Quantum Higher-Spin Gravity

HSTH-VII

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Intro Comments

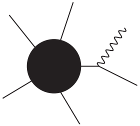
- Recent results on higher spin gravities in AdS and 50 years old results in flat space start to converge to each other and the basic statements about problems in flat space have a direct analogy in the AdS case

- The plan is to review what these general statements are and in which sense AdS $\sim$ flat for higher spins

- Then we (review how to) construct and quantize a model of higher spin gravity in flat space and discuss the main features that eventually lead to a consistent quantum higher spin theory

Flat Space: HSGRA cannot exist

It has been long known that massless particles with $s > 2$ are somewhat special (do not want to exist). One of most powerful no-go theorems against HSGRA is the Weinberg low energy theorem:



- $s = 1$ we get charge conservation $\sum q_i$=0
- $s = 2$ we get equivalence principle $\sum g_i\, p^i_\mu$=0
- $s > 2$ we get too many conservation laws

$$\sum_i g_i\, p^i_{\mu_1} ... p^i_{\mu_{s-1}} = 0$$

May be massless higher spin fields confine? or do not exist?

Flat Space: HSGRA cannot exist

Coleman-Mandula theorem constrains the symmetries of nontrivial S-matrix to be a direct product of Poincare and inner symmetries.

$$\text{argument :} \qquad Q_{\mu_1\ldots\mu_{s-1}} \sim \sum_i p^i_{\mu_1}\ldots p^i_{\mu_{s-1}} \sim 0$$

so that we again get too many conservation laws.

Exceptions: SUSY and 2d.

Flat Space: HSGRA cannot exist

Another local and description-dependent no-go is due to Deser and Aragone. If we use Fronsdal fields

$$\delta\Phi_{\mu_1\dots\mu_s} = \partial_{\mu_1}\xi_{\mu_2\dots\mu_s} + \text{permutations}$$

then the standard spell $\partial \to \nabla$ in the two-derivative $\int(\partial\Phi)^2$-type action does not work: $[\nabla, \nabla]$ will bring the four-index Riemann tensor.

This is avoided by low spins, $s = 0, \frac{1}{2}, 1$, and results only in the Ricci-part for $s = \frac{3}{2}, 2$.

Flat Space: HSGRA may exist

The two no-go theorems constrain the physics at infinity by stating that $S = 1$ (more or less) once at least one massless higher spin particle is present

However, they have little to say about possible local interactions

Neither do they imply that an example can be constructed within the local field theory framework

Long ago some local cubic interactions were found by Brink, Bengtsson[2], Linden using the light-cone approach. How these local effects comply with global restrictions?

Let's now move to AdS HiSGRA and see what is the difference

HiSGRA in AdS

The most basic higher-spin AdS/CFT duality conjecture Klebanov, Polyakov; Sezgin, Sundell says that

- free vector model (fancy name for free scalars) should be dual to a higher-spin theory whose spectrum contains totally-symmetric massless fields
- critical vector model (Wilson-Fisher) should be dual to the same theory for $\Delta = 2$ boundary conditions on $\Phi(x)$. This duality is kinematically related to the first one (Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad).

$$J_{a_1\ldots a_s} = \phi\partial_{a_1}\ldots\partial_{a_s}\phi \qquad \leftrightarrow \qquad \delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s}$$

HS Current Conservation implies Free CFT, i.e. given a CFT with stress-tensor J_2 and at least one higher-spin current J_s, one can prove Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab, Stanev that

- there are infinitely many higher-spin currents and spin is unbounded;
- correlation function (higher-spin algebra) corresponds to free CFT (which CFT, depends on the spectrum)

This essentially proves the duality no matter how the bulk theory is realized. Loops still need to be shown to vanish (be proportional to the tree result)

This is a generalization of the Coleman-Mandula theorem to AdS/CFT: HS symmetries imply free CFT, i.e. $S = 1$

AdS versus Flat

With a 50 years delay we see that asymptotic higher spin symmetries

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s}$$

always completely fix (holographic) S-matrix to be

$$\text{S-matrix} = \begin{cases} 1, & \text{flat space} \\ \text{free CFT}, & \text{asymptotic AdS} \\ ???, & \text{some other space} \end{cases}$$

There is not much difference between flat and AdS space: S-matrix is already known and the theories should exhibit some sort of 'pathological' non-locality (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight; Ponomarev).

Flat Space

Let's move back to flat space since AdS complicates things without bringing anything significantly new

Unless one gives S-matrix right away, the light-cone approach seems to be the most fundamental approach to local dynamics

The idea of the light-cone approach is that QFT is about writing explicitly P^A and J^{AB}

$$\begin{aligned}
[P^A, P^B] &= 0 \\
[J^{AB}, P^C] &= P^A\eta^{BC} - P^B\eta^{AC} \\
[J^{AB}, J^{CD}] &= J^{AD}\eta^{BC} - J^{BD}\eta^{AC} - J^{AC}\eta^{BD} + J^{BC}\eta^{AD}
\end{aligned}$$

where the generators are field-dependent ($p = (p^-, p^+, p_\perp)$), e.g.

$$H \equiv P^- = \int \Phi(-p)\frac{\vec{p}_\perp^2}{2p^+}\Phi(p) + \mathcal{O}(\Phi^3)$$

Flat Space

+ no extra assumptions, just study the interactions of a given set of particles;
+ manifestly Poincare-invariant S-matrix;

- not manifestly Lorentz-covariant expressions;

+ independent of the description: gauge potentials/dual gauge potentials/curvatures/set of auxiliary fields;

- quantum computations are harder than in the covariant methods;
- most of the covariant structures, e.g. diffeomorphisms, get lost;

+ more fundamental is only S-matrix itself;
+ manifest unitarity, control over degrees of freedom;

Flat Space

Most of the generators stay free and one has to solve for

$$[H, J^{a-}] = 0$$

or perturbatively

$$[H_2, \delta J^{a-}] = [J_2^{a-}, \delta H]$$

which looks like one equation for two functions:

$$\delta J^{a-} \sim \frac{[J_2^{a-}, \delta H]}{\displaystyle\sum_i \frac{(p_\perp^i)^2}{2p^+}}$$

Looks like we have one equation for two functions:

$$\delta J^{a-} \sim \frac{[J_2^{a-}, \delta H]}{\displaystyle\sum_i \frac{(p_\perp^i)^2}{2p^+}}$$

Imposing locality is crucial! Light-cone approach becomes nontrivial when we avoid transverse derivatives, $p_\perp$ in denominators.

Unless locality is imposed, any δH looks like an ok formal deformation and gives some δJ!

If we need just correct free limit and formal consistency, then any δH is ok. This works the same way in AdS.

Flat Space

In $4d$ a massless spin-$|\lambda|$ field equals two scalars, $\Phi^{\pm\lambda}$.

Brink, Bengtsson^2, Linden; Metsaev showed that there exists δH:

$$\delta H \sim C^{\lambda_1,\lambda_2,\lambda_3} \int V^{\lambda_1,\lambda_2,\lambda_3} \Phi^{\lambda_1} \Phi^{\lambda_2} \Phi^{\lambda_3} + c.c.$$

$$V^{\lambda_1,\lambda_2,\lambda_3} = \frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} \sim [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[13]^{\lambda_1+\lambda_3-\lambda_2}$$

where $\beta \equiv p^+$ and $\mathbb{P}_{12} = p_1\beta_2 - p_2\beta_1$ and similarly for the complex conjugate.

$C^{\lambda_1,\lambda_2,\lambda_3}$ and $\bar{C}^{\lambda_1,\lambda_2,\lambda_3}$ are any numbers so far.

Flat Space

Now, $(+s, -s, 2)$ gives a two-derivative coupling to gravity

$$V^{\lambda_1,\lambda_2,\lambda_3} = \frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + c.c.$$

so we can avoid the Deser-Aragone argument. Also, there are no higher-spin gauge symmetries, so the Coleman-Mandula theorem is avoided.

$C^{\lambda_1,\lambda_2,\lambda_3}$ and $\bar{C}^{\lambda_1,\lambda_2,\lambda_3}$ are any numbers so far.

But the existence of cubic vertices does not yet entail existence of any theory (Example: for YM, cubic vertices exist for any anti-symmetric f_{ijk} and it is the quartic closure of the Poincare algebra that imposes Jacobi identity)

We need to go to the quartic order and higher

Flat Space

One can rediscover the equivalence principle by trying to couple, say scalar to gravity ($C^{0,0,2} = C^{2,2,-2}$):

$$H_3 = \Phi^2 \Phi^2 \Phi^{-2} \bar{\mathbb{P}}^2 C^{2,2,-2} + \Phi^0 \Phi^0 \Phi^2 \bar{\mathbb{P}}^2 C^{0,0,2}$$

Analogously, one can see that the equivalence principle extends to all spins

$$s - s - 2: \qquad\qquad C^{s,-s,2} = C^{2,2,-2} = g\, l_{pl}$$

It was shown by Metsaev that the necessary condition for the quartic closure is

$$C^{\lambda_1,\lambda_2,\lambda_3} = \frac{g(l_{pl})^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma[\lambda_1+\lambda_2+\lambda_3]}$$

and the same for $\bar{C}$ if we want a parity even theory.

Complete chiral HiSGRA is obtained by setting $\bar{C}=0$ (Ponomarev, E.S.):

$$S=\sum_\lambda \int \Phi^{-\lambda}\, p^2\, \Phi^{\lambda} + \sum_{\lambda_i} C^{\lambda_1,\lambda_2,\lambda_3} \int V^{\lambda_1,\lambda_2,\lambda_3} \Phi^{\lambda_1} \Phi^{\lambda_2} \Phi^{\lambda_3}$$

where the couplings discriminate negative helicities

$$C^{\lambda_1,\lambda_2,\lambda_3}=\frac{g(l_{pl})^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma[\lambda_1+\lambda_2+\lambda_3]}$$

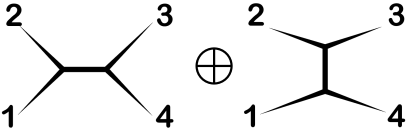
One can also add color (Metsaev) leading to higher-spin glue. The theory is nontrivial and contains parts of YM and EH actions.

Once we have a complete theory, it is interesting to quantize it and see how it complies with the no-go's

Chiral Higher Spin Gravity

First, let's have a look at trees. Using higher-spin glue allows us to look at color-ordered amplitudes only.

The four-point



$$\sim \frac{(\bar{\mathbb{P}}_{12}+\bar{\mathbb{P}}_{34})^{\Lambda_4-2}\beta_2\,\boldsymbol{p}_4^2}{\Gamma(\Lambda_4-1)\beta_4\mathbb{P}_{12}\mathbb{P}_{23}}$$

vanishes on-shell

Chiral Higher Spin Gravity

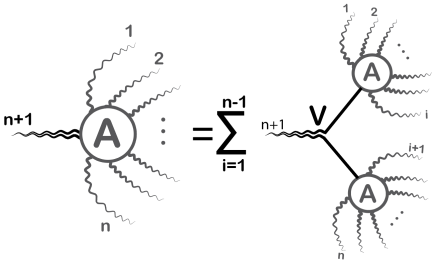
First, let's have a look a trees. Using higher-spin glue allows us to look at color-ordered amplitudes only.

The five-point

$$+ \ldots \sim \frac{(\bar{\mathbb{P}}_{45}+\bar{\mathbb{P}}_{13}+\bar{\mathbb{P}}_{12}+\bar{\mathbb{P}}_{23})^{\Lambda-3}}{\Gamma(\Lambda_5-2)\prod_{i=1}^{5}\beta_i^{\lambda_i-1}}\frac{\beta_2\beta_3\,\boldsymbol{p}_5^2}{8\beta_5\mathbb{P}_{12}\mathbb{P}_{23}\mathbb{P}_{34}}$$

vanishes on-shell

Now we can use an obvious identity (Berends, Giele)



which gives

$$A_n \sim \frac{1}{\Gamma(\Lambda_n-(n-3))\prod_{i=1}^{n}\beta_i^{\lambda_i-1}}\frac{\alpha_n^{\Lambda_n-(n-2)}\beta_2 ... \beta_{n-2}\,\boldsymbol{p}_n^2}{\beta_n\mathbb{P}_{12}...\mathbb{P}_{n-2,n-1}}$$

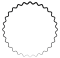
$\alpha_n = \sum_{i<j}^{n-2}\bar{\mathbb{P}}_{ij} + \bar{\mathbb{P}}_{n-1,n}$

Chiral Higher Spin Gravity

At least at the tree-level we do not see any signs of higher spin interactions in S-matrix (at infinity) due to the coupling conspiracy. This is in agreement with the no-go's

Chiral Higher Spin Gravity

The simplest loop corrections are vacuum diagrams. There is a difference between one-loop and higher loops.

 :

$$Z_{\text{1-loop}} = \frac{1}{(z_0)^{1/2}} \prod_{s>0} \frac{(z_{s-1})^{1/2}}{(z_s)^{1/2}}\,,$$

This should count the total number of degrees of freedom $Z_{\text{1-loop}} = (z_0)^{\nu_0/2}$. It was argued (Tseytlin, Beccaria) that it should be understood as

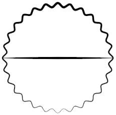
$$\nu_0 = \sum_\lambda 1 = 1 + 2\sum_{s=1}^{\infty} 1 = 1 + 2\zeta(0) = 0\,,$$

Much more nontrivial examples of one-loop det's in AdS (Klebanov, Giombi, Tseytlin, Beccaria, Bekaert, Joung, Lal, E.S., Gunaydin, Tung, ...) show that the above prescription is correct.

Chiral Higher Spin Gravity

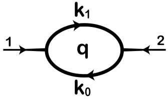
The simplest loop corrections are vacuum diagrams. There is a difference between one-loop and higher loops.

Higher vacuum loops vanish due to the coupling conspiracy: sum over all helicities must be zero, but in order for a vertex to contribute the sum must be positive. For example,

 $= 0$

since both $(\lambda_1 + \lambda_2 + \lambda_3)$ and $-(\lambda_1 + \lambda_2 + \lambda_3)$ cannot be positive

The legged diagrams are supposed to be the most difficult ones. Vanishing of tree amplitudes should improve the behaviour of loop diagrams.



$$= \frac{\nu_0 (l_p)^{\Lambda_2-2}}{\Gamma[\Lambda_2-1]} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{\mathbb{P}}^2_{k_0-q,p}\delta_{\Lambda_2,2}}{(q-k_0)^2(q-k_1)^2}\,,$$

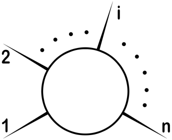
where most importantly we have an overall factor

$$\nu_0 = \sum_\lambda 1 = 0$$

which is known to vanish. Anyway, the integral can be regularized and shown to be finite.

Chiral Higher Spin Gravity

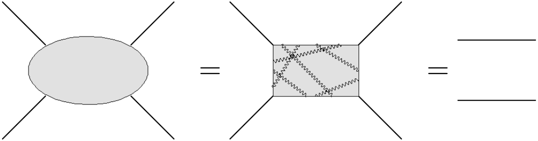
General loop diagram can be decomposed into elementary sunrise diagrams



$$= \frac{\nu_0 (l_p)^{\Lambda_2 - 2}}{\Gamma[\Lambda_2 - 1]} \int \frac{d^4 q}{(2\pi)^4} \frac{\bar{\mathbb{P}}^2_{k_0 - q, p} \delta_{\Lambda_2, 2}}{(q - k_0)^2 (q - k_1)^2} \,,$$

Crucially, they all have an overall factor of $\nu_0 = 0$.

Therefore, all loops vanish! We have **coupling conspiracy**



Flat space summary

- Really many no-go's
- Light-cone allows to avoid all of them in $4d$
- Quantum Chiral HiSGRA does exist
- The only way out seems to have **coupling conspiracy**: local interactions conspire to get $S=1$
- Some stringy features are still present in the form of $\sum_\lambda 1 = 0$
- non-chiral HiSGRA is unlikely to exist (recent: Roiban, Tseytlin; Taronna; Ponomarev, E.S.) in the usual sense: parity preserving interactions will face non-localities. One could try to achieve $S=1$ with some sort of non-locality — flat space reconstruction.
- Locality+parity=no HiSGRA in flat space

Summary

- Asymptotic higher spin symmetry works the same way both in flat and AdS spaces: completely fixes the S-matrix;
- Nevertheless it is (was) unclear if a concrete example can be constructed and why then it would comply with the no-go's. Chiral HiSGRA gives an (the only) example (very close is the conformal HiSGRA);
- Both in flat and AdS the non-chiral theory can be (re)constructed by inverting $S = 1$ or $S =$ free CFT at the price of some (not field theoretical) non-locality;
- What are other interesting observables? (since S-matrix is already known). What is the meaning of the finite (Gross-Mende) loop amplitudes in the Chiral Theory?

Thank you for your attention!