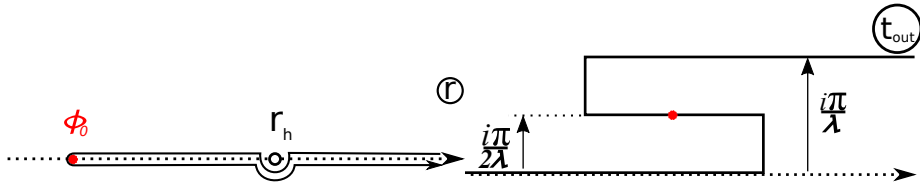
$$S = -m \int d\tau \sqrt{-g_{\mu\nu} \frac{dy^\mu}{d\tau} \frac{dy^\nu}{d\tau}} + S_{gravity}$$

Junction condition on shell:  $\left(\frac{dr}{d\tau}\right)^2 - \left(\frac{M}{m} + \frac{m}{8\lambda} e^{-2\lambda r}\right)^2 + 1 = 0$

Well-known analytic continuation:  $M \mapsto M + i\varepsilon, \quad \varepsilon \rightarrow +0$

*ArXiv:9907001 [hep-th] M. Parikh, F. Wilczek, 2000*

*ArXiv:1503.07181 [hep-th] F. Bezrukov, D. Levkov, S. Sibiriakov, 2015*

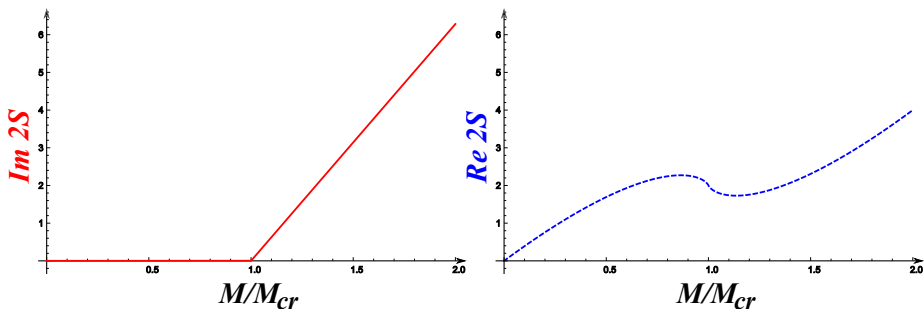


Can be generalized to field theory.

# “Shell” model: result

$$2\mathfrak{I}m S_{tot} = S_{BH} - S_{cr}, \quad S = \frac{M}{T_H} = \frac{2\pi}{\lambda} M$$

Plots for  $m = 0$ :



Probability:

$$\mathcal{P}_{fi} \approx \exp(-S_{BH} + S_{cr}).$$

Non-entropic suppression: Nature abhors discontinuity.

# Conclusions

- Mean field theory:
  - ▶ Either not a good approximation...
  - ▶ ...or models with stiff boundaries are not self-consistent.
- Complex semiclassical method:
  - ▶ Reliable analytic continuation for shells.
  - ▶ Relevant solutions for fields.
  - ▶ Non-entropic suppression.

Thank you for attention!