Higher Spin Theory
and Holography-7
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BRST-BV approach to continuous spin field

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Plan

1) **Massless and massive continuous spin field in $R^{d,1}$** metric-like formulation in terms of double-traceless fields

2) **Metric-like formulation in terms of traceless fields**

3) **BRST-BV formulation** (+ interrelation with metric-like approach)

BRST Lagrangian for continuous massless spin in $R^{3,1}$

$\mathcal{L} \neq \langle\Phi|Q|\Phi\rangle$
$\mathcal{L} = \langle\Phi|Q|\Phi\rangle + \Lambda|\Phi\rangle$ **A. Bengtsson (2013)**

Metric-like Lagrangian for continuous massless spin
bosonic in $R^{3,1}$ Schuster and Toro (2014)

fermionic in $R^{3,1}$
Bekaert, Najafizadeh, Setare, (2016)

bosonic and fermionic massless and massive in $R^{d,1}$ (and AdS too)
RRM, (2016,2017)

Metric-like approach

Continuous spin field via deformation of tower decoupled Fang-Fronsdal fields

method by Zinoviev 2001

Field content

Totally symmetric double-traceless field in $R^{d,1}$

$$\phi^{a_1 \dots a_n}(x)\,, \qquad n = 0, 1, \dots \infty$$

$$\phi^{aabba_5 \dots a_n} = 0$$

Lagrangian for decoupled Fang-Fronsdal fields

$$\mathcal{L} = \sum_{n=0}^{\infty} \mathcal{L}_{\mathbf{n}}$$

$\mathcal{L}_{\mathbf{n}}$ - Fronsdal Lagrangian for spin n-field

$$[\bar{\alpha}^a, \alpha^b] = \eta^{ab}, \qquad [\bar{v}, v] = 1$$

$$\bar{\alpha}^a|0\rangle = 0 \qquad \bar{v}|0\rangle = 0$$

$$|\phi\rangle = \phi(x,\alpha,\upsilon)|0\rangle$$

$$\phi(x,\alpha,\upsilon) = \sum_{n=0}^{\infty} \upsilon^n \alpha^{a_1} \ldots \alpha^{a_n} \phi^{a_1 \ldots a_n}(x)$$

$$(N_\alpha - N_\upsilon)|\phi\rangle = 0$$

$$N_\alpha \equiv \alpha^a \bar{\alpha}^a \qquad N_\upsilon = \upsilon\bar{\upsilon}$$

gauge transformation parameters

$$\xi^{a_1 \ldots a_n}(x)\,, \qquad n = 0, 1, \ldots \infty$$

$$|\xi\rangle = \sum_{n=0}^{\infty} \upsilon^{n+1} \alpha^{a_1} \ldots \alpha^{a_n} \xi^{a_1 \ldots a_n}(x)|0\rangle$$

traceless

$$\xi^{aaa_3 \ldots a_n} = 0$$

Lagrangian for massless fields in $R^{d,1}$

$$\mathcal{L} = \langle\phi|(1 - \frac{1}{4}\alpha^2\bar{\alpha}^2)\Box|\phi\rangle$$

$$+ \langle\bar{L}\phi|\bar{L}\phi\rangle$$

$$\bar{L} = \alpha\partial - \frac{1}{2}\alpha\partial\bar{\alpha}^2$$

$$\alpha\partial = \alpha^a\partial^a \qquad \alpha^2 = \alpha^a\alpha^a$$

$$\bar{\alpha}\partial = \bar{\alpha}^a\partial^a \qquad \bar{\alpha}^2 = \bar{\alpha}^a\bar{\alpha}^a$$

massless in flat: gauge transformations

$$\delta|\phi\rangle = \alpha\partial|\xi\rangle$$

massive in flat

$$\phi(x,\alpha,\upsilon) = \sum_{n=0}^{s} \upsilon^n \alpha^{a_1} \ldots \alpha^{a_n} \phi^{a_1 \ldots a_n}(x)$$

$$\mathcal{L} = \langle\phi|(1-\frac{1}{4}\alpha^2\bar{\alpha}^2)(\Box - \mathbf{m^2})|\phi\rangle$$

$$+ \langle\bar{L}_{\mathbf{m}}\phi|\bar{L}_{\mathbf{m}}|\phi\rangle$$

$$\bar{L}_{\mathbf{m}} = \bar{L} + \mathbf{\Delta\bar{L}_m}$$

$$\mathbf{\Delta\bar{L}_m} = (1-\alpha^{\mathbf{2}}\frac{1}{2(2N_\alpha+d)}\bar{\alpha}^{\mathbf{2}})\mathbf{\bar{e}_m}$$

$$+ \bar{\alpha}^{\mathbf{2}}\mathbf{e_m}$$

$$\delta|\phi\rangle = \mathbf{G}|\xi\rangle$$

$$\mathbf{G} = \alpha\partial - \mathbf{e_m} + \alpha^{\mathbf{2}}\bar{\mathbf{e}}_{\mathbf{m}}$$

$$
\begin{aligned}
\delta\phi^{a_1\dots a_n} &= \partial^{(a_1}\xi^{a_2\dots a_n)} \\
&+ \mathbf{e_m}\xi^{\mathbf{a_1\dots a_n}} \\
&+ \mathbf{\bar{e}_m}\eta^{(\mathbf{a_1 a_2}\xi^{\mathbf{a_3\dots a_n}})}
\end{aligned}
$$

deformation procedure **Zinoviev 2001**

massive in flat

$$\mathbf{e_m} = f\bar{\upsilon} \qquad \bar{\mathbf{e}}_\mathbf{m} = \upsilon f$$

$$f = \sqrt{F_\upsilon}$$

$$F_\upsilon = (s - N_\upsilon)m^2$$

Continuous massless in flat

$$\mathbf{e_m} = f\bar{\upsilon} \qquad \bar{\mathbf{e}}_\mathbf{m} = \upsilon f$$

$$f = \sqrt{F_\upsilon}$$

$$F_\upsilon = \kappa^2$$

Continuous massive in flat

$$\mathbf{e_m} = f\bar{\upsilon} \qquad \bar{\mathbf{e}}_\mathbf{m} = \upsilon f$$

$$f = \sqrt{F_\upsilon}$$

$$F_\upsilon = \kappa^2 - N_\upsilon(N_\upsilon + d - 3)m^2$$

κ and $m-$ dimensionfull parameters

Classical unitarity

1)

$$\mathcal{L} = \langle \phi | \Box | \phi \rangle + \ldots$$

2)

$$\mathcal{L} = \mathcal{L}^\dagger$$

2) $\Longrightarrow$

$$2a) \quad \mathbf{e}_{\mathbf{m}}^\dagger = \bar{\mathbf{e}}_{\mathbf{m}}$$

$$2b) \quad \mathbf{F}_v \geq 0$$

All previously known classically unitary systems turns out to be associated with unitary representations of space-time symmetry algebras

$\kappa^2 > 0$ **massless continuous**

unitary and irreducible

$\kappa^2 > 0\,, \quad m^2 < 0$ **tachyonic continuous**

unitary and irreducible

conjecture

massive classically unitary continuous spin field

is associated with

tachyonic UIR of Poincaré algebra

reducible case

$$F_v(s) = 0 \quad \Longrightarrow$$

$$\kappa^2 = s(s + d - 3)m^2$$

$$F_v = (s - N_v)(s + d - 3 + N_v)m^2$$

$$|\phi\rangle = |\phi^{0,s}\rangle + |\phi^{s+1,\infty}\rangle$$

$$|\phi^{M,N}\rangle \equiv \sum_{n=M}^{N} \upsilon^n \alpha^{a_1} \ldots \alpha^{a_n} \phi^{a_1 \ldots a_n} |0\rangle$$

$$\mathcal{L}(\phi) = \mathcal{L}(\phi^{0,s}) + \mathcal{L}(\phi^{s+1,\infty})$$

$$\mathbf{m^2 > 0}$$

$\phi^{0,s}$ – classically **unitary**

$\phi^{s+1,\infty}$ – classically **non-unitary**

Metric like formulation in terms of traceless fields

$$\phi^{a_1\ldots a_s} \quad = \quad \phi_{I}^{a_1\ldots a_s} \oplus \phi_{II}^{a_1\ldots a_{s-2}}$$

double-traceless = traceless ⊕ traceless

$$|\phi\rangle = |\phi_{I}\rangle + \alpha^2 |\phi_{II}\rangle$$

$$\mathcal{L} = \langle\phi_{\mathbf{I}}|(\Box - m^2)|\phi_{\mathbf{I}}\rangle - \langle\phi_{\mathbf{II}}|(\Box - m^2)|\phi_{\mathbf{II}}\rangle$$

$$+ \langle\bar{L}_m\phi|\bar{L}_m\phi\rangle$$

$$\bar{L}_m|\phi\rangle = \bar{L}_I|\phi_{\mathbf{I}}\rangle + L_{II}|\phi_{\mathbf{II}}\rangle$$

BRST-BV formulation in terms of

traceless fields

Oscillators in **metric-like approach**

$$\alpha^{\mathbf{a}}, \upsilon$$

Fields in **metric-like approach**

$$\phi(\alpha^{\mathbf{a}}, \upsilon)$$

Oscillators in **BRST-BV** approach

$$\alpha^{\mathbf{a}}, \upsilon, \theta, \eta, \rho$$

Fields and anti-fields in **BRST-BV** approach

$$\Phi(\alpha^{\mathbf{a}}, \upsilon, \theta, \eta, \rho)$$

θ – Grassmann odd coordinate

η ρ Grassmann odd oscillators (creation operators)

$$\{\bar{\rho}, \eta\} = \mathbf{1}\,, \qquad \{\bar{\eta}, \rho\} = \mathbf{1}$$

$$\bar{\eta}|0\rangle = 0 \qquad \bar{\rho}|0\rangle = 0$$

$$|\Phi\rangle = \Phi(\alpha^{\mathbf{a}}, \upsilon, \theta, \eta, \rho)|0\rangle$$

$$(\mathbf{N}_\alpha + \mathbf{N}_\eta + \mathbf{N}_\rho - \mathbf{N}_\upsilon)|\Phi\rangle = 0$$

$$\bar{\alpha}^2|\Phi\rangle = 0$$

traceless

$$|\Phi\rangle = |\phi\rangle + \theta|\phi_*\rangle \,,$$

$$|\phi\rangle = |\phi_{\mathbf{I}}\rangle + \rho|\mathbf{c}\rangle + \eta|\bar{\mathbf{c}}\rangle + \rho\eta|\phi_{\mathbf{II}}\rangle$$

$$|\phi_*\rangle = |\phi_{\mathbf{I}*}\rangle + \rho|\bar{\mathbf{c}}_*\rangle + \eta|\mathbf{c}_*\rangle + \rho\eta|\phi_{\mathbf{II}*}\rangle$$

$$|\phi\rangle - \textbf{fields}$$

$$|\phi_*\rangle - \textbf{antifields}$$

Batalin-Vilkovisky bracket

$$(\phi(\mathbf{x}), \phi_*(\mathbf{y})) = \delta(x-y)$$

BRST-BV Lagrangian

Siegel 1986

$$S = \int d^d x \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2} \int \mathbf{d}\theta \langle \mathbf{\Phi} | \mathbf{Q} | \mathbf{\Phi} \rangle$$

BRST charge

$$\mathbf{Q} = \theta(\Box - \mathbf{M}^2) + \mathbf{M}^{\eta \mathbf{a}} \partial^{\mathbf{a}} + \mathbf{M}^{\eta} + \frac{1}{2} \mathbf{M}^{\eta\eta} \partial_{\theta}$$

$$\mathbf{Q^2 = 0} \qquad \Longrightarrow$$

$$\{M^{\eta a}, M^{\eta b}\} = -\eta^{ab} M^{\eta\eta}$$

$$\{M^{\eta}, M^{\eta}\} = M^2 M^{\eta\eta}$$

$$[M^2, M^{\eta a}] = 0\,, \qquad [M^2, M^{\eta}] = 0$$

$$[M^2, M^{\eta\eta}] = 0$$

$$\{M^{\eta a}, M^{\eta}\} = 0\,, \qquad [M^{\eta a}, M^{\eta\eta}] = 0$$

$$[M^{\eta}, M^{\eta\eta}] = 0$$

$$\delta|\Phi\rangle = Q|\Xi\rangle$$

$$|\Xi\rangle = \Xi(\alpha, \upsilon, \theta, \eta, \rho)|0\rangle$$

$$(\mathbf{N}_\alpha + \mathbf{N}_\eta + \mathbf{N}_\rho - \mathbf{N}_\upsilon)|\Xi\rangle = 0$$

$$\bar{\alpha}^2|\Xi\rangle = 0$$ **traceless**

$$M^{\eta a} = \eta \mathbf{g}_{\rho\upsilon}\bar{\alpha}^a + A^a\bar{\mathbf{g}}_{\eta\upsilon}\bar{\eta}\,,$$

$$M^{\eta\eta} = 2\eta\bar{\eta}\,,$$

$$M^{\eta} = \eta\upsilon\mathbf{l}_{\rho\upsilon} + \bar{\mathbf{l}}_{\eta\upsilon}\bar{\upsilon}\bar{\eta}\,,$$

$$M^2 = m^2\,,$$

$$A^a = \alpha^a - \alpha^2\frac{1}{2N_\alpha + d}\bar{\alpha}^a$$

$$\mathbf{g}_{\rho\upsilon} = \left(\frac{2N_\upsilon + d - 4 - 2N_\rho}{2N_\upsilon + d - 4}\right)^{1/2},$$

$$\bar{\mathbf{g}}_{\eta\upsilon} = -\left(\frac{2N_\upsilon + d - 4 - 2N_\eta}{2N_\upsilon + d - 4}\right)^{1/2},$$

$$\mathbf{l}_{\rho\upsilon} = \mathbf{f}\left(\frac{2N_\upsilon + d - 2 - 2N_\rho}{2N_\upsilon + d - 2}\right)^{-1/2},$$

$$\bar{\mathbf{l}}_{\eta\upsilon} = \mathbf{f}\left(\frac{2N_\upsilon + d - 2 - 2N_\eta}{2N_\upsilon + d - 2}\right)^{-1/2},$$

$$\mathbf{f} = \left(\frac{1}{(N_\upsilon + 1)(2N_\upsilon + d - 2)}\mathbf{F}_\upsilon\right)^{\mathbf{1/2}}$$

$$\mathbf{F}_\upsilon = \kappa^{\mathbf{2}} - \mathbf{N}_\upsilon(\mathbf{N}_\upsilon + \mathbf{d} - \mathbf{3})\mathbf{m}^{\mathbf{2}}$$

BRST-BV $\Longrightarrow$ metric-like formulation.

Fields and antifields with nonzero ghost number = 0

$$|c\rangle = 0, \qquad |\bar{c}\rangle = 0$$

$$|\phi_{*I}\rangle = 0, \qquad |c_*\rangle = 0, \qquad |\phi_{*II}\rangle = 0$$

$$\mathcal{L} \;=\; \langle\phi_I|(\Box - m^2)|\phi_I\rangle - \langle\phi_{II}|(\Box - m^2)|\phi_{II}\rangle$$
$$-\;\; \langle\bar{c}_*|\bar{L}_I|\phi_I\rangle - \langle\bar{c}_*|L_{II}|\phi_{II}\rangle - \frac{1}{2}\langle\bar{c}_*||\bar{c}_*\rangle\,,$$

$$-|\bar{c}_*\rangle = \bar{L}_I|\phi_I\rangle + |L_{II}|\phi_{II}\rangle\,,$$

Siegel gauge, global BRST and antiBRST transformations

$|\phi_*\rangle = 0$ **Siegel gauge**

$$\mathcal{L} = \frac{1}{2}\langle\phi_I|(\Box - m^2)|\phi_I\rangle - \frac{1}{2}\langle\phi_{II}|(\Box - m^2)|\phi_{II}\rangle$$

$$+ \langle\bar{c}|(\Box - m^2)|c\rangle\,,$$

BRST

$$\mathsf{s}|\phi_I\rangle = G_I|c\rangle\,, \quad \mathsf{s}|\phi_{II}\rangle = \bar{G}_{II}|c\rangle$$

$$\mathsf{s}|c\rangle = 0 \qquad \mathsf{s}|\bar{c}\rangle = \bar{L}_I|\phi_I\rangle + L_{II}|\phi_{II}\rangle\,,$$

antiBRST

$$\bar{\mathsf{s}}|\phi_I\rangle = G_I|c\rangle\,, \quad \bar{\mathsf{s}}|\phi_{II}\rangle = \bar{G}_{II}|\bar{c}\rangle\,,$$

$$\bar{\mathsf{s}}|\bar{c}\rangle = 0 \qquad \bar{\mathsf{s}}|c\rangle = -\bar{L}_I|\phi_I\rangle - L_{II}|\phi_{II}\rangle$$

$$\mathsf{s}^2 = 0\,, \qquad \bar{\mathsf{s}}^2 = 0\,, \qquad \mathsf{s}\bar{\mathsf{s}} + \bar{\mathsf{s}}\mathsf{s} = 0$$

$$(\Box - m^2)|c\rangle = 0$$