On non-abelian interactions of self-dual antisymmetric tensors in 6 dimensions

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• 6d model of a set of self-dual 2-form B-fields
interacting with a non-abelian vector A-field in 5d plane

• may be related to (non-local) interacting theory of B-fields

• compute divergent part of one-loop effective action:
$(DF)^2 + F^3$ structure

• discuss possible cancellation

Motivation:

• interacting theory of non-abelian B-fields:
6d CFT's, theory of multiplet M5-branes

• single M5-brane: 11d sugra solution –
free 6d CFT – $(2,0)$ tensor multiplet as w-volume theory:
selfdual $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$, 5 ϕ_r and 2 Weyl ψ_a

• analogy with multiple D-branes connected by open strings:
N^3 (2,0) multiplets to match N^3 scaling in 11d sugra

• N^3 scaling of observables of multiple M5-branes
explained (?) in terms of M2-branes ending on 3 M5-branes:
triple M5-brane connections by "pants-like" membrane surfaces
provide dominant contribution [Klebanov, AT 96]

suggests $B^{ijk}_{\mu\nu}$ of $(2,0)$ tensor multiplets
in 3-tensor rep of $SU(N)$ or $SO(N)$ [Bastianelli, Frolov, AT 99]

• interacting (2,0) tensor multiplets as low-energy limit of tensionless 6d string – closed strings carrying 3-plet indices from virtual membranes connecting 3 parallel M5-branes [cf. $H^{ijk}_{\mu\nu\lambda} = dB^{ijk}_{\mu\nu} + ...$ and $F^{ij}_{\mu\nu}$ in open string (adjoint) case]

• earlier discussions:

"tensionless 6d strings" [Witten; Strominger 95]

in fact, strongly coupled $(2,0)$ or $(1,0)$ CFTs [Seiberg 96]

implicit constructions as decoupling limits of string theory

• non-Lagrangian? no perturbative description of RG flow leading to 6d CFT?

related to interacting $L = (H^{ijk}_{\mu\nu\lambda})^2 + ...$

only at quantum level – interacting fixed point?

• A-gauge theory in 6d: conf inv requires $\phi F_{\mu\nu} F_{\mu\nu} + (\partial\phi)^2$ or non-unitary $F_{\mu\nu}\partial^2 F_{\mu\nu}$ for renormalizability

• attempts to construct classical theories of 6d B-fields: consider tensor hierarchy of 1-, 2-, 3-form fields
e.g. non-abelian (1,0) t.m. [Samtleben, Sezgin, Wimmer 11]

$$L = \phi(B_{\mu\nu} + F_{\mu\nu})^2 + (C_{\lambda\mu\nu} + \partial_{[\lambda} B_{\mu\nu]})^2 + \partial\phi\partial\phi + ...$$

• may be natural to start with coupled system of gauge fields A and B

• self-duality of B: unusual properties – lack of manifest Lorentz symmetry and/or locality?

• first step: study bosonic system of B in some rep of G
coupled "minimally" to gauge vector A

• particular model [Ho, Huang, Matsuo 11]
consider both non-chiral and chiral (=selfdual H) versions

• consistent gauge-invariant coupling is possible
provided one keeps only 5d part of 6d Lorentz symmetry

• action is quadratic in B and local in particular gauge
with A-field restricted to 5d subspace of 6d space
[alternative: A is expressed in terms of B
leading to a non-local interacting theory of B-fields only]

• aim to study this (B, A) model at the quantum level
in one-loop approximation where B is integrated out
and A is treated as a background

• $(DF)^2 + F^3$ logarithmic UV divergences in eff action Γ
breaking classical scale invariance

• similar divergent terms appear in Γ also for
free scalar, spinor or YM coupled to 6d vector:
attempt to cancel these divergences
adding other fields (e.g., imposing supersymmetry)?
not clear how to do this but may be need to relax unitarity

• self-dual B-field model: a priori expect also
anomalous (gauge-symmetry breaking) terms
in P-odd part of effective action
(cf. Weyl spinor or grav. anomaly for single self-dual B
[Alvarez-Gaume, Witten 83])

• does not happen in the present case: as A-field
is restricted to 5 dimensions Γ has no P-odd part
– no gauge anomaly as in 5d theory

Non-abelian B-field coupled to gauge vector

• tensor field gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu$$

residual $\delta\epsilon_\mu = \partial_\mu \eta$ important for degrees of freedom count
non-abelian generalization to admit analog of residual symm

• to construct such model relax condition
of 6d Lorentz covariance (and locality)

• fields: 2-form field $B_{\mu\nu}$ in some rep (e.g. adjoint) of G
and gauge vector A_μ
notation: $D_\mu ... = \partial_\mu ... + [A_\mu, ...]$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$

• define non-abelian gauge transformations [Ho, Huang, Matsuo 11]

$$\delta A_\mu = D_\mu \lambda$$

$$\delta B_{\mu\nu} = D_\mu \epsilon_\nu - D_\nu \epsilon_\mu - [F_{\mu\nu}, (n^\rho \partial_\rho)^{-1} (n^\sigma \epsilon_\sigma)] + [B_{\mu\nu}, \lambda]$$

• λ: A_μ gauge transf., ϵ_μ: $B_{\mu\nu}$ gauge transf.
both taking values in algebra of G

• n_μ constant unit vector – breaking $O(6)$ symmetry to $O(5)$

• if assume $n^\mu A_\mu = 0$ – get non-abelian generalization of residual gauge symmetry under which $\delta B_{\mu\nu}$ is invariant:

$$\delta\epsilon_\mu = D_\mu \eta \ , \qquad\qquad \delta\lambda = 0$$

• if further impose $n^\mu \partial_\mu A_\nu = 0$, i.e. A_μ depends only on 5 of the 6 coordinates then gauge algebra closes

$$[\delta_1, \delta_2] = \delta_3 \ , \qquad \lambda_3 = [\lambda_1, \lambda_2] \ , \qquad \epsilon_{\mu 3} = [\lambda_1, \epsilon_{\mu 2}] - [\lambda_2, \epsilon_{\mu 1}]$$

• corresponding covariant field strength of $B_{\mu\nu}$

$$H_{\mu\nu\lambda} = D_\mu B_{\nu\lambda} + [F_{\mu\nu}, (n^\rho \partial_\rho)^{-1} (n^\sigma B_{\lambda\sigma})] + \text{cycle}$$

$$\delta_\epsilon H_{\mu\nu\sigma} = 0 \ , \qquad\qquad \delta_\lambda H_{\mu\nu\sigma} = [H_{\mu\nu\sigma}, \lambda]$$

• thus one can couple non-abelian antisymmetric tensor to non-abelian vector restricted to a codimension-1 subspace; effective "non-locality" along the "bulk" direction is gauge artifact: action is local in a gauge

• choose n_μ in the 6th direction: $n_\mu = (0,0,0,0,0,1)$

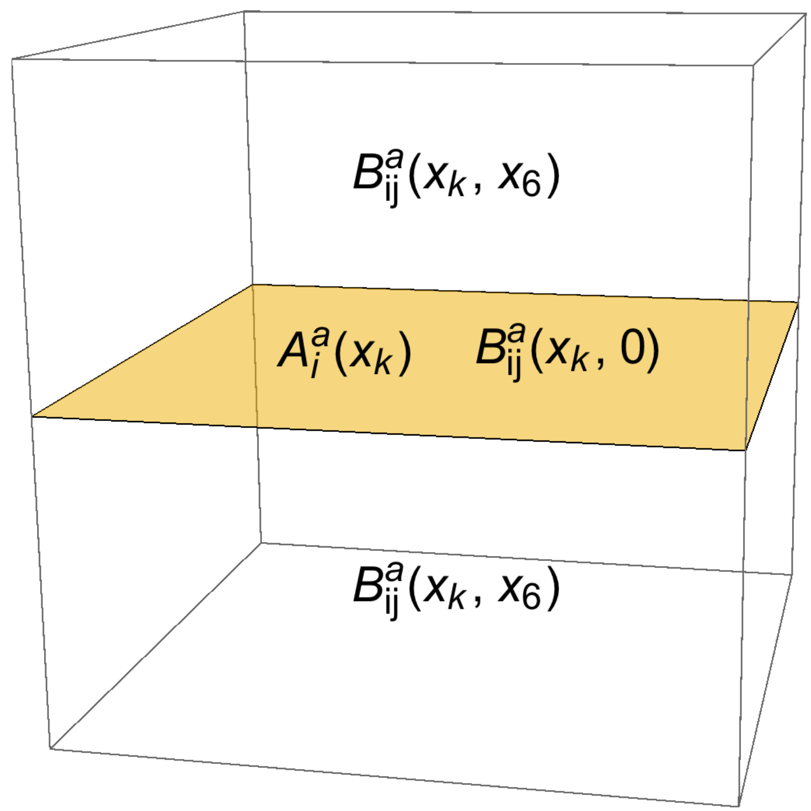
$$A_\mu = \{A_i(x^k,0),\ A_6 = 0\}\ , \qquad\qquad D_6 = \partial_6$$

$$F_{i6} = 0\ , \qquad\qquad F_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j]$$

• (B, A) model as intermediate step towards model (non-Lorentz-invariant, non-local) of self-interacting B-fields ?

- e.g. relate A to B by non-local condition:
  $A_i \equiv \int dx^6\, B_{i6} = 2\pi R B_{i6}(x^k, 0)$
- or treat A_i as independent and then integrate it out $\rightarrow$ effective non-local model of self-coupled B-fields

• A_i as background, quantum B with only $SO(5)$ of $SO(1,5)$


$B^a_{ij}(x_k, x_6)$
$A^a_i(x_k)$
$B^a_{ij}(x_k, 0)$
$B^a_{ij}(x_k, x_6)$

Classical action and gauge fixing

- 6d field $B_{\mu\nu}(x^\mu)$ (e.g. adjoint) coupled to 5d gauge field $A_i(x^j)$

$$S(B,A) = \frac{1}{6}\int d^6x \, \mathrm{Tr}\,(H_{\mu\nu\lambda}H^{\mu\nu\lambda})$$

- renormalizable model: add $S(A) = \int d^6x\,[c_1(DF)^2 + c_2F^3]$
- explicit form:

$$H_{ij6} = \partial_6 B_{ij} + D_i B_{j6} - D_j B_{i6}\,, \qquad H_{ijk} = D_i B_{jk} + [F_{ij}, \partial_6^{-1} B_{k6}] + \text{ cycle}$$

$$\delta B_{ij} = D_i\bar{\epsilon}_j - D_j\bar{\epsilon}_i + [B_{ij},\lambda]\,, \quad \delta B_{i6} = -\partial_6\bar{\epsilon}_i + [B_{i6},\lambda]\,, \quad \delta A_i = D_i\lambda$$

$$\lambda, \quad \bar{\epsilon}_i = \epsilon_i - D_i\partial_6^{-1}\epsilon_6 \quad \text{depend on } x^i$$

- fix $\bar{\epsilon}_i$ gauge freedom by "axial" gauge $B_{i6} = 0$

$$B_{i6} = 0: \qquad H_{ijk} = D_i B_{jk} + D_j B_{ki} + D_k B_{ij}\,, \qquad H_{ij6} = \partial_6 B_{ij}$$

$$S = \frac{1}{2}\int d^6x \,\mathrm{Tr}\left[(\partial_6 B_{ij})^2 + \frac{1}{3}H^{ijk}H_{ijk}\right] = \frac{1}{2}\int d^6x \,\mathrm{Tr}\left(B^{ij}\Delta_{ij}^{mn}B_{mn}\right)$$

$$\Delta_{ij}^{mn} = -\delta_{ij}^{mn}(\partial_6^2 + D^2) + 2\delta_{[i}^{[m}D^{n]}D_{j]}$$

$$D_i B_{jk} \equiv \partial_i B_{jk} + [A_i, B_{jk}], \quad D^2 \equiv D^i D_i \;, \quad \delta_{ij}^{mn} = \delta_{[i}^{m}\delta_{j]}^{n}$$

- classically scale invariant: overall dimensionless constant
- aim: compute logarithmic div part of eff action in $\Gamma[A_i]$

$$\Gamma = \frac{1}{2}\log\det\Delta_{ij}^{mn}(A)$$

- residual 5d local gauge transformations ():

$$B'_{ij} = UB_{ij}U^{-1}\;, \qquad A'_i = UA_iU^{-1} + U\partial_i U^{-1}\;, \qquad U(x^i) \in G$$

Γ: gauge-invariant combinations of F_{ij} and D_i

Free-theory partition function

- $L = H^2$: covariant Feynman-like gauge [Schwarz 79]

$$Z = \Big[\frac{(\det\Delta_1)^2}{\det\Delta_2\,(\det\Delta_0)^3}\Big]^{1/2}$$

$\Delta_n = -\partial^2$ on rank n antisymmetric tensors

- equivalent form in transv. (Landau-like) gauge

$$Z = \Big[\frac{\det\Delta_{1\perp}}{\det\Delta_{2\perp}\,\det\Delta_0}\Big]^{1/2},$$

$\Delta_{n\perp}$ on transv: $\det\Delta_1 = \Delta_{1\perp}\det\Delta_0$, $\det\Delta_2 = \det\Delta_{2\perp}\det\Delta_{1\perp}$

- d.o.f. of rank 2 tensor in d dim: $Z{=}\big[\det\Delta_0\big]^{-\nu/2}$

$$\nu_2(d) = C^2_{d-2} = \tfrac{1}{2}(d-2)(d-3)\,, \qquad \nu_2(6) = 6$$

• equivalent result in "axial" gauge $B_{i6} = 0$ $(i = 1, ..., 5)$

$H_{6ij} = \partial_6 B_{ij}, \quad H_{ijk} = 3\partial_{[i} B_{jk]}, \qquad\qquad B_{ij} = B^{\perp}_{ij} + \partial_i b_j - \partial_j b_i$

• int. over b_i: det cancels against the ghost and Jacobian factors

$$Z = \frac{1}{\left(\det \Delta_{\perp}\right)^{1/2}}$$

$\Delta_{\perp}$ is 6d Laplacian on $B^{\perp}_{ij}$:

same $\nu_2 = \frac{1}{2} \times 4 \times 5 - (5 - 1) = 6$ d.o.f.

• in self-dual case get "square root" of this Z

self-dual tensor in 6d: $\nu_{2,+} = 3$

Self-dual B-field model

analog of $S(B,A)$ action with self-dual $H = dB + ...$?

free-field case of single B:

$$H_{\mu\nu\lambda} = \tfrac{1}{6}\epsilon_{\mu\nu\lambda\sigma\rho\delta}H^{\sigma\rho\delta}$$

• way to find action: relax manifest Lorentz

start with phase-space path integral for $H^2_{\mu\nu\lambda}$ in gauge $B_{i0} = 0$

trade momenta for another 2-form field, impose self-duality

end up with "$\mathcal{E}\mathcal{B} - \mathcal{B}\mathcal{B}$" type action [Henneaux, Teitelboim]

• Euclidean notation: $x^0 \to ix^6$, gauge $B_{i6} = 0,\, i, j, ... = 1, ..., 5$

$$\tilde{S}_+ = \int d^6x\ \tfrac{1}{2}i\,\epsilon_{ijkpq}\partial_k B_{pq}\big(\partial_6 B_{ij} + \tfrac{1}{2}i\,\epsilon_{ijrmn}\partial_r B_{mn}\big)$$

• equation of motion

$$\partial_{[k}\mathcal{O}_+ B_{ij]} = 0\,, \qquad\qquad (\mathcal{O}_\pm)_{ij,mn} \equiv \delta_{ij,mn}\partial_6 \pm \tfrac{1}{2}i\,\epsilon_{ijrmn}\partial_r$$

• solved by

$$\mathcal{O}_+ B_{ij} = \partial_i q_j(x_i) - \partial_j q_i(x_i) + f_{ij}(x^6)$$

q_i part $\to$ redefinition of B_{ij}; impose b.c.:
self-duality $\mathcal{O}_+ B_{ij} = 0$ is satisfied at $|x^i| = \infty$
gives $f_{ij} = 0$ and thus $\mathcal{O}_+ B_{ij} = 0$ everywhere
• integrating over B_{ij} in path integral get partition function

$$Z_+ = \left(\det \mathcal{O}_+^\perp\right)^{-1/2}$$

$\mathcal{O}_+^\perp$ acts on transverse $B_{ij}^\perp$
$B_{ij}^\perp$: $\frac{1}{2} \times 4 \times 5 - (5-1)$=6
real but $\mathcal{O}_+$ is 1-st order – 3 d.o.f

• same results from alternative "$\mathcal{E}\mathcal{B} - \mathcal{E}\mathcal{E}$" action:

$$S_+ = \int d^6x\, \partial_6 B_{ij} \big(\partial_6 B_{ij} + \tfrac{1}{2} i\, \epsilon_{ijkmn} \partial_k B_{mn} \big)$$

• eom $\partial_6(\mathcal{O}_+ B_{ij}) = 0$ reduce to $\mathcal{O}_+ B_{ij} = f_{ij}(x^k)$;
if self-duality $\mathcal{O}_+ B_{ij} = 0$ at $|x^6| = \infty$, then everywhere

• B_{ij} path integral measure has extra $(\det \partial_6)^{1/2}$
ensures 6d Lorentz – get same Z_+

• free non-chiral model = self-dual + anti self-dual:

$$(\partial_6 B_{ij})^2 + \tfrac{1}{3} H^{ijk} H_{ijk} = \mathcal{O}_+ B_{ij}\, \mathcal{O}_- B_{ij}$$

corresponding partition function

$$Z = (\det \Delta_\perp)^{-1/2} = Z_+ Z_-$$

Non-abelian self-dual actions

• self-duality condition on non-abelian H in $B_{i6} = 0$ gauge

$$\hat{\mathcal{O}}_+ B_{ij} = 0\,, \qquad (\hat{\mathcal{O}}_\pm)_{ij,mn} \equiv \delta_{ij,mn}\partial_6 \pm \tfrac{1}{2} i\, \epsilon_{ijkmn} D_k(A)$$

• expect to follow (under b.c.) from analogs of free actions

$$S_+ = \int d^6x\, \mathrm{Tr}\Big[\,\partial_6 B_{ij}\big(\partial_6 B_{ij} + \tfrac{1}{2} i\, \epsilon_{ijkmn} D_k B_{mn}\big)\Big]$$

$$\tilde{S}_+ = \int d^6x\, \mathrm{Tr}\Big[\,\tfrac{1}{2} i\, \epsilon_{ijrpq} D_r B_{pq}\big(\partial_6 B_{ij} + \tfrac{1}{2} i\, \epsilon_{ijkmn} D_k B_{mn}\big)\Big]$$

• here will use the simplest action S_+

as definition of non-abelian self-dual B-field model

• partition function: direct analog of free one $\mathcal{O}_+ \to \hat{\mathcal{O}}_+(A)$

$$Z_+ = \big(\det \hat{\mathcal{O}}_+\big)^{-1/2}$$

∂_6 factorizes, ignore constant factors (no restriction to $B^\perp$)

• $\Delta(A)$ in non-chiral action factorizes

$$\Delta_{ij}^{mn}(A) = -\hat{\mathcal{O}}_{+\,pq}^{\;mn}(A)\,\hat{\mathcal{O}}_{-\,ij}^{\;pq}(A)$$

thus non-chiral B-model effective action is sum of chiral ones

$$\Gamma = \Gamma_+ + \Gamma_- \,, \qquad\qquad \Gamma_\pm = \tfrac{1}{2}\log\det\hat{\mathcal{O}}_\pm(A)$$

• Γ is P-even; $\Gamma_\pm$ a priori contain Im P-odd parts (anomaly)
but for 5d field A_i the eff actions $\Gamma_\pm$ are P-even and equal:
$\partial_6 \to -\partial_6$, $\epsilon_5 \to -\epsilon_5$ symmetry of classical and eff. action
P-odd part of $\Gamma_\pm$: odd number of ϵ_5 and p_6 factors: $\int dp_6(...)$=0
• no anom (5d gauge field background) part of $\Gamma_\pm$
both Γ of non-chiral theory and Γ_+ of self-dual theory
are invariant under residual gauge symmetry of A-field

$$\Gamma = 2\Gamma_+ \,, \qquad\qquad \Gamma_+ = \Gamma_- = \tfrac{1}{2}\log\det\hat{\mathcal{O}}_+(A)$$

Structure of divergent part of effective action

• div part of 1-loop 6d eff action for YM vectors, scalars, fermions in background gauge field A:

B_6 heat kernel coeff [Gilkey 75; Fradkin, AT 82]

$$\Gamma_\infty = -B_6 \log \Lambda \,, \qquad \Lambda \to \infty$$

$$B_6 = -\frac{1}{180(4\pi)^3} \int d^6x \Big[3\beta_2 \, \mathrm{tr}(D_m F_{mn} \, D_k F_{kn}) - 2\beta_3 \, \mathrm{tr}(F_{mn} F_{nk} F_{km}) \Big]$$

• only two independent invariants:

$\mathrm{tr}(D_m F_{kn} \, D_m F_{kn}) = 2 \, \mathrm{tr}(D_m F_{mn} \, D_k F_{kn}) - 4 \, \mathrm{tr}(F_{mn} F_{nk} F_{km}) + \mathrm{div}$,

$\mathrm{tr}(D_m F_{kn} \, D_k F_{mn}) = \frac{1}{2} \, \mathrm{tr}(D_m F_{kn} \, D_m F_{kn})$

• in adj rep (in general, $\mathrm{tr}(t^a t^b) = T_R \delta^{ab}$, $C_2 \to T_R$)

$$\mathrm{tr}(D_m F_{mn} D_k F_{kn}) = -C_2 D_m F^a_{mn} D_k F^a_{kn} \,, \qquad f_{acd} f_{bcd} = C_2 \delta_{ab}$$

$$\mathrm{tr}(F_{mn} F_{nk} F_{km}) = -\frac{1}{2} C_2 f^{abc} F^a_{mn} F^b_{nk} F^c_{km}$$

- N_1 YM vectors, N_0 real scalars, $N_{\frac{1}{2}}$ Weyl fermions

$$\beta_2 = -36N_1 + N_0 + 16N_{\frac{1}{2}}\ , \qquad\qquad \beta_3 = 4N_1 + N_0 - 4N_{\frac{1}{2}}$$

- $\beta_2 = \beta_3 = 0$ for $N_1 = 1,\ N_0 = 4,\ N_{\frac{1}{2}} = 2$

i.e. in maximally (1,1) susy 6d SYM (reduction of 10d SYM)
or in (1,0) SYM coupled to one adjoint 6d hypermultiplet

- expression for β_3 same as number of effective d.o.f.:

$\beta_3 = 0$ also in (1,0) 6d SYM $N_1 = 1, N_0 = 0, N_{\frac{1}{2}} = 1$
consistent with F^3 ruled out by (1,0) susy [Ivanov, Smilga, Zupnik 05]

- self-dual B: $\beta_2 = -27,\ \beta_3 = -57;$ non-chiral B: twice
- N_T self-dual tensors + vectors+scalars+fermions in adj rep

$$\beta_2 = -27N_T - 36N_1 + N_0 + 16N_{\frac{1}{2}}$$

$$\beta_3 = -57N_T + 4N_1 + N_0 - 4N_{\frac{1}{2}}$$

Calculation of one-loop divergences

• dimensional regularization: $6 = 1 + 5 \to 1 + d,\ d = 5 - 2\varepsilon$
6-th direction treated separately in action and gauge $B_{i6} = 0$:
mom algebra in 6d, scalar integrals in d dim: d.o.f. unchanged
• β_2, β_3 coefficients: from A^2 and A^3 terms in Γ_∞

Self-dual B-field model

$$\Gamma_+ = \tfrac{1}{2} \log \det \Delta_+ \,, \quad \Delta_+ B_{ij} = -\partial_6 \hat{\mathcal{O}}_+ B_{ij} = -\partial_6 (\partial_6 B_{ij} + \tfrac{i}{2} \epsilon_{ijkmn} D_k B_{mn})$$

$$\Delta_+ = \Delta^{(0)} + \Delta^{(1)} \,, \qquad [\Delta^{(0)}]^{ab}_{ij,mn} = -\delta^{ab} \big(\delta_{ij,mn} \partial_6^2 + \tfrac{i}{2} \epsilon_{ijkmn} \partial_6 \partial_k \big)$$

$$[\Delta^{(1)}]^{ab}_{ij,mn} = -\tfrac{i}{2} f^{acb} \epsilon_{ijkmn} A^c_k \partial_6$$

$$\Gamma_+ = \Gamma_2 + \Gamma_3 + \dots \,, \qquad \Gamma_2 = -\tfrac{1}{4} \,\mathrm{tr} \big[(\Delta^{(0)})^{-1} \Delta^{(1)} (\Delta^{(0)})^{-1} \Delta^{(1)} \big]$$

$$\Gamma_3 = \tfrac{1}{6} \,\mathrm{tr} \big[(\Delta^{(0)})^{-1} \Delta^{(1)} (\Delta^{(0)})^{-1} \Delta^{(1)} (\Delta^{(0)})^{-1} \Delta^{(1)} \big]$$

• A_i independent of x_6: $L_6 = \int dx_6$ factorizes
terms with odd number of ∂_6 vanish
and symmetry under $\partial_6 \to -\partial_6$, $\epsilon_5 \to -\epsilon_5$: $\quad \Gamma_+$ is P-even

• momentum space $A_i^a(x_k) = \int \frac{d^5 s}{(2\pi)^5} \tilde{A}_i^a(s) e^{i s_k x_k}$.

free B-field propagator $\langle p | (\Delta^{(0)})^{-1} | p \rangle \;\to\; \delta^{ab}\, P_{mn}^{jk}(p_i, p_6)$

$$P_{mn}^{jk}(p_i, p_6) \equiv \frac{1}{(p_i^2 + p_6^2)} \left(\delta_{mn}^{jk} - \frac{i}{2} \frac{\epsilon_{mnq}{}^{jk} p_q}{p_6} + 2 \frac{p^{[j} p_{[m} \delta_{n]}^{k]}}{p_6^2} \right)$$

interaction ABB vertex

$$\langle p + s | \Delta^{(1)} | p \rangle \;\to\; V_{ij}^{ab\; mn}(s_i, p_6) \equiv \frac{1}{2} f^{acb} \epsilon_{ij}{}^{kmn} p_6 \tilde{A}_k^c(s_i)$$

A^2 term

$$\Gamma_2 = L_6 \int \frac{d^5 s}{(2\pi)^5}\, \mathcal{G}_2(s)$$

$$\mathcal{G}_2 = \int \frac{dp_6 d^d p}{(2\pi)^{d+1}} V^{cd\ j_1 j_2}_{i_1 i_2}(s_i, p_6) P^{k_1 k_2}_{j_1 j_2}(p_i, p_6) V^{dc\ l_1 l_2}_{k_1 k_2}(-s_i, p_6) P^{i_1 i_2}_{l_1 l_2}(p_i + s_i, p_6)$$

$$\Gamma_2 = \tfrac{1}{4} C_2\, L_6 \int \frac{d^5 s}{(2\pi)^5}\, \tilde{A}^a_i(s) \left(\delta_{ij} s^2 - s_i s_j\right) \Pi(s^2)\, \tilde{A}^a_j(-s)$$

$$\Pi(s^2) = \int_0^1 dy \int \frac{dp_6 d^d p}{(2\pi)^{d+1}}\, \frac{(1-y)[(1-12y)\, p_6^2 - 2y\, s^2]}{2 p_6^2 [p_i^2 + p_6^2 + y(1-y)\, s^2]^2}$$

standard integrals give log divergent part as

$$\Gamma_{2\,\infty} = \frac{1}{d-5}\, \frac{9\, C_2}{5 \times 2^8 \pi^3} L_6 \int \frac{d^5 s}{(2\pi)^5}\, \tilde{A}^a_i(s)\, s^2 \left(s^i s^j - \delta^{ij} s^2\right) \tilde{A}^a_j(-s)$$

compare to $\Gamma_\infty = -B_6 \log \Lambda$, $\frac{1}{d-5} = -\log \Lambda$

$$B_6 = \frac{1}{(4\pi)^3} \int d^6x \Big[-\frac{1}{60}\beta_2 \, \mathrm{tr}(D_m F_{mn} \, D_k F_{kn}) + \frac{1}{90}\beta_3 \, \mathrm{tr}(F_{mn} F_{nk} F_{km}) \Big]$$

$$\beta_2 = -27$$

A^3 term

$$\Gamma_3 = L_6 \int \frac{d^5 s_1}{(2\pi)^5} \frac{d^5 s_2}{(2\pi)^5} \frac{d^5 s_3}{(2\pi)^5} \, \mathcal{G}_3(s_1, s_2, s_3) \, \delta^{(5)}(s_1 + s_2 + s_3)$$

$$\mathcal{G}_3 = \frac{1}{6} \int \frac{dp_6 d^d p}{(2\pi)^{d+1}} \; \mathrm{tr} \Big[V^{i_1 i_2}_{j_5 j_6}(s_{1i}, p_6) \, P^{j_1 j_2}_{i_1 i_2}(p_i, p_6) \, V^{i_3 i_4}_{j_1 j_2}(s_{2i}, p_6)$$
$$\times \; P^{j_3 j_4}_{i_3 i_4}(p_i + s_{2i}, p_6) V^{i_5 i_6}_{j_3 j_4}(s_{3i}, p_6) \, P^{j_5 j_6}_{i_5 i_6}(p_i + s_{2i} + s_{3i}, p_6) \Big]$$

$$\mathrm{tr}(t^a \, t^b \, t^c) = \tfrac{1}{2} T_R \, f^{abc} + \tfrac{1}{2} \mathrm{A}_R \, d^{abc} \,, \qquad T_{\mathrm{adj}} = C_2, \; \mathrm{A}_{\mathrm{adj}} = 0$$

- P-odd part $\sim \epsilon_5 \, d^{abc}$ vanished identically

- Feynman parametrization and momentum integration:

$$\mathcal{G}_{3\,\infty} = \frac{1}{d-5}\,\frac{i}{15\times 2^8\pi^3}\,C_2\,f^{a_1a_2a_3}\;K^{a_1a_2a_3}(s_1,s_2,s_3)$$

in transverse background gauge $s_i\tilde{A}^a_i(s)=0$:

$$\begin{aligned}
K^{a_1a_2a_3} &= -19\,s_3\cdot\tilde{A}^{a_1}(s_1)\;s_1\cdot\tilde{A}^{a_3}(s_3)\;(s_1-s_3)\cdot\tilde{A}^{a_2}(s_2)\\
&+\left[18(s_1^2+s_2^2)-s_3^2\right]\;\tilde{A}^{a_1}(s_1)\cdot\tilde{A}^{a_2}(s_2)\;s_1\cdot\tilde{A}^{a_3}(s_3)\\
&+\left[18(s_2^2+s_3^2)-s_1^2\right]\;\tilde{A}^{a_2}(s_2)\cdot\tilde{A}^{a_3}(s_3)\;s_2\cdot\tilde{A}^{a_1}(s_3)\\
&+\left[18(s_3^2+s_1^2)-s_2^2\right]\;\tilde{A}^{a_3}(s_3)\cdot\tilde{A}^{a_1}(s_1)\;s_3\cdot\tilde{A}^{a_2}(s_2)\;.
\end{aligned}$$

- comparing to $DFDF+FFF$ terms in Γ_∞ and using β_2

$$\beta_3 = -57$$

- same found taking $A=$const and computing $\mathrm{tr}([A,A])^3$ in Γ

Non-chiral B-field model

$$\Gamma = \tfrac{1}{2}\ln\det\Delta\,, \qquad \Delta B_{ij} = -(\partial_6^2 + D^2)B_{ij} + 2\delta_{[i}^{[m} D^{n]} D_{j]} B_{mn}$$

$$\Delta = \Delta^{(0)} + \Delta^{(1)} + \Delta^{(2)}$$

$$[\Delta^{(0)}] = \delta^{ac}\big[-\delta_{ij,mn}(\partial_i^2 + \partial_6^2) + 2\delta_{[m[i}\partial_{j]}\partial_{n]}\big]$$

$$[\Delta^{(1)}] = f^{abc}\big[-\delta_{ij,mn}(\partial_k A_k^b + 2A_k^b\partial_k) + 2\delta_{[i[m}(A_{n]}^b\partial_{j]} + \partial_{n]}A_{j]}^b + A_{j]}^b\partial_{n]})\big]$$

$$[\Delta^{(2)}] = f^{ade}f^{ebc}\big[-\delta_{ij,mn}A_k^d A_k^b + 2\delta_{[i[m}A_{n]}^d A_{j]}^b\big]$$

$$\Gamma_2 = \tfrac{1}{2}\,\mathrm{tr}\,\big[(\Delta^{(0)})^{-1}\Delta^{(2)}\big] - \tfrac{1}{4}\,\mathrm{tr}\,\big[(\Delta^{(0)})^{-1}\Delta^{(1)}(\Delta^{(0)})^{-1}\Delta^{(1)}\big],$$

$$\Gamma_3 = \mathrm{tr}\,\big[-\tfrac{1}{2}(\Delta^{(0)})^{-1}\Delta^{(2)}(\Delta^{(0)})^{-1}\Delta^{(1)} + \tfrac{1}{6}(\Delta^{(0)})^{-1}\Delta^{(1)}(\Delta^{(0)})^{-1}\Delta^{(1)}(\Delta^{(0)})$$

$$P_{mn}^{ij}(p_k, p_6) = \frac{1}{(p_i^2+p_6^2)}\Big(\delta_{mn}^{ij} + 2\frac{p^{[i}p_{[m}\delta_{n]}^{j]}}{p_6^2}\Big)$$

$$V^{(1)ab\ mn}_{ij}(p_k, s_k) = -if^{acb}\big[\delta^{mn}_{ij}\tilde{A}^c_k(s_k + 2p_k) + 2\delta^{[m}_{[j}(\tilde{A}^c_{i]}s^{n]} + \tilde{A}^{n]c}p_{i]} + \tilde{A}^c_{i]}p$$

$$V^{(2)ab\ mn}_{ij}(p_k, s_{1k}, s_{2k}) = f^{ade}f^{bce}\Big(\delta^{mn}_{ij}\tilde{A}^d_k\tilde{A}^c_k + 2\delta^{[m}_{[j}\tilde{A}^{n]d}\tilde{A}^c_{i]}\Big).$$

A^2 term

$$\Gamma_2 = L_6 \int \frac{d^5 s}{(2\pi)^5}\,\mathcal{G}_2(s)$$

$$\begin{aligned}\mathcal{G}_2 = -\tfrac{1}{4}\int \frac{dp_6 d^d p}{(2\pi)^{d+1}} & V^{(1)cd\ j_1 j_2}_{i_1 i_2}(s_i, p_6)\,P^{k_1 k_2}_{j_1 j_2}(p_i, p_6) \\ & \times V^{(1)dc\ l_1 l_2}_{k_1 k_2}(p_i + s_i, -s_i)\,P^{i_1 i_2}_{l_1 l_2}(p_i + s_i, p_6)\end{aligned}$$

$$\mathcal{G}_2 = -\tfrac{3}{2}C_2\int_0^1 dy\int \frac{dp_6 d^d p}{(2\pi)^{d+1}}\,Q(s_i, p_k, p_6, y)$$

$$\begin{aligned}Q = \Big(&\big[\tfrac{1}{2} - y(1-y)\big]s^2 + y^2(1-y)^2\frac{s^4}{p_6^2} + \tfrac{8}{5}p^2 \\ &+ \big[5 - 26y(1-y)\big]\frac{s^2p^2}{10p_6^2} + \frac{12p^4}{5p_6^2}\Big)\tilde{A}^a(s)\cdot\tilde{A}^a(-s) \\ -\Big[&\tfrac{1}{2} - y^2(1-y)^2\frac{s^2}{p_6^2} - \big[1 - 18y(1-y)\big]\frac{p^2}{10p_6^2}\Big]\,s\cdot\tilde{A}^a(s)\,s\cdot\tilde{A}^a(-s)\end{aligned}$$

$$\Gamma_{2\,\infty} = \frac{1}{d-5}\,\frac{9\,C_2}{5\times 2^8\pi^3}L_6\int \tfrac{d^5 s}{(2\pi)^5}\;\tilde{A}^a_i(s)\,s^2\big(s^i s^j - \delta^{ij}s^2\big)\,\tilde{A}^a_j(-s)$$

$$\beta_2 = -54 = 2\beta_2^{\text{self-dual}}$$

A^3 term

$$\Gamma_3 = L_6\int \frac{d^5s_1}{(2\pi)^5}\frac{d^5s_2}{(2\pi)^5}\frac{d^5s_3}{(2\pi)^5}\;\mathcal{G}_3(s_1,s_2,s_3)\,\delta^{(5)}(s_1+s_2+s_3)$$

$$\begin{aligned}\mathcal{G}_3 = \int \tfrac{dp_6 d^dp}{(2\pi)^{d+1}}\,\Big[&-\tfrac{1}{2}V^{(2)}{}^{cd\;\;j_1j_2}_{i_1i_2}(p_i,s_{1i},s_{2i})\,P^{k_1k_2}_{j_1j_2}(p_i,p_6)\\ &\qquad\times V^{(1)}{}^{dc\;\;l_1l_2}_{k_1k_2}(q_i,s_{3i})\,P^{i_1i_2}_{l_1l_2}(q_i,p_6)\Big|_{q=p-s_3}\\ &+\tfrac{1}{6}V^{(1)}{}^{de\;\;i_1i_2}_{j_5j_6}(p_i,s_{2,i})\,P^{j_1j_2}_{i_1i_2}(p_i,p_6)\,V^{(1)}{}^{ef\;\;i_3i_4}_{j_1j_2}(q_i,s_{1i})\\ &\times\;P^{j_3j_4}_{i_3i_4}(q_i,p_6)\,V^{(1)}{}^{fd\;\;i_5i_6}_{j_3j_4}(r_i,s_{3i})\,P^{j_5j_6}_{i_5i_6}(r_i,p_6)\Big|_{q=p-s_1,\,r=p-s_1-s_3}\Big]\end{aligned}$$

comparison of $\frac{1}{d-5}$ part to $(DF)^2 + FFF$ gives

$$\beta_3 = -114 = 2\beta_3^{\text{self-dual}}$$

Concluding remarks

• studied model of 6d 2-form B in some rep of G
coupled consistently to gauge field A in 5d subspace

• 1-loop log div integrating out B-field with A as background

• $\Gamma_\infty \sim \log\Lambda \int \mathrm{tr}[3\beta_2(D_m F_{mn})^2 - 2\beta_3 F_{mn}F_{nk}F_{km}]$
$\beta_2 = -27,\ \beta_3 = -57$ in self-dual model and twice in non-chiral

• implies $c_1(DF)^2 + c_2 F^3$ should be added to bare 6d action
classical 6d scale inv, but broken at loop level unless div cancel

• cancel adding other fields – imposing supersymmetry?

• for N_T self-dual tensors, N_1 YM vectors, N_0 real scalars
and $N_{\frac{1}{2}}$ Weyl fermions in 6d coupled to gauge field A

$$\beta_2 = -27N_T - 36N_1 + N_0 + 16N_{\frac{1}{2}}, \quad \beta_3 = -57N_T + 4N_1 + N_0 - 4N_{\frac{1}{2}}$$

• unexpected feature: minimally coupled B-field contributes to
β_3 with opposite in sign to standard 2-derivative bosonic fields

• naive expectation could be that $\beta_3 \sim \nu$= no. of d.o.f.

$$\nu = 3N_T + 4N_1 + N_0 - 4N_{\frac{1}{2}}$$

simplest 6d supermultiplet containing self-dual B:
(1,0) tensor multiplet: $N_T = 1,\ N_1 = 0,\ N_0 = 1,\ N_{\frac{1}{2}} = 1$
natural coupling to (1,0) SYM ($N_1 = 1, N_0 = 0, N_{\frac{1}{2}} = 1$)
$\beta_3 = 0$ would be consistent with F^3 not having susy extension
[cf. (1,0) classically conformal (non-unitary) gauge theory:
$\int d^6x \,\mathrm{tr}\left[(DF)^2 + \psi D^3\psi + \phi D^2\phi + ...\right]$ [Ivanov, Smilga, Zupnik 06]
has $\beta_2 \neq 0$ (non-conf) and also gauge anomaly]

• (1,0) tensor multiplet: actually get

$$\beta_3 = 2\beta_2 = -60$$

• (2,0) tensor multiplet: $N_T = 1, N_1 = 0, N_0 = 5, N_{\frac{1}{2}} = 2$:

$$\beta_2 = -\tfrac{1}{6}\beta_3 = 10.$$

- $\beta_3 \neq 0$: non-abelian (B, A) model has no (1,0) susy version; may be not surprising due to lack of 6d Lorentz

- applications/extensions of this non-abelian (B, A) model?
  - may be related to some intersecting brane configuration with 5d gauge field living on a 5d brane "defect"
  - 5d A-field may play an auxiliary role: eliminating it get effective interacting theory of B-fields
  - generalization with 6d A-field and 6d Lorentz inv but non-local classical action?